THERMAL EMF DUE TO ELECTRON-MAGNON SCATTERING IN FERROMAGNETIC METALS

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The thermal emf of ferromagnetic metals at temperatures considerably above 1°K but much below the Curie point is considered for the case when electron scattering on spin waves and on defects is important. It is shown that when scattering on defects predominates scattering of electrons on spin waves gives rise, even in the zeroth approximation in degeneracy, to a thermal emf that is inversely proportional to the defect concentration and can exceed the usual first-approximation thermal emf.

1. INTRODUCTION

We study in the present article the thermal emf produced in ferromagnetic metals by scattering of electrons by magnons at temperatures (T) much below the Curie point (T_C).

The scattering of electrons by spin waves in ferromagnetic metals was considered in [1-3] in connection with the possible contribution of this process to the resistivity. This contribution, according to (4.10), is indistinguishable in order of magnitude and in temperature dependence from the contribution due to the electron-electron interaction, whereas, as will be shown, the contribution to the thermal emf can be appreciable. The relaxation of electrons on magnons differs materially, in some respects, from their relaxation on phonons. An electron in an arbitrary state is capable of both absorbing and emitting phonons, the transition to the new state having the same spin direction and satisfying the energy-momentum conservation laws. To the contrary, exchange interaction between electrons and the spin waves produces only electron transitions in which the spin changes direction but the total spin of the system is conserved. Therefore an electron with a magnetic moment opposite to the magnetization [(−) electron] can reverse its moment only by emitting a magnon, while the inverse transition [of the (+) electron] is possible only by absorption of a magnon. Consequently, on going from phonon to magnon scattering of electrons it is necessary to "split" the expression for the phonon collision integrals into two parts, each having a lower symmetry than their sum.

It is easy to understand that the symmetry properties of the collision operator are quite important for thermoelectric phenomena in metals. The electron density above and below the Fermi level ζ increases with and against the direction of the temperature gradient, respectively, so that the former diffuse against the temperature gradient and the latter with the gradient, and their currents subtract. The dynamic characteristics of both electron groups differ by a small quantity of the order of the relative smearing of the Fermi surface (the degeneracy parameter T/ζ). If the difference in the relaxation time is also small (or else is appreciable but is an even function of $\epsilon - \zeta$), then the order of magnitude of the resultant current will be smaller than the current of each electron group by the ratio T/ζ. In the zeroth order of degeneracy we therefore have for the thermal emf $\alpha^{(0)} = 0$, and in first order, as is well known [4]

$$\alpha^{(1)} \approx T/e\zeta. \quad (1.1)$$

The exchange interaction of the conduction electrons with the magnetization leads to a splitting of the energy spectrum of the (±) electrons by some amount I, so that

$$\varepsilon_{\pm}(p) = \varepsilon_0(p) \mp \tfrac{1}{2} I. \quad (1.2)$$

The relative difference of their Fermi momenta is

$$(p_+ - p_-)/p = \gamma = I/pv \approx I/\zeta \ll 1 \quad (1.3)$$

(v — electron velocity). The momentum q of the magnon connected with the (±) transition should therefore be $\gtrsim \gamma p$, which corresponds to temperatures

$$T \gg T_0 \approx \gamma^2 T_C.$$

We consider the temperature region $T \gg T_0$. If $\gamma^2 \approx 10^{-3}$, then $T_0 \approx 1°K$; when $T \approx T_0$ the hitherto disregarded relativistic interactions become significant; when $T \ll T_0$ the electron-magnon interaction is exponentially small.

The properties of the electron-ferromagnon collision integral are such that (a) the relaxation time for electrons with definite spin can vary appreciably in the intervals $|(\epsilon - \zeta)T| \lesssim 1$, without being an even function of $(\epsilon - \zeta)$, and (b) the relaxation time of the (+) electrons at some distance above (below) the Fermi surface is equal to the relaxation time of the (−) electrons at the same distance below (above) the Fermi surface. The corresponding drift velocities are equal in the presence of an electric field and opposite in the presence of a temperature gradient. Consequently, the thermal emf $\alpha^{(0)}$ differs from zero but arises only in the first approximation in the splitting parameter γ (1.3).

We have considered the thermal emf $\alpha^{(0)}$ for two cases: (a) when the electrons are scattered only by spin waves and (b) when the scattering of the electrons by the defects is the strongest. In the first case $\alpha^{(0)}$ turns out to be of the same order as $\alpha^{(1)}$. This is connected with the fact that the drift velocity of the electrons, determined by the "long" momentum relaxation time $\tau_s^{(p)}$, which, as will be shown, does not depend on the energy, leads to a current only in the first order of degeneracy. There exists, however, a drift velocity component connected with the energy relaxation, with a much smaller relaxation time τ_s ($\approx \tau_s^{(p)} T/T_C$), which leads to $\alpha^{(0)} \neq 0$. For this reason, $\alpha^{(0)}$ is proportional (in addition to γ) to the ratio $\tau_s/\tau_s^{(p)}$ and when $I \approx T_C$ it has the same order of magnitude as $\gamma T/eT_C$, i.e., the same order as $\alpha^{(1)}$.

In the second case the drift velocity of the electrons is determined essentially by the scattering on the defects, with a relaxation time $\tau_d \ll \tau_s$. However, this part of the drift velocity leads only to $\alpha^{(1)}$. On the other hand, the part connected with the scattering by spin waves is smaller than the first by a factor τ_d/τ_s so that (with account of the parameter γ) $\alpha^{(0)} \approx \gamma\tau_d/e\tau_s$, which can be comparable with in magnitude or larger than $\alpha^{(1)}$ for not too small a value of τ_d/τ_s. Although $\tau_s^{-1} \sim T$, and therefore $\alpha^{(0)}$ and $\alpha^{(1)}$ have the same temperature dependence, $\alpha^{(0)}$ can be distinguished by its appreciable dependence on the time of relaxation on defects; in particular, $\alpha^{(0)}$ is inversely proportional to the concentration of the defects.

For a quantitative examination of the problem, we have chosen to solve the kinetic equation in the presence not of a temperature gradient, but of an electric field, and have calculated $\alpha^{(0)}$ with the aid of the Onsager relation[5], which has in the isotropic case the form

$$\alpha = \frac{1}{T}\frac{Q}{j}, \tag{1.4}$$

where Q and j are the heat flux and the electric current in the electric field **E**.

2. ELECTRON-FERROMAGNON COLLISION INTEGRAL

The interaction between the conduction electrons and the spin waves can be described phenomenologically, for our purposes, by the expression

$$H_{es} = I\boldsymbol{\sigma}\frac{\mathbf{M}}{M_0} = \frac{I}{M_0}\left[\sigma_z M_z + \frac{1}{2}(\sigma_+ M_- + \sigma_- M_+)\right], \tag{2.1}$$

where I will be regarded as a constant that describes effectively the exchange interaction and satisfies the inequality

$$\zeta \gg I \gg T, \tag{2.2}$$

$\boldsymbol{\sigma}$ is the electron spin operator, **M** the magnetization operator per unit volume, and M_0 the saturation magnetization.

$$\sigma_\pm = \sigma_x \pm i\sigma_y, \quad M_\pm = M_x \pm iM_y.$$

At temperatures satisfying the inequality

$$T_C \gg T \gg 2\pi\beta M_0 \approx 1° \text{K} \tag{2.3}$$

(β — Bohr magneton) we can put [6] $M_z = M_0$ and

$$M_+ = (2\beta M_0)^{1/2} V^{-1/2} \sum_{\mathbf{q}} \alpha^+_{-\mathbf{q}} \exp(i\mathbf{qr}/\hbar),$$
$$M_- = (2\beta M_0)^{1/2} V^{-1/2} \sum_{\mathbf{q}} \alpha_{\mathbf{q}} \exp(i\mathbf{qr}/\hbar),$$

where $\alpha_{\mathbf{q}}$ and $\alpha^+_{\mathbf{q}}$ are the operators for the annihilation and creation of magnons with momentum **q** (V — volume of the sample).

Then, in the second-quantization representation

$$H_{es} = \frac{I}{2}\sum_{p}(a^+_{\mathbf{p}(-)}a_{\mathbf{p}(-)} - a^+_{\mathbf{p}(+)}a_{\mathbf{p}(+)})$$
$$+ I\left(\frac{\beta}{2M_0 V}\right)^{1/2}\sum_{\mathbf{pq}}(a^+_{\mathbf{p+q}(-)}a_{\mathbf{p}(+)}\alpha_{\mathbf{q}} + a^+_{\mathbf{p+q}(+)}a_{\mathbf{p}(-)}\alpha^+_{-\mathbf{q}}). \tag{2.4}$$

The (±) subscripts of the electron operators a and a^+ designate whether the magnetic moment of the electron is parallel or antiparallel to the equilibrium magnetization.

The diagonal part of H_{es} leads to a splitting of (1.2), while the nondiagonal part leads to scattering of the electrons by the magnons. We have confined ourselves to one zone (when $q \ll p$ the interzone matrix element H_{es} vanishes).

In the presence of an external directional perturbation (electric field, temperature gradient) we can represent the electron distribution function $n_{\pm}(\mathbf{p})$ in the form

$$n_{\pm}(\mathbf{p}) = n_0(\varepsilon) - \frac{\partial n_0}{\partial \varepsilon} \mathbf{p} \mathbf{u}_{\pm}(\varepsilon), \qquad (2.5)$$

where the vectors $\mathbf{u}_{\pm}$ have the meanings of electron drift velocities. Then the interaction (2.4) leads to the collision integral

$$\left(\frac{\partial n_+(\mathbf{p})}{\partial t}\right)_{\text{col}} = I^2 \frac{\pi\beta}{\hbar M_0} \frac{1}{T} (2\pi\hbar)^{-3} \int d^3q\, \delta(\varepsilon_{\mathbf{p}}^{(+)} - \varepsilon_{\mathbf{p+q}}^{(-)} + \hbar\omega_{\mathbf{q}})$$
$$\times \frac{e^{x+\xi}}{(e^{x+\xi}+1)(e^x+1)(e^{\xi}-1)} [(p_u + q_u)\, u_-(x+\xi)$$
$$- p_u u_+(x)];$$

$$\left(\frac{\partial n_-(\mathbf{p})}{\partial t}\right)_{\text{col}} = I^2 \frac{\pi\beta}{\hbar M_0} \frac{1}{T} (2\pi\hbar)^{-3} \int d^3q\, \delta(\varepsilon_{\mathbf{p}}^{(-)} - \varepsilon_{\mathbf{p+q}}^{(+)} - \hbar\omega_{\mathbf{q}})$$
$$\times \frac{e^{x}}{(e^{x-\xi}+1)(e^x+1)(e^{\xi}-1)} [(p_u + q_u)\, u_+(x-\xi)$$
$$- p_u u_-(x)]. \qquad (2.6)$$

Here $x = (\epsilon - \zeta)/T$; $\xi = \hbar\omega_q/T$. The magnon energy is $\hbar\omega_q = q^2/2\mu$, where μ is the effective mass of the magnon, with order of magnitude $\hbar^2/T_C a^2$ (a — lattice constant); $p_u(q_u)$ — projection of the vector $\mathbf{p}(\mathbf{q})$ on the direction of the vector $\mathbf{u}$.

The energy conservation law leads in the first and second expressions of (2.6) to

$$-\cos(\mathbf{q}, \mathbf{p})$$
$$+ \frac{1}{2}\cos^2(\mathbf{q}, \mathbf{p})\left(1 - \frac{p}{v}\frac{dv}{dp}\right) - \frac{q}{2p}\left(1 \mp \frac{p}{\mu v}\right) \mp \frac{I}{qv} = 0.$$

Since $q/p \ll 1$ and $p/\mu v \approx T_C/\zeta \ll 1$, we have $\cos(\mathbf{q}, \mathbf{p}) = -q/2p \mp I/qv$, hence $\gamma p \leq q \leq 2p$.

The upper limit of q can be replaced by infinity, since the integrals converge when $q \ll p$. The lower limit signifies that the magnon momentum should be sufficient to ensure transfer of the energy I connected with the spin reorientation. If the minimum momentum of the magnons interacting with the electrons (γp) exceeds the average momentum q_T, then the interaction is exponentially small. We consider the case when $\gamma p \ll q_T$ so that

$$\xi_0 = (\gamma p)^2/2\mu T \ll 1, \qquad (2.7)$$

i.e., when almost all the thermal magnons interact with the electrons. This corresponds to temperatures

$$T \gg \gamma^2 p^2/2\mu \approx \gamma^2 T_C. \qquad (2.8)$$

After integrating (2.6) over the angles we get

$$\left(\frac{\partial n_+(\mathbf{p})}{\partial t}\right)_{\text{col}} = \frac{p_u}{T}\left(\frac{\partial u_+}{\partial t}\right)_{\text{col}} = \frac{p_u}{T}\tau_s^{-1}\int dx' L_+(x, x')$$
$$\times \{[u_-(x') - u_+(x)] - \gamma'(x' - x)\, u_-(x') - \gamma u_-(x')\};$$
$$\left(\frac{\partial n_-(\mathbf{p})}{\partial t}\right)_{\text{col}} = \frac{p_u}{T}\left(\frac{\partial u_-}{\partial t}\right)_{\text{col}} = \frac{p_u}{T}\tau_s^{-1}\int dx' L_-(x, x')$$
$$\times \{[u_+(x') - u_-(x)] - \gamma'(x - x')\, u_+(x') + \gamma u_+(x')\}. \qquad (2.9)$$

Here

$$\tau_s^{-1} = \gamma^2 T p^2 v \mu\beta/4\pi\hbar^4 M_0 \approx \gamma^2 \zeta T/\hbar T_C, \qquad (2.10)$$
$$\gamma' = \mu T/p^2 \approx T/T_C \ll 1, \qquad \gamma' = \gamma^2/2\xi_0 \gg \gamma^2, \qquad (2.11)$$
$$L_+(x, x') = \frac{e^{x'}}{(e^x+1)(e^{x'}+1)(e^{x'-x}-1)}\,\theta(x' - x - \xi_0),$$
$$L_-(x, x') = \frac{e^{x}}{(e^{x'}+1)(e^{x}+1)(e^{x-x'}-1)}\,\theta(x - x' - \xi_0),$$
$$\theta(x) = 1 \text{ for } x > 0, \quad \theta(x) = 0 \text{ for } x < 0. \qquad (2.12)$$

In (2.9) the terms which do not contain the small parameters γ and γ' remain only if the change in electron energy due to the magnon collision (which is of the order of T) is taken into account, but the small change in momentum is disregarded ($q \ll p$). Consequently, these terms represent the energy relaxation of the electrons on the magnons, with a characteristic time τ_s. On the other hand, the terms containing the small parameters γ and γ' are connected with the momentum relaxation and, as will be shown, determine the electric conductivity.

L_+ and L_- have the following symmetry properties;

$$L_{\pm}(x, x') = L_{\pm}(-x', -x),$$
$$L_+(x, x') = L_-(x', x) = L_-(-x, -x'). \qquad (2.13)$$

3. SYMMETRY OF THE COLLISION OPERATOR AND SOLUTION OF THE KINETIC EQUATION

The heat flux carried by the electrons is

$$\mathbf{Q} = -(2\pi\hbar)^{-3}\int d^3p \frac{\partial n_0}{\partial \varepsilon}(\varepsilon - \zeta)\, \mathbf{v}\, [\mathbf{p}\mathbf{u}_+ + \mathbf{p}\mathbf{u}_-]$$
$$= -\frac{1}{3}T\int_0^{\infty} dx \frac{\partial n_0}{\partial x} x\, [(\rho v p)_+(x)\, \mathbf{u}_+(x) + (\rho v p)_-(x)\, \mathbf{u}_-(x)$$
$$- (\rho v p)_+(-x)\, \mathbf{u}_+(-x) - (\rho v p)_-(-x)\, \mathbf{u}_-(-x)]$$

ρ — density of states).

In the zeroth approximation in the degeneracy parameter, the quantities $(\rho v p)_{\pm}(x)$ should be taken for $x = 0$, and then

$$\mathbf{Q} = -\frac{1}{3}T\int_0^\infty dx \frac{\partial n_0}{\partial x} x \Big\{(\rho v p)\,\{[\mathbf{u}_+(x) - \mathbf{u}_-(-x)] - [\mathbf{u}_+(-x) - \mathbf{u}_-(x)]\} + \frac{1}{2}\gamma p \frac{d}{dp}(\rho v p)\,\{[\mathbf{u}_+(x) + \mathbf{u}_-(-x)] - [\mathbf{u}_+(-x) + \mathbf{u}_-(x)]\}\Big\}. \quad (3.1)$$

Analogously, the electric current density is

$$\mathbf{j} = -\frac{1}{3}e\int_0^\infty dx \frac{\partial n_0}{\partial x}\Big\{(\rho v p)\,\{[\mathbf{u}_+(x) + \mathbf{u}_-(-x)] + [\mathbf{u}_+(-x) + \mathbf{u}_-(x)]\} + \frac{1}{2}\gamma p \frac{d}{dp}(\rho v p)\,\{[\mathbf{u}_+(x) - \mathbf{u}_-(-x)] + [\mathbf{u}_+(-x) - \mathbf{u}_-(x)]\}\Big\}. \quad (3.2)$$

In (3.1) and (3.2) the quantity $(\rho v p)$ pertains to the "unperturbed" Fermi surface $(\epsilon_0(p) = \xi)$.

We define a function as quasi-even or quasi-odd if it satisfies respectively the relations

$$u_+(x) = \pm u_-(-x). \quad (3.3)$$

We introduce quasi-even and quasi-odd parts $w_\pm$, defined as

$$w_\pm(x) = \frac{1}{2}[u_-(x) \pm u_+(-x)]. \quad (3.4)$$

For a quasi-even (quasi-odd) function we have $w_- \equiv 0$ $(w_+ \equiv 0)$.

Formulas (3.1) and (3.2) lead to the following conclusions:

1. A contribution to the heat flux (current) is made in the zeroth order of degeneracy only by the odd (even) part of the drift velocity $\mathbf{u}$.

2. The contribution to the heat flux (current) from the quasi-even (quasi-odd) part of the drift velocity has an order of magnitude γ times as small as the contribution from the quasi-odd (quasi-even) part.

The kinetic equation in the presence of an external perturbation is of the form

$$-R_+(x) = \int dx'\,\{L_{++}(x, x')\,u_+(x') + L_{+-}(x, x')\,u_-(x')\};$$
$$-R_-(x) = \int dx'\,\{L_{-+}(x, x')\,u_+(x') + L_{--}(x, x')\,u_-(x')\}. \quad (3.5)$$

R is the inhomogeneity due to the external perturbation

$$R_\pm(x) = R(x) \pm \gamma R'(x), \quad (3.6)$$

with R and R′ of the same order of magnitude (see Sec. 4). In the zeroth approximation in the degeneracy parameter we have in the case of an electric field

$$R_\pm^{(E)}(x) = R_\pm^{(E)}(-x) \quad (3.7)$$

[see (4.2)], and in the case of a temperature gradient

$$R_\pm^{(T)}(x) = -R_\pm^{(T)}(-x). \quad (3.8)$$

Thus, it follows from (3.6)—(3.8) that the quasi-odd part in $R^{(E)}$ has an order of magnitude γ times as small as the quasi-even part, while in $R^{(T)}$ the order of magnitude of the quasi-even part is γ times as small as the quasi-odd part.

If the operator L_{ab} $(a, b = \pm)$ is even, i.e.,

$$L_{ab}(x, x') = L_{ab}(-x, -x') \quad (3.9)$$

(such as the electron-phonon collision operator, for example, [4]), then the solution of the system (3.5) has the same parity as the inhomogeneity. If the operator L_{ab} is quasi-even, i.e.,

$$L_{++}(x, x') = L_{--}(-x, -x'),$$
$$L_{+-}(x, x') = L_{-+}(-x, -x'), \quad (3.10)$$

then the solution has the same quasi-parity as the inhomogeneity.

The collision integral (2.9) can be reduced to the form (3.5) by putting

$$L_{++}(x, x') = -\delta(x - x')\int L_+(x, x'')\,dx'',$$
$$L_{--}(x, x') = -\delta(x - x')\int L_-(x, x'')\,dx'',$$
$$L_{+-}(x, x') = L_+(x, x')\,[1 - \gamma'(x' - x) - \gamma],$$
$$L_{-+}(x, x') = L_-(x, x')\,[1 - \gamma'(x - x') + \gamma]. \quad (3.11)$$

The symmetry relations for $L_\pm$ (2.13) allow us to conclude that

$$L_{++}(x, x') = L_{--}(-x, -x'),$$
$$L_{+-}(x, x'; \gamma) = L_{-+}(-x, -x'; -\gamma). \quad (3.12)$$

Thus, the electron–spin wave collision integral does not satisfy the parity conditions (3.9), and therefore the solution of the kinetic equation does not have the same parity as the inhomogeneity. The quasi-parity condition (3.10) is satisfied by the operator L_{ab} accurate to the parameter γ. Because of this, the solution of the equation will be such that the part possessing a quasi-parity opposite that of the inhomogeneity will have an order of magnitude γ times as small as the part whose quasi-parity coincides with that of the inhomogeneity.

The absence of parity in the electron–spin wave collision integral, as can be readily seen, is due to the fact that this operator consists of individual parts, one with magnon emission only and another with magnon absorption only. Failure to satisfy the quasi-parity condition is connected with the fact that in the $(-) \rightarrow (+)$ transitions the spin orientation energy is changed by $(-I)$, while in the $(+) \rightarrow (-)$ transitions it changes by (I).

4. SOLUTION OF THE KINETIC EQUATION IN THE CASE OF THE SCATTERING OF ELECTRONS BY SPIN WAVES

If in the presence of an external electric field the kinetic equation has the form

$$\frac{\partial n_0}{\partial x}\frac{eEv}{p}\tau_s = \tau_s\left(\frac{\partial u_\pm}{\partial t}\right)_{\text{col}}, \qquad (4.1)$$

where $(\partial u_\pm/\partial t)_{\text{col}}$ is determined by (2.9). In the zeroth approximation in the degeneracy parameter all the quantities except $u_\pm$ must be taken at the Fermi value of the momentum, which is different for the $(+)$ and $(-)$ electrons. We then obtain [compare with (3.6)—(3.8)]

$$-[R^{(E)}x \pm \gamma R'^{(E)}(x)] = \frac{\partial n_0}{\partial x}\frac{eEv}{p}\tau_s\left[1 \pm \frac{1}{2}\gamma p\frac{d}{dp}\ln\left(\frac{v}{p}\tau_s\right)\right] = \tau_s\left(\frac{\partial u_\pm}{\partial t}\right)_{\text{col}}. \qquad (4.2)$$

The solution of the system (4.2) can be represented in the form of a sum of two terms, due to the quasi-even and the quasi-odd parts of the inhomogeneity, respectively.

Solution of the system (4.2) with quasi-even inhomogeneity. Taking the half-sum and the half-difference of the first and second equations of (4.2) for the point $(-x)$, we obtain for $w_\pm$ (3.4), using the asymmetry properties (2.13) of $L_\pm$:

$$-R^{(E)}(x) = \int dx' L_+(x, x')\{[w_+(x') - w_+(-x)] - \gamma'(x'-x)w_+(x') - \gamma w_-(x')\}, \qquad (4.3a)$$

$$0 = \int dx' L_+(x, x')\{[w_-(x') + w_-(-x)] - \gamma'(x'-x)w_-(x') - \gamma w_+(x')\}. \qquad (4.3b)$$

As can be seen from (4.3b), $w_- \approx \gamma w_+$ and, in particular, when $\gamma = 0$ the equations for w_+ and w_- are not coupled and $w_- = 0$. This is the consequence of the fact that when $\gamma = 0$ relations (3.12) are transformed into the quasi-parity conditions (3.10).

If we discard terms containing the small parameters γ and γ' and connected with the change in momentum, we obtain for w_+ an equation of the form

$$F(x) = \int L_+(x, x')[w_+(x') - w_+(-x)]\,dx'. \qquad (4.4)$$

This equation has a solution only if $\int F(x)\,dx = 0$, for by virtue of (2.13) we have

$$\iint L_+(x, x')[w_+(x') - w_+(-x)]\,dx'dx \equiv 0.$$

Since

$$\int F(x)\,dx = \int R^{(E)}(x)\,dx \neq 0$$

in the case of an electric field, Eq. (4.4) has no solution and the terms with γ and γ' cannot be discarded. This means that in the first equation the "large" part of the operator acting on w_+ gives a result of the same order as the "small" part. Consequently, w_+ should contain a "large" part ($w_+^{(0)}$) which yields zero when acted upon by the "large" part of the operator, and a small part $w_+^{(1)}$, i.e., $w_+ = w_+^{(0)} + w_+^{(1)}$ (and also $w_- = w_-^{(0)} + w_-^{(1)}$), where

$$\int dx' L_+(x, x')[w_+^{(0)}(x') - w_+^{(0)}(-x)] = 0,$$

i.e., $w_+^{(0)} = \text{const}$. For $w_+^{(1)}$ we obtain the equation

$$F(x) = -R^{(E)}(x) + \int dx' L_+(x, x')[\gamma'(x'-x)w_+^{(0)} + \gamma w_-^{(0)}(x')] = \int dx' L_+(x, x')[w_+^{(1)}(x') - w_+^{(1)}(-x)]. \qquad (4.5)$$

Equation (4.5) is of the same type as (4.4), so that the condition $\int F(x)\,dx = 0$ under which it has a solution leads to one of the equations for $w_\pm^{(0)}$

$$\int R^{(E)}(x)\,dx = \iint dx dx' L_+(x, x')[\gamma'(x'-x)w_+^{(0)} + \gamma w_-^{(0)}(x')]. \qquad (4.6a)$$

The second equation for $w_\pm^{(0)}$ is obtained by substituting $w_+^{(0)}$ in the last term of (4.3b) and discarding the term with γ'

$$\int dx' L_+(x, x')[w_-^{(0)}(x') + w_-^{(0)}(-x)] = w_+^{(0)}\gamma\int dx' L_+(x, x'). \qquad (4.6b)$$

It is easy to see that (4.6b) is satisfied if

$$w_-^{(0)} = \tfrac{1}{2}\gamma w_+^{(0)} = \text{const}. \qquad (4.7)$$

Then (4.6a) yields

$$w_+^{(0)} = \int R^{(E)}(x)\,dx \Big/ \left[\gamma'\iint dx' dx L_+(x, x')(x'-x) + \tfrac{1}{2}\gamma^2\iint dx' dx L_+(x, x')\right]. \qquad (4.8)$$

If $\xi_0 \ll 1$, the ratio of the second term in the denominator of (4.8) to the first is $3\xi_0\pi^{-2}\ln(e/\xi_0) \ll 1$, so that

$$w_+^{(0)} = \frac{3}{\pi^2\gamma'}\int R^{(E)}(x)\,dx. \qquad (4.9)$$

The electron–spin wave momentum relaxation time is thus

$$\tau_s^{(p)} = \frac{3}{\pi^2\gamma'}\tau_s \approx \gamma^{-2}\frac{\hbar}{\zeta}\left(\frac{T_C}{T}\right)^2. \qquad (4.10)$$

If I is comparable with T_C, then $\tau_s^{(p)}$ is of the same order as the electron-electron relaxation time[7].

Since $w_+^{(0)}$ and $w_-^{(0)}$ are constant, they make no contribution to the heat flux in the zeroth approximation in the degeneracy, so that we estimate the next approximation of $w_\pm^{(1)}$ in $\tau_s/\tau_s^{(p)}$. As follows from (4.5), if we disregard the weak (logarithmic) dependence of $\int dx' L_+(x,x')$ on ξ_0, then

$$w_+^{(1)} \approx R^{(E)} \approx \gamma' w_+^{(0)}. \qquad (4.11)$$

$w_-^{(1)}$ is determined by iteration of (4.3b):

$$\int dx' L_+(x,x')[w_-^{(1)}(x') + w_-^{(1)}(-x)]$$
$$= w_-^{(0)}\gamma'\int dx' L_+(x,x')(x'-x)$$
$$+\gamma\int dx' L_+(x,x')\,w_+^{(1)}(x'), \qquad (4.12)$$

i.e., using (4.7)

$$w_-^{(1)} = \tfrac{1}{2}x\gamma'\gamma w_+^{(0)} + \Delta w_-^{(1)}, \qquad (4.13)$$

where $\Delta w_-^{(1)}$ is determined from (4.12), in the inhomogeneity of which only the second term is left. With the aid of (4.11) we can conclude that $\Delta w_-^{(1)} \approx \gamma'\gamma w_-^{(0)}$, i.e.,

$$w_-^{(1)} \approx \gamma' w_-^{(0)} \approx \gamma'\gamma w_+^{(0)}. \qquad (4.14)$$

We note that although $w_-^{(1)} \approx \gamma w_+^{(1)}$ [see (4.11) and (4.14)], the contribution from $w_\pm^{(1)}$ to the heat flux will, in accordance with (3.1), be of the same order of magnitude, and since the parity conditions (3.9) are not satisfied, the even and odd parts $w_\pm^{(1)}$ have the same order of magnitude, in spite of the inhomogeneity being even [see, for example, the first term in (4.13)].

Solution of the system (4.3) with quasi-odd inhomogeneity. In this case the system (4.2) has, after transformation to the system for $w_\pm$, the form

$$0 = \int dx' L_+(x,x')\{[w_+(x') - w_+(-x)]$$
$$-\gamma'(x'-x)w_+(x') - \gamma w_-(x')\};$$
$$-\gamma R'^{(E)}(x) = \int dx' L_+(x,x')\{[w_-(x') + w_-(-x)]$$
$$-\gamma'(x'-x)w_-(x') - \gamma w_+(x')\}. \qquad (4.15)$$

In analogy with the preceding case, the solution is sought in the form $w_\pm = w_\pm^{(0)} + w_\pm^{(1)}$, where $w_+^{(0)}$ = const and is determined from the condition

$$w_+^{(0)}\gamma'\iint dx'dx\,L_+(x,x')(x'-x)$$
$$+\gamma\iint dx'dx\,L_+(x,x')\,w_-^{(0)}(x') = 0. \qquad (4.16)$$

$w_+^{(1)}$ and $w_-^{(0)}$ are determined by the equations

$$-\gamma R'^{(E)}(x) = \int dx' L_+(x,x')[w_-^{(0)}(x') + w_-^{(0)}(-x)], \qquad (4.17)$$

$$\int dx' L_+(x,x')[w_+^{(1)}(x') - w_+^{(1)}(-x)]$$
$$= \int dx' L_+(x,x')[\gamma'(x'-x)w_+^{(0)} + \gamma w_-^{(0)}(x')]. \qquad (4.18)$$

There is no sense in rewriting the equation for $w_-^{(1)}$, for $w_-^{(0)}$ itself contributes to the heat flux. By virtue of (4.17) we have

$$w_-^{(0)} \approx \gamma R^{(E)}, \qquad (4.19)$$

i.e., it is of the same order as (4.14). From (4.16) and (4.18) we conclude that

$$w_+^{(1)} \approx \gamma w_-^{(0)}. \qquad (4.20)$$

Consequently, the contribution to the heat flux from (4.20) will have an order of magnitude γ^2 times as small as the contribution from (4.19) ($w_+^{(0)}$ is constant and therefore does not contribute to the heat flux in the zeroth order of the degeneracy).

Thus, in the zeroth order of the degeneracy the heat flux $Q^{(0)}$ is determined by the drift velocity (4.11), (4.14), (4.19). It is estimated at

$$Q^{(0)} \approx \gamma'\gamma T N e E\frac{v}{p}\tau_s^{(p)} \qquad (4.21)$$

(N — electron concentration). In the first approximation in the degeneracy, the heat flux $Q^{(1)}$ is determined by $w_+^{(0)}$ (4.9) and its order of magnitude is

$$Q^{(1)} \approx T\frac{T}{\zeta}NeE\frac{v}{p}\tau_s^{(p)}. \qquad (4.22)$$

The temperature dependence of (4.21) and (4.22) is the same, and their ratio is

$$Q^{(0)}/Q^{(1)} \approx I/T_C \approx 1.$$

Consequently, although the heat flux (and therefore also the thermal emf) arises in the zeroth approximation in the degeneracy, it has the same order of magnitude as $\alpha^{(1)}$. The reasons for it are as follows.

1. The flux $Q^{(0)}$ does not vanish, because the collision operator $L(x,x')$ does not satisfy the parity condition (3.9).

2. The operator $L(x,x')$ is quasi-even, apart from the parameter γ (3.12), and consequently $Q^{(0)}$ contains the parameter γ.

3. The drift velocity in the zeroth approximation ($w_+^{(0)}$) is determined by the momentum relaxation $\tau_s^{(p)}$ (4.10), and is therefore independent of the energy and makes no contribution to $Q^{(0)}$. In the first approximation it is determined by the energy relaxation time τ_s, which is of the order of $\gamma'\tau_s^{(p)}$ Therefore $Q^{(0)}$ contains a parameter γ' (4.21).

5. THERMAL EMF DUE TO THE SCATTERING OF THE ELECTRONS BY MAGNONS, WITH SCATTERING ON DEFECTS PREDOMINATING

In the presence of defects, the kinetic equation (4.1) is of the form

$$\frac{\partial n_0}{\partial x}\frac{eEv}{p} = \frac{\partial n_0}{\partial x} u_\pm \tau_d^{-1} + \left(\frac{\partial u_\pm}{\partial t}\right)_s, \qquad (5.1)$$

where $(\partial u_\pm/\partial t)_s$ is the electron-magnon collision integral (2.9). The solution of (5.1) can be represented in the form ($\tau_d \ll \tau_s$)

$$u_\pm = u_\pm^{(d)} + u'_\pm, \quad u_\pm^{(d)} = \tau_d eEv/p, \qquad (5.2)$$

and $u_\pm \ll u_\pm^{(d)}$, are determined by the iteration

$$-\frac{\partial n_0}{\partial x} u'_\pm = \tau_d \left(\frac{\partial u_\pm^{(d)}}{\partial t}\right)_s. \qquad (5.3)$$

The expressions become more convenient if they are written out directly for $w'_\pm$. Putting $u_\pm^{(d)}$ = const (in the zeroth approximation in the degeneracy), we have with account of (2.13)

$$-\frac{\partial n_0}{\partial x} w'_- = \frac{\tau_d}{\tau_s}\left[(u_+^{(d)} - u_-^{(d)}) + \gamma u^{(d)}\right]\int L_-(x, x')\,dx'$$
$$= \gamma \frac{\tau_d}{\tau_s}\frac{d}{dp}(pu^{(d)})\int L_-(x, x')\,dx', \qquad (5.4)$$

$$-\frac{\partial n_0}{\partial x} w'_+(x) = -\frac{1}{2}\gamma' \frac{\tau_d}{\tau_s} u^{(d)} \int L_-(x, x')(x - x')\,dx'. \qquad (5.5)$$

In (5.4) and (5.5) we have discarded terms of order $\gamma\gamma'$ and of order $\gamma^2 \ll \gamma'$ (2.11). The heat flux connected with w'_+ is negligibly small, for according to (3.1) and (3.5) it is proportional to $\gamma\gamma'$. Then (3.1) and (5.4) yield in the zeroth approximation in the degeneracy (for $\xi_0 \ll 1$)

$$Q^{(0)} = \frac{2}{3} T \int_0^\infty x\,dx \int_{-\infty}^{\infty} dx' \,[L_-(x, x') - L_-(-x, x')]\,\gamma \frac{\tau_d}{\tau_s}(\rho v p)\frac{d}{dp}(pu^{(d)})$$
$$= \frac{\pi^2}{9}\gamma \frac{\tau_d}{\tau_s}(\rho v p)\frac{d}{dp}(pu^{(d)}). \qquad (5.6)$$

The electric current is

$$j = \frac{2}{3} e\,(\rho v p)\, u^{(d)}, \qquad (5.7)$$

so that by using (1.4) we obtain for the thermal emf $\alpha^{(0)}$

$$\alpha^{(0)} = \frac{1}{T}\frac{Q^{(0)}}{j} = \frac{\pi^2}{6}\frac{1}{e}\gamma\frac{\tau_d}{\tau_s}\left[1 + p\frac{d(\ln u^{(d)})}{dp}\right]. \qquad (5.8)$$

The drift velocity $u^{(d)}$ leads to a thermal emf

$$\alpha^{(1)} = \frac{\pi^2}{3}\frac{1}{e}\frac{T}{v}\frac{d}{dp}(\ln(\rho v p u^{(d)})). \qquad (5.9)$$

The temperature dependence of (5.8) and (5.9) is the same. In order of magnitude, on the other hand,

$$\frac{\alpha^{(0)}}{\alpha^{(1)}} \approx \frac{\zeta}{T}\gamma\frac{\tau_d}{\tau_s},$$

and if τ_d/τ_s is not very small, $\alpha^{(0)}$ can be comparable with or even larger than $\alpha^{(1)}$. The thermal emf $\alpha^{(0)}$, unlike $\alpha^{(1)}$, depends on the time of relaxation on the defects and, in particular, $\alpha^{(0)}$ is inversely proportional to the concentration of the defects, where $\alpha^{(1)}$ is independent of this concentration.

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