PHASE-SHIFT ANALYSIS OF ELASTIC pp SCATTERING AT 660 MeV WITH ACCOUNT OF RELATIVISTIC EFFECTS

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A modified phase shift analysis of elastic pp scattering is performed by taking the relativistic spin rotation effect into account. The unique solution $\chi^2 = 41.5$ in the interval $\overline{\chi^2} \leq \chi^2 \leq 2\overline{\chi^2} = 56$ is obtained under the assumption that meson production occurs at the indicated energy from $^3P_0$, $^3P_1$, $^3P_2$ and $^1D_2$ states of the pp system.

INTRODUCTION

EXTENSIVE experimental information has been accumulated by now on elastic NN scattering in a wide range of energies below the pion production threshold. This has made it possible to make a first serious attempt to analyze this material, namely a phase shift analysis (psa) of elastic NN interaction in the indicated energy interval. The latter has led to unique solutions in practically the entire range of the investigated energies. It also showed what must be assumed in order to eliminate the ambiguities that still arise in the analysis in some cases. The next step in this subthreshold energy region could be the construction of phenomenological NN potentials using the obtained energy dependences of the phase shifts of the elastic NN scattering. The construction of these potentials would complete in a certain sense the phenomenological picture of the NN interaction.

Extensive experimental material on the interaction between nucleons and nucleons has also been accumulated at energies above the pion production threshold. In this energy region, the process investigated in greatest detail is pp scattering at an energy near 660 MeV, for which a set of experiments, proposed by L. Puzikov, R. Ryndin, and Ya. Smorodinskiĭ[1] has been carried out at the Joint Institute for Nuclear Research, and for which some characteristics of inelastic processes occurring in pp collisions have been studied. This has brought about a situation wherein a sufficiently complete and rigorous phenomenological description, say, of pp scattering cannot be made as yet for this energy, owing to the incompleteness of the experimental data[1)]. On the other hand, it is not yet clear which experiments at what angles, and with what accuracy, must be performed first in order to ensure such a phenomenological description, and in particular to enable one to describe the scattering process by means of a set of phase shifts of partial waves participating in the scattering. From the experience with phase shift analysis below the threshold of meson production it is seen that for a possible elimination of the ambiguity in the solutions the experiments must be set up at such angles at which the measured quantities are most sensitive to the different variants of the solutions. At the same time, these regions of angles and these experiments can be predicted reliably beforehand, prior to the performance of the phase shift analysis.

Thus, at a definite stage of accumulation of experimental information, it becomes necessary to undertake a more or less rigorously founded planning of the experiment.

Such planning can follow two lines. The first is connected with the results of [1,3,4] and consists of a further consecutive performance of the "complete experiment" program; the second consists of attempting a phase shift analysis based on the already available experimental material, under assumptions which could simplify the formulation of the phase shift analysis problem without distorting it appreciably. The well-known experimental dif-

[1)]The available experimental material for elastic pp scattering at this energy was used in [2] to determine only the moduli of the coefficients of the scattering matrix M in the interval $54° \leq \vartheta \leq 126°$ and to find the real parts of the phase shifts of the waves of states $^1S_0$ and $^1D_2$.

ficulties do not promise any noticeable success in the near future in the realization of the first line of approach. For a further study of nucleon-nucleon interactions a more acceptable and effective method is the second approach, in favor of which a few remarks are appropriate.

The "complete set of experiments," at least in the energy range below the threshold of meson production, carries not only information which is sufficient from the point of view of the possible determination of the complex coefficients of the matrix M, but, under certain conditions, also excess information, from the point of view of the possible determination of the scattering phase shifts. This question has already been discussed in detail (see, for example, [5]). In addition, in the performance of the modified phase shift analysis, not all the parameters must be determined by experiment. Thus, for elastic pp scattering we can assume that $5(L_{max} - l_{max})/2$ parameters are known[6] (L_{max} and l_{max} are assumed odd here). This circumstance decisively facilitates the analysis. The possibility of finding a value of l_{max} above which the scattering can be described by the pole term of the one-meson Feynman diagram now makes the modified analysis more meaningful.

In the energy region where inelastic processes, for example, meson production, play a noticeable role along with the elastic ones[2)], the number of parameters necessary for the description of the elastic scattering by the partial-wave method increases because of the appearance of additional reaction channels. In particular, analysis of the experimental data on meson production in pp collisions near 660 MeV[7] indicates that a possible first approximation, which does not contradict the available experimental data in the phase shift analysis of elastic pp scattering, is the assumption that meson production in this energy region proceeds essentially from the initial $^1D_2$ and $^3P_{0,1,2}$ states of two protons. If meson production is confined to these states, then the available experimental information is sufficient for a modified analysis of elastic pp scattering at 660 MeV.

In our earlier communication[8] we mentioned that five solutions were obtained in the interval $\overline{\chi^2} \leq \chi^2 \leq 3\overline{\chi^2}$. In the present paper we refine these solutions with account of several relativistic corrections. In addition, the results of a search for solutions with $l_{max} = 5$ are reported.

2)We disregard inelastic processes of electromagnetic character.

CHOICE OF FORMULAS AND EXPERIMENTAL DATA

1. The analysis was performed with account of the relativistic spin rotation effects only for the parameters I and R using the Stapp-Sprung formulas[9,10].

2. The connection between the total cross section σ_{tot} and the imaginary part of the spinless scattering amplitude $M_{ss}(0) + M_{00}(0) + 2M_{11}(0)$ for 0° was determined from a formula derived from the optical theorem for the elastic scattering process:

$$\sigma_{tot} = \frac{\pi}{k} \operatorname{Im} [M_{ss}(0) + M_{00}(0) + 2M_{11}(0)],$$

where k is the wave vector of the proton in the center of mass system.

3. In the region of the scattering-particle energies, where the elastic scattering is the only possible reaction channel, the symmetry and unitarity conditions for the S matrix lead as is well known, for example, to the Stapp parametrization [11] for the matrices α (here and throughout δ stands for $\bar{\delta}$ in [11]):

$$\alpha_l = \exp(2i\delta_l) - \exp(2i\Phi_l) \text{ for singlet}$$
$$\alpha_{l,j} = \exp(2i\delta_{l,j}) - \exp(2i\Phi_l) \text{for } l = j,$$

where Φ_l — Coulomb phase shift, and to the formulas

$$\alpha_{j\neq 1} = \cos 2\varepsilon_j \exp[2i\delta_{j\pm 1,j}] - \exp[2i\Phi_{j\pm 1}],$$
$$\alpha_j = i \sin 2\varepsilon_j \exp[i(\delta_{j+1,j} + \delta_{j-1,j})]$$

for triplet states with $l = j \pm 1$.

We use here the notation of Stapp et al for the matrix elements and the phase shifts, and also take into account the Coulomb scattering phase shifts.

In the case of interest to us, when intense inelastic reaction channels open up along with the elastic channels, the elastic scattering matrix S_{el} will not be unitary (for the given total angular momentum and parity; only the entire S matrix is unitary). As a result, the moduli of the matrix elements of the states from which meson production proceeds in pp interaction will differ from unity. The symmetry conditions lead in this case to the following possible parametrization:

$$\delta_l \equiv \delta_l^R + \delta_l^I \text{ for singlet}$$
$$\delta_{l,j} \equiv \delta_{l,j}^R + i\delta_{l,j}^I \text{ for triplet with } l = j.$$

There now exists only one relation for the four complex matrix elements of the matrix S_{el}, which describes the transitions between states with $j = l \pm 1$. This makes it necessary to employ six real parameters. Without contradicting the condition for the unitarity of the S matrix, we can conveniently put

$$\delta_{j\pm1,j} \equiv \delta^R_{j\pm1,j} + i\delta^I_{j\pm1,j}, \qquad \varepsilon_j \equiv \varepsilon^R_j + i\varepsilon^I_j.$$

4. We used in the phase shift analysis the following experimental quantities characterizing elastic pp scattering in the energy region under consideration:

a) the differential cross section $\sigma(\vartheta)$ for 657 MeV [12];

b) the polarization angular distribution $P(\vartheta)$ of the protons with initial energy 635 ± 15 MeV [13];

c) the angular dependences of the Wolfenstein parameters $D(\vartheta)$ and $R(\vartheta)$ for triple scattering, obtained for 635 ± 15 MeV [14-15];

d) the angular dependence of the correlation coefficient $C_{nn}(\vartheta)$ for 640 MeV [1,16] and the value of the parameter C_{kp} for 90° in the c.m.s., obtained for an incident-proton energy of 660 MeV [17];

e) The total scattering cross section σ_{tot} was assumed to be 40.6 ± 0.6 mb. This value of σ_{tot} was obtained by averaging the measurement results for 635 and 660 MeV [18];

f) the imaginary part of the phase shift in the state $^1D_2$ was determined from the results of Soroko [19] and assumed equal to 18.24°.

CONTRIBUTION OF THE POLE TERMS TO THE SCATTERING AMPLITUDE

The maximum value of the orbital angular momentum, l_{max}, was obtained following Chamberlain et al [5] by expanding $\sigma(\vartheta)P(\vartheta)$ in Legendre polynomials. This procedure, which justified itself at energies below meson-production threshold, led to $l_{max} \geq 3$.

In the present work we calculated the contribution of the pole terms to the scattering amplitude with an interaction constant $f^2 = 0.080$ 3). Although the value of f^2 obtained from experiments on np scattering at 630 MeV by extrapolation to the unphysical region is not in good agreement with the value of f^2 determined from πN-scattering experiments, the difference between the two quantities is evidently connected with the still large experimental errors.

SEARCH FOR SOLUTIONS AND DISCUSSION OF RESULTS AT $l_{max} = 4$

The phase shifts were determined by the method of least squares. The minima of the functional χ^2

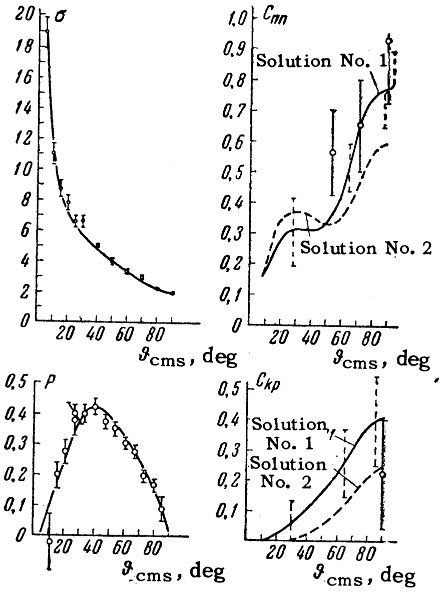

FIG. 1

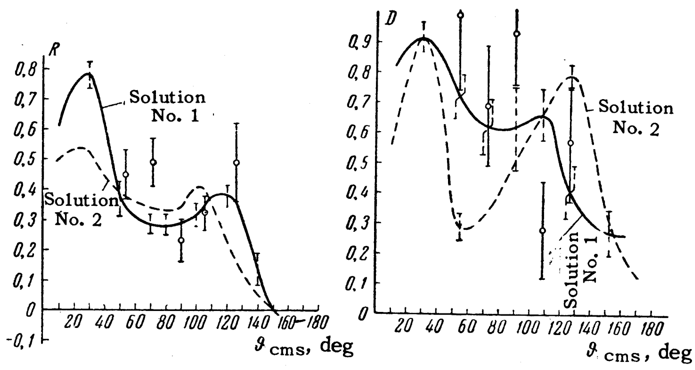

FIG. 2

were sought for by the linearization method using the electronic computer of the Joint Institute for Nuclear Research [21].

A total of 130 searches for solutions with random initial conditions ($l_{max} = 4$; $\overline{\chi^2} = 28$) yielded one solution ($\chi^2 = 41.5$; solution No. 1) in the interval $\overline{\chi^2} \leq \chi^2 \leq 2\overline{\chi^2}$ and another solution, which was 50 times less probable ($\chi^2 = 68.5$; solution No. 2) in the interval $2\overline{\chi^2} \leq \chi^2 \leq 3\overline{\chi^2}$. The obtained sets of phase shifts are listed in Table I. Figures 1—3 illustrate the angular dependences of the experimentally measured quantities, calculated for both solutions (the points are the experimental data and the dashed segments show the predicted error corridor).

In order to ascertain the stability of the solutions, the following pairs of parameters were additionally varied; $\delta^I(^1S_0)$ and $\delta^I(^1D_2)$; $\delta^I(^3F_2)$ and ϵ^I_2; $\delta^I(^3F_2)$ and $\delta^I(^3F_3)$ with $\epsilon^R_2 = 0$. As a result, with a practically constant value of the agreement criterion $\chi^2/\overline{\chi^2}$, the assumed conditions were well confirmed, and the phase shift

3)Owing to an error that has crept into the computation program (concerning which see [20]) our previous analysis[8] was actually carried out with $f^2 = 0.053$. However, owing to the large value of l_{max} used in the searches of the solutions, this inaccuracy did not influence noticeably the previously obtained [8] values of the phase shifts.

Table I. Phase Shifts, Degrees

	Solution No. 1	Solution No. 2		Solution No. 1	Solution No. 2
$\delta^R(^1S_0)$	−20.46+8.93	11.00+4.22	$\delta^R(^3F_4)$	−3.43+1.10	4.51+1.02
$\delta^R(^3P_0)$	−37.26+8.25	−25.57+2.85	$\delta^R(^1G_4)$	7.99+1.00	−6.93+1.08
$\delta^R(^3P_1)$	−15.32+4.75	1.12+3.12	$\delta^I(^1S_0)$	—	—
$\delta^R(^3P_2)$	55.23+8.62	−69.32+7.28	$\delta^I(^3P_0)$	4.38+9.63	−9.80+4.63
$\delta^R(^1D)_2$	3.56+2.62	1.88+3.82	$\delta^I(^3P_1)$	−1.86+3.21	2.20+3.87
ε_2^R	−0.16+3.70	−0.89+3.19	$\delta^I(^3P_2)$	26.31+6.93	30.62+4.98
$\delta^R(^3F_2)$	−5.61+1.14	−3.55+0.66	$\delta^I(^1D_2)$	18.24	18.24
$\delta^R(^3F_3)$	2.39+1.61	8.24+1.39			

Table II. Values of χ^2 Criterion

	Solutions from [8]				
	№ 1	№ 2	№ 3	№ 4	№ 5
$l_{max}=4$, $f^2=0.053$, Nonrelativistic formulas	47	62.1	67.2	82.7	83.1
$l_{max}=4$, $f^2=0.053$, Relativistic formulas	44.8	—	67.0	85.0	78.3
$l_{max}=4$, $f^2=0.080$, Relativistic formulas	41.5	Converges to solution No. 1	68.2	85.4	99.1
$f^2=0.08$, $l_{max}=5$, Relativistic formulas	30.5	—	45; 46	—	43

$\delta^I(^1D_2)$ turned out to be 16.00 ± 4.58°. No great change in $\chi^2/\overline{\chi^2}$ was observed likewise when l_{max} was increased from 4 to 5 (see Fig. 4).

The results of these tests show that if the introduced number of parameters is too low, it is so to an insignificant degree. On the other hand, the fact that the agreement criterion remains constantly somewhat larger than unity is possibly evidence of exaggerated accuracy of some of the experimentally obtained quantities.

It is important to note that the number of solutions was small at the already existing accuracy and volume of experimental material. The magnitude of the phase shifts $\delta^R(^1F_0)$ and $\delta^R(^1D_2)$ obtained in [2] (solution a) for pp scattering at 650 MeV, as well as the value of $A(\pi/2)$ predicted in [16], agree with the corresponding values given by solution No. 1 of the present work.

The fact that in both solutions the phase shifts $\delta^I(\ D_2)$ and $\delta^I(\ P_2)$, which correspond to states with j = 2, are singled out from among the other states, while the phase shift $\delta^I(^3F_2)$ does not exceed ~ 2°, is in our opinion very instructive. In principle, however, it can also indicate that some of our assumptions are not well founded.

The point is that, by virtue of the assumptions made, the solutions were sought in the obviously limited 13-dimensional phase-shift space. It might have happened that stable solutions were actually obtained in this space; however, the physical reality is more closely represented by a set of phase shifts obtained in a phase-shift space with more dimensions. With the existing experimental information on pp scattering and its accuracy, it would hardly be meaningful to expand appreciably this space. However, as further experimental information is accumulated, for example after the measurements of the angular dependence $A(\vartheta)$ are completed, the search for solutions in a space with higher dimensionality should be carried out.

It is of interest to trace the extent to which the solutions become distorted as relativistic corrections are taken into account and the constant f^2 is varied. It was found that relativistic corrections do not distort noticeably the phase shift sets previously obtained [8], and the values of χ^2 have a distribution as shown in Table II.

Refinement of the previously obtained solutions,

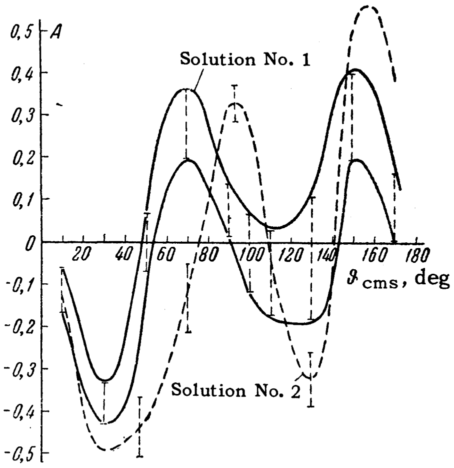

FIG. 3

carried out with a constant $f^2 = 0.080$, has shown that only solution No. 2 of [8] changes appreciably, going into solution No. 1. For the remaining four sets (sets 1, 3, 4, and 5) the phase shifts remain practically constant and only the χ^2 change insignificantly, as seen from Table II. Variation of f^2 for solution No. 1 yielded $f^2 = 0.093 \pm 0.032$.

In [8] we also refined the set of phase shifts obtained by Hoshizaki and Machida[22]. The remarks made in [8] with respect to this set remain in force, as shown by control calculations, also for the case with $f^2 = 0.080$. This set, which we obtained for $l_{max} = 4$ with account of the Coulomb effect and relativistic spin rotation has that peculiarity, that the phase shifts for the states $^3P_0$ and $^3P_1$ are essentially negative, contradicting the unitarity of the S matrix.

SEARCH FOR SOLUTIONS WITH $l_{max} = 5$

Supplementing the 125 solution searches with $l_{max} = 4$, we carried out a search with variation of $\delta^R(^3H_{4,5,6})$ and of the mixing parameter ϵ_4 under the previous assumption[4)]. After 40 "throws" we found five solutions with the following values of χ^2: 30.49, 40.25, 42.70, 44.88, and 46.18, where $\overline{\chi^2} \le \chi^2 \le 2\overline{\chi^2} = 48$.

In many cases the errors of the phase shifts in these sets are quite large. However, the set with $\chi^2 = 30.49$ (reliability ~ 20%) can be identified with confidence with solution No. 1 obtained with $l_{max} = 4$. Likewise reminiscent of solution No. 1 is the set with $\chi^2 = 40.25$ (reliability ~ 2%). The last two sets ($\chi^2 = 44.88$ and 46.18) are very similar, in the mean values of the phase shifts, to solution No. 2 ($l_{max} = 4$), while the set with $\chi^2 = 42.70$ can be identified with solution No. 5 of our previous paper[8].

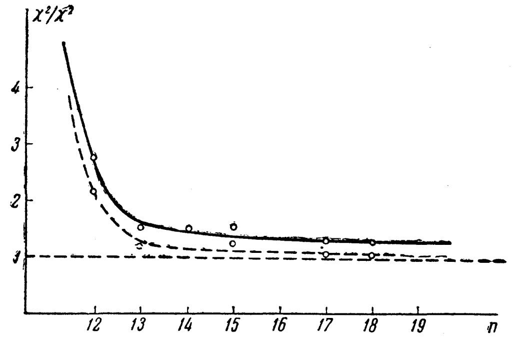

FIG. 4. Dependence of $\chi^2/\overline{\chi^2}$ on the number of the varied parameters. The dashed curve shows the same dependence without the point R (72°) taken into account.

Thus, insofar as the statistics of these searches allows, we can consider solution No. 1 ($l_{max} = 4$; $f^2 = 0.08$) to have the lowest lying minimum of χ^2 even when the phase shifts are varied up to $l_{max} = 5$. We note here that the greatest contribution to χ^2 of solution No. 1, which reaches 7.5 at $l_{max} = 4$ and 6.5 at $l_{max} = 5$, is made by the point R (72°).

CONCLUSIONS

1. Assuming that meson production in the pp collisions proceeds only from initial $^1D_2$ and $^3P_{0,1,2}$ states, a single solution (solution No. 1, $\chi^2 = 41.5$) is obtained in the interval $\overline{\chi^2} \le \chi^2 \le 2\overline{\chi^2}$.

2. At the existing accuracy and volume of the experimental material and under the assumptions made, set no. 1 is sufficiently stable against an increase in the number of the varied parameters.

3. Elastic pp scattering at 660 MeV is well described by a 1-meson Feynman diagram, starting with $l_{max} \ge 5$.

4. The imaginary parts of the phase shifts clearly stand out for the states $^1D_2$ and $^3P_2$ of the pp system.

5. On the basis of the obtained set of phase shifts, we calculated the angular dependences of the different parameters characterizing elastic pp scattering at 660 MeV. Further experiments on pp scattering at this energy are best planned with the result of the present paper taken into account.

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