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ON THE THEORY OF THE NEUTRAL VECTOR FIELD WITH SPIN 1

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The most general form of the interaction between a neutral vector field with spin 1 and a spinor field is derived. Such interactions turn out to be necessarily invariant with respect to certain phase transformations.

1. An interacting vector field has spin 1 in gauge invariant theories and in theories in which the Lorentz condition is satisfied (theories of class A) [1-3]. By definition, in theories of class A[3] the equations of motion imply that

$$\partial_\mu A_\mu = \begin{cases} 0, & \text{if } m^2 \neq 0, \quad (1) \\ \text{arbitrary}, & \text{if } m^2 = 0, \quad (2) \end{cases}$$

where m is the mass of the vector field A_μ. What kind of interactions are selected by this requirement? In the present article we investigate the most general Lagrangian of class A with dimensionless coupling constants (in units $\hbar = c = 1$), which describes the interaction of a vector field with a spinor field.

Interactions of the class A with dimensionless coupling constants will be called minimal interactions. As we shall see, in electrodynamics such a definition of minimality singles out uniquely the interaction $j_\mu A_\mu$. In this respect this definition is better than the usual recipe of replacing in the free Lagrangian ∂_μ by $\partial_\mu - ieA_\mu$, which, as has been remarked by Glashow and Gell-Mann[4] is ambiguous.

The condition (2) is an alternate method of singling out the spin-1 part. It is equivalent to the condition of gauge invariance, the role of which reduces to singling out the spin 1 (of the vector field) and which has no relation to the mass of the vector field[1,2]. The utilization of different conditions (1) and (2) for vanishing and nonvanishing masses, respectively, is dictated solely by considerations of practical convenience. On the basis of an analysis of dynamical models, Schwinger[5] has also arrived at the conclusion, (derived earlier[1]), that gauge invariance does not imply the vanishing of the mass of the vector field. Finally, quite recently the same statement has been repeated by Feldman and Matthews[6].

2. The most general Lagrangian with dimensionless coupling constants for one neutral vector field A_μ and one spinor field ψ is

$$\begin{aligned} \mathscr{L} = & -{}^1/_4 F_{\mu\nu}F_{\mu\nu} - {}^1/_2 m^2 A_\mu A_\mu + \alpha\partial_\nu A_\mu \cdot A_\nu A_\mu + \beta A_\mu A_\mu A_\nu A_\nu \\ & - \bar{\psi}(\gamma\partial + M)\psi + i\bar{\psi}\gamma_\mu(g_1 + \gamma_5 g_2)\psi A_\mu \\ & + {}^1/_2 if\bar{\psi}\gamma_\mu\gamma_5\psi_C A_\mu + {}^1/_2 if^*\bar{\psi}_C\gamma_\mu\gamma_5\psi A_\mu, \end{aligned} \tag{3}$$

where $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$, ψ_C is the charge-conjugate spinor: $\psi_C = C\bar{\psi}$. The free part of the Lagrangian for A_μ and ψ is written in the standard form so that in the free-field case the equation for A_μ should imply the conditions (1) or (2). In writing the Lagrangian (3) we require only relativistic invariance and do not impose a priori any restrictions connected with conservation of parity or the number of spinor particles. The term $\bar{\psi}\gamma_\mu\psi_C A_\mu$ (together with its Hermitean conjugate) is absent, since it vanishes identically, owing to the Fermi statistics of the spinor field and the properties of the charge conjugation matrix C.

The Lagrangian (3) implies the following equations of motion

$$\begin{aligned} & \Box A_\mu - \partial_\mu\partial_\nu A_\nu - m^2 A_\mu + \alpha\partial_\mu A_\nu \cdot A_\nu - \alpha A_\mu \partial_\nu A_\nu + 4\beta A_\nu A_\nu A_\mu \\ & \quad + i\bar{\psi}\gamma_\mu(g_1 + \gamma_5 g_2)\psi + {}^1/_2 if\bar{\psi}\gamma_\mu\gamma_5\psi_C + {}^1/_2 if^*\bar{\psi}_C\gamma_\mu\gamma_5\psi = 0, \end{aligned} \tag{4}$$

$$(\gamma\partial + M)\psi - i\gamma_\mu(g_1 + \gamma_5 g_2)\psi A_\mu - if\gamma_\mu\gamma_5\psi_C A_\mu = 0. \tag{5}$$

We take the divergence of Eq. (4) and replace in the expression so obtained $\Box A_\mu$ according to Eq. (4) and the derivatives of ψ according to Eq. (5). The result is

$$\begin{aligned} & -m^2\partial_\mu A_\mu + \alpha\partial_\mu A_\nu \cdot \partial_\mu A_\nu - \alpha\partial_\mu A_\mu \cdot \partial_\nu A_\nu - \alpha A_\mu \cdot \partial_\mu\partial_\nu A_\nu \\ & \quad + 8\beta\partial_\mu A_\nu \cdot A_\mu A_\mu + 4\beta A_\nu A_\nu \partial_\mu A_\mu + \alpha A_\mu \{m^2 A_\mu + \partial_\mu\partial_\nu A_\nu \\ & \quad - \alpha\partial_\mu A_\nu \cdot A_\nu + \alpha A_\mu \partial_\nu A_\nu - 4\beta A_\nu A_\nu A_\mu - i\bar{\psi}\gamma_\mu(g_1 + \gamma_5 g_2)\psi \\ & \quad - {}^1/_2 if\bar{\psi}\gamma_\mu\gamma_5\psi_C - {}^1/_2 if^*\bar{\psi}_C\gamma_\mu\gamma_5\psi\} \\ & \quad + iM\{2g_2\bar{\psi}\gamma_5\psi + f\bar{\psi}\gamma_5\psi_C + f^*\bar{\psi}_C\gamma_5\psi\} = 0. \end{aligned} \tag{6}$$

3. We consider first a massive field ($m \neq 0$). We require that Eq. (6) imply the Lorentz condition (1). We show that a necessary and sufficient condition for this is:

$$\alpha = \beta = 0, \quad Mg_2 = Mf = 0. \tag{7}$$

The sufficiency of conditions (7) is obvious. Let us prove their necessity. Let $\partial_\mu A = 0$. Then Eq. (6) can be rewritten in the form

$$\alpha\partial_\mu A_\nu\cdot\partial_\mu A_\nu + 8\beta\partial_\mu A_\nu\cdot A_\nu A_\mu + \alpha A_\mu [m^2 A_\mu - \alpha\partial_\mu A_\nu\cdot A_\nu - 4\beta A_\nu A_\nu A_\mu - i\bar\psi\gamma_\mu(g_1+\gamma_5 g_2)\psi - {}^1/_2 if\bar\psi\gamma_\mu\gamma_5\psi_c - {}^1/_2 if^*\bar\psi_c\gamma_\mu\gamma_5\psi] + iM[2g_2\bar\psi\gamma_5\psi + f\bar\psi\gamma_5\psi_c + f^*\bar\psi_c\gamma_5\psi] = 0, \tag{8}$$

in which we have retained only derivatives of the field A_μ not higher than the first order and no derivatives of the field ψ. If Eq. (8) were not identically satisfied, it would represent an additional supplementary condition (in addition to the condition (1) and to the condition into which Eq. (4) turns when $\mu = 4$). This would mean that the A_μ and ψ have a smaller number of degrees of freedom than is necessary for the description of spins 1 and $^1\!/_2$, which is inadmissible. Therefore, each term with independent structure must vanish individually in Eq. (8), i.e., the conditions (7) must be fulfilled. Thus there is but one term of the form $\alpha\partial_\mu A_\nu\cdot\partial_\mu A_\nu$. It must vanish; therefore it follows that $\alpha = 0$. Elimination of the terms with α leaves one term of the form $8\beta A_\nu A_\mu\partial_\mu A_\nu$ and consequently $\beta = 0$. By continuing this analysis we conclude the proof of the necessity of the conditions (7).

4. Not all the A_μ components are independent. Therefore the question can arise whether Eq. (8) excessively restricts the number of degrees of freedom, why it is necessary to equate the individual terms to zero, and why the vanishing of the individual terms implies that $\alpha = 0$ and $\beta = 0$.

As an answer to this question we show that in classical field theory it would be impossible, in the presence of Eq. (8) with α, β, g_2 and $f \neq 0$, to specify arbitrary initial values (Cauchy conditions) for the field components $\psi(\mathbf{x}, 0)$ and $A_m(\mathbf{x}, 0)$ ($m = 1,2,3$), and for the canonically conjugate momenta of the latter $\Pi_m(\mathbf{x}, 0)$:

$$\Pi_m = \partial_4 A_m - \partial_m A_4 + \alpha A_m A_4. \tag{9}$$

First we select the initial conditions in the form[1]:

$$\psi(\mathbf{x}, 0) = 0, \qquad A_m(\mathbf{x}, 0) = 0,$$
$$\Pi_m(\mathbf{x}, 0) = ia_m \sin \mathbf{kx}, \qquad (\mathbf{ak}) = 0, \qquad \mathbf{a}^2 = 1. \tag{10}$$

Then the supplementary condition (4) with $\mu = 4$ takes on the form

$$-m^2 A_4 + 4\beta A_4^3 = 0,$$

and taking this relation into account, the condition (8) can be written as $\alpha\Pi_m(\mathbf{x}, 0)\Pi_m(\mathbf{x}, 0) = 0$, which makes it clear that α must vanish.

The set of supplementary conditions [for $\psi(\mathbf{x}, 0) = 0$] then has the form:

$$-\partial_m\Pi_m - m^2 A_4 + 4\beta A_\mu A_\mu A_4 = 0, \tag{4'}$$
$$\beta A_\nu\partial_\nu(A_\mu A_\mu) = 0, \qquad \partial_\mu A_\mu = 0. \tag{8'}$$

Now we specify initial conditions such that

$$A_m(\mathbf{x}, 0) A_m(\mathbf{x}, 0) = \text{const} \neq 0, \; \partial_m\Pi_m(\mathbf{x}, 0) = \text{const} \neq 0,$$
$$\partial_m A_m(\mathbf{x}, 0) = 0, \tag{11}$$

for instance, in the form

$$\mathbf{A}(\mathbf{x}, 0) = \mathbf{a}\cos([\mathbf{ab}]\,\mathbf{x}) + \mathbf{b}\sin([\mathbf{ab}]\,\mathbf{x}),$$
$$(\mathbf{ab}) = 0, \quad \mathbf{a}^2 = \mathbf{b}^2 = 1, \quad \Pi(\mathbf{x}, 0) = \mathbf{x}. \tag{12*}$$

By virtue of (11), we can conclude from (4′) that $A_4(\mathbf{x}, 0) = \text{const} \neq 0$, and that therefore $A_\mu(\mathbf{x}, 0)\times A_\mu(\mathbf{x}, 0) = \text{const}$. Then Eq. (8′) has the form

$$2\beta\Pi_m(\mathbf{x}, 0) A_m(\mathbf{x}, 0) = 0,$$

i.e., β must also vanish. By specifying appropriate nonvanishing initial conditions for ψ we can verify the necessity of the rest of the relations (7).

Similarly, in quantum field theory the condition that the equations of motion, the Lorentz condition (1), and the equal-time commutation relations (which replace the initial conditions) be consistent would also lead to the relations (7).

5. So far the analysis pertained to a massive vector field ($m \neq 0$). For $m = 0$ we must replace the Lorentz condition by the requirement that $\partial_\mu A_\mu$ be completely arbitrary, i.e., condition (2). From Eq. (6) it is completely obvious that the conditions (7) must again be satisfied.

6. Thus it has been proved that in theories of the class A the neutral vector field a) is coupled to a conserved current, b) does not have self-interaction, and, depending on the mass values, the following possibilities are realized.

1) The masses of the spinor field and of the vector field do not vanish: $M \neq 0$ and $m \neq 0$. The Lagrangian has the form

$$\mathcal{L} = -{}^1/_4 F_{\mu\nu}F_{\mu\nu} - {}^1/_2 m^2 A_\mu A_\mu - \bar\psi(\gamma\partial + M)\psi + ig_1\bar\psi\gamma_\mu\psi A_\mu. \tag{13}$$

[1]For us it is essential only that $\partial_m\Pi_m(\mathbf{x}, 0) = 0$.

*$[\mathbf{ab}] = \mathbf{a}\times\mathbf{b}$.

It is remarkable that for the Lorentz condition (1) to be satisfied in this case it was necessary besides a) and b) that: c) the number of spinor particles be conserved and that the theory be invariant with respect to the transformation $\psi \rightarrow e^{i\alpha}\psi$, and that: d) parity be conserved.

2) $M \neq 0$, $m = 0$. The Lagrangian has again the form (13), but with $m = 0$. Such a theory is Maxwellian electrodynamics. The 4-divergence $\partial_\mu A_\mu$ is in this case completely arbitrary, and accordingly gauge invariance appears. Conclusions a)—d) are valid, as before[2)].

3) $M = 0$, $m \neq 0$. Terms with g_2 and f are added to the Lagrangian:

$$\mathcal{L} = -{}^1/_4 F_{\mu\nu}F_{\mu\nu} - {}^1/_2 m^2 A_\mu A_\mu - \bar{\psi}\gamma\partial\psi + j_\mu A_\mu,$$

$$j_\mu = i\bar{\psi}\gamma_\mu (g_1 + \gamma_5 g_2)\psi + {}^1/_2 if\bar{\psi}\gamma_\mu\gamma_5\psi_C + {}^1/_2 if^*\bar{\psi}_C\gamma_\mu\gamma_5\psi. \quad (14)$$

The conclusions c) and d) now lose their validity: neither parity nor the number of spinor particles need be conserved.

The conservation of the current j_μ corresponds to a curious generalization of phase-invariance, namely invariance with respect to a one-parameter transformation group (a subgroup of the Pauli-Gürsey group)

$$\psi' = e^{i\omega g_2\gamma_5}\{[\cos(\omega a) + i\sin\varphi\sin(\omega a)] \times \psi + if|f|^{-1}\gamma_5\cos\varphi\sin(\omega a)\psi_C\}, \quad (15)$$

where ω is the transformation parameter and a and φ are functions of the coupling constant:

$$a = \sqrt{g_1^2 + |f|^2}; \quad \sin\varphi = g_1/a; \quad \cos\varphi = |f|/a.$$

4) $M = 0$, $m = 0$. Everything said with respect to case 3) applies here, too. This case is interesting because it opens up a certain specific possibility for the interaction of massless particles with an electromagnetic field.

2)Solov'ev[7] has indicated that parity conservation is a consequence of gauge invariance under the assumption of renormalizability, i.e., of the fact that the coupling constants are actually dimensionless and there is no interaction of the type of an anomalous magnetic moment.

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