GRAVITATIONAL TRANSMUTATIONS OF FERMIONS

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The density of the interaction Lagrangian is constructed for gravitational, spinor and electromagnetic fields. The effective cross sections for annihilation of fermion pairs into two gravitons or into a photon and a graviton, and also the Compton effect of a photon on a fermion with transmutation into a graviton, are calculated on the basis of the quantization of the weak gravitational field.

1. INTRODUCTION

In our previous work,[1] a calculation and an analysis of the differential and total effective cross sections of the process of pair annihilation of spinor particles into two transverse quanta (gravitons) of the linearized gravitational field was carried out.1) Use was made of the interaction Hamiltonian introduced in the works of Gupta:[10,11]

$$H = \frac{i\sqrt{\varkappa}}{4}\left(h_{\mu\nu} - \frac{1}{2}\delta_{\mu\nu}h\right)\left(\overline{\Psi}\gamma_\mu\frac{\partial\Psi}{\partial x_\nu} - \frac{\partial\overline{\Psi}}{\partial x_\nu}\gamma_\mu\Psi\right). \quad (1)$$

In the present work, the form of interaction of the gravitational and spinor fields is made more precise. It follows from this development that the use of Eq. (1) in the calculation of the different effects gives insufficiently complete results for the following reasons.

a) In the derivation of the density of the interaction Lagrangian, Gupta probably started from the Lagrangian of the spinor field in curved space, and from the non-compactness of the Lagrangian. In the calculations of the S matrix, it is necessary to take under the integral sign the Hamiltonian density (1), which will lead to additional terms because of the expansion of $\sqrt{-g}$ in powers of $\sqrt{\kappa}$. A similar inaccuracy is contained in [2,3,12], but because of the specific nature of the scalar field it does not change the order of the cross section of gravitational annihilation of two scalar particles.

b) In the calculation of processes of second order and higher in $\sqrt{\kappa}$ on the basis of perturbation-theory methods, the matrix element of the process contains additional terms of the same order, brought about by the nonlinearity of the gravitational field.[9]

c) For a correct description of the interaction of the spinor and gravitational fields it is necessary to use the tetrad formalism,[13] without the use of which it is impossible to obtain the complete Hamiltonian or Lagrangian of the interaction.

2. THE DENSITY OF THE INTERACTION LAGRANGIAN OF SPINOR AND GRAVITATIONAL FIELDS

The density of the Lagrangian of the spinor field in curved space has the form

$$\mathscr{L} = \tfrac{1}{2}\sqrt{-g}\,[-i\overline{\Psi}\gamma^\mu\,(\nabla_\mu\Psi) + i\,\overline{(\nabla_\mu\Psi)}\,\gamma^\mu\Psi + 2m\overline{\Psi}\Psi], \quad (2)$$

where ∇_μ denotes covariant differentiation. The covariant derivative of a spinor can be written only with the help of the tetrad formalism:[13]

$$\nabla_\mu\Psi = \partial\Psi/\partial x^\mu - \tfrac{1}{4}\Delta_\mu(\alpha,\beta)\,\gamma(\alpha)\,\gamma(\beta)\,\Psi,$$

$$\overline{\nabla_\mu\Psi} = \partial\overline{\Psi}/\partial x^\mu - \tfrac{1}{4}\overline{\Psi}\Delta_\mu(\alpha,\beta)\,\gamma(\beta)\,\gamma(\alpha), \quad (3)$$

where $\Delta_\mu(\alpha,\beta)$ are the Ricci rotation coefficients

$$\Delta_\mu(\alpha,\beta) = \Delta_{\mu,\nu\sigma}h^\nu(\alpha)\,h^\sigma(\beta). \quad (4)$$

Then (2) takes the form

1)The hypothesis of mutual transmutations of ordinary matter and the gravitational field, suggested by Ivanenko in 1947, was used by Ivanenko and Sokolov[2,3] in the calculation of the mutual transmutations of scalar particles into gravitons. The transmutations of photons and gravitons was considered by Piĭr,[4] and also by Korkina[5] on the basis of the linear gravitational theory of Birkhoff, as developed by Borgardt. However, the Birkhoff theory, which is constructed on the basis of a series of postulates, is in our view too artificial and represents a step backward in comparison with the Einstein theory (even in the special formulation of Gupta). The mutual transmutations of neutrinos and gravitons were considered by Wheeler and Brill[6,7] and by Kobzarev and Okun'.[8] Recently, the transmutation problem has been considered by Feynman.[9]

$$\mathscr{L} = -\tfrac{1}{2}\sqrt{-g}\Big[-i\overline{\Psi}\gamma(\alpha)\,h^{\mu}(\alpha)\frac{\partial\Psi}{\partial x^{\mu}} + i\frac{\partial\overline{\Psi}}{\partial x^{\mu}}\gamma(\alpha)\,h^{\mu}(\alpha)\,\Psi + 2m\overline{\Psi}\Psi + \frac{i}{4}\overline{\Psi}\,(\gamma(\lambda)\,\gamma(\alpha)\,\gamma(\beta) - \gamma(\beta)\,\gamma(\alpha)\,\gamma(\lambda))\,\Psi h^{\mu}(\lambda)\,h^{\nu}(\alpha)\,h^{\sigma}(\beta)\,\Delta_{\mu,\nu\sigma}\Big]. \quad (5)$$

We expand (5) in a power series in $\sqrt{\kappa}$, and limit ourselves to terms of order not higher than κ. The quantities entering into (5) are expanded in a series of the form

$$\sqrt{-g}\,g^{\mu\nu} = \varepsilon^{\mu\nu} - \sqrt{\varkappa}\,h^{\mu\nu} \quad (6)$$

(the initial formula for the determination of $h^{\mu\nu}$ was developed by Gupta; $\epsilon^{\mu\nu}$ is the metric of flat space-time),

$$\sqrt{-g} = 1 - \tfrac{1}{2}\sqrt{\varkappa}\varepsilon_{\mu\nu}h^{\mu\nu} - \tfrac{1}{4}\varkappa\varepsilon_{\mu\rho}\varepsilon_{\nu\lambda}h^{\mu\nu}h^{\lambda\rho} + \tfrac{1}{8}\varkappa\varepsilon_{\mu\nu}\varepsilon_{\lambda\rho}h^{\mu\nu}h^{\lambda\rho} + O(\varkappa^{3/2}); \quad (7)$$

$$g^{\mu\nu} = \varepsilon^{\mu\nu} - \sqrt{\varkappa}h^{\mu\nu} + \tfrac{1}{2}\sqrt{\varkappa}\varepsilon^{\mu\nu}\varepsilon_{\alpha\beta}h^{\alpha\beta} - \tfrac{1}{2}\varkappa\varepsilon_{\alpha\beta}h^{\alpha\beta}h^{\mu\nu} + \tfrac{1}{4}\varkappa\varepsilon^{\mu\nu}\varepsilon_{\alpha\rho}\varepsilon_{\beta\lambda}h^{\alpha\beta}h^{\lambda\rho} + \tfrac{1}{8}\varkappa\varepsilon^{\mu\nu}\varepsilon_{\alpha\beta}\varepsilon_{\lambda\rho}h^{\alpha\beta}h^{\lambda\rho} + O(\varkappa^{3/2}); \quad (8)$$

$$g_{\mu\nu} = \varepsilon_{\mu\nu} - \tfrac{1}{2}\sqrt{\varkappa}\,\varepsilon_{\mu\nu}\varepsilon_{\alpha\beta}h^{\alpha\beta} + \sqrt{\varkappa}\,\varepsilon_{\mu\alpha}\varepsilon_{\nu\beta}h^{\alpha\beta} + \varkappa\varepsilon_{\alpha\mu}\varepsilon_{\lambda\nu}\varepsilon_{\beta\rho}h^{\alpha\beta}h^{\lambda\rho} - \tfrac{1}{2}\varkappa\varepsilon_{\alpha\mu}\varepsilon_{\beta\nu}\varepsilon_{\lambda\rho}h^{\alpha\beta}h^{\lambda\rho} + \tfrac{1}{8}\varkappa\varepsilon_{\mu\nu}\varepsilon_{\alpha\beta}\varepsilon_{\lambda\rho}h^{\alpha\beta}h^{\lambda\rho} - \tfrac{1}{4}\varkappa\varepsilon_{\mu\nu}\varepsilon_{\alpha\lambda}\varepsilon_{\beta\rho}h^{\alpha\beta}h^{\lambda\rho} + O(\varkappa^{3/2}). \quad (9)$$

By making use of the definitions of $h^{\mu}(\alpha)$ and $h_{\nu}(\alpha)$

$$g^{\mu\nu} = h^{\mu}(\alpha)\,h^{\nu}(\alpha) \text{ and } g_{\mu\nu} = h_{\mu}(\alpha)\,h_{\nu}(\alpha),$$

we find

$$h^{\mu}(\alpha) = \varepsilon^{\mu}(\alpha) + \tfrac{1}{4}\sqrt{\varkappa}\,(\varepsilon_{\omega\varepsilon}\varepsilon^{\mu}(\alpha) - 2\delta^{\mu}_{\varepsilon}\varepsilon_{\omega}(\alpha))\,h^{\omega\varepsilon} + \tfrac{1}{32}\varkappa\,(\varepsilon_{\omega\varepsilon}\varepsilon_{\sigma\tau}\varepsilon^{\mu}(\alpha) + 4\varepsilon_{\omega\sigma}\varepsilon_{\varepsilon\tau}\varepsilon^{\mu}(\alpha) - 4\varepsilon_{\omega\varepsilon}\delta^{\mu}_{\sigma}\varepsilon_{\tau}(\alpha) - 4\varepsilon_{\omega\sigma}\delta^{\mu}_{\varepsilon}\varepsilon_{\tau}(\alpha))\,h^{\omega\varepsilon}h^{\sigma\tau} + O(\varkappa^{3/2}), \quad (10)$$

$$h_{\nu}(\alpha) = \varepsilon_{\nu}(\alpha) - \tfrac{1}{4}\sqrt{\varkappa}\,(\varepsilon_{\tau\beta}\varepsilon_{\nu}(\alpha) - 2\varepsilon_{\nu\beta}\varepsilon_{\tau}(\alpha))\,h^{\tau\beta} + \tfrac{1}{32}\varkappa\,(12\varepsilon_{\tau}(\alpha)\varepsilon_{\lambda\nu}\varepsilon_{\beta\rho} - 4\varepsilon_{\tau}(\alpha)\varepsilon_{\beta\nu}\varepsilon_{\lambda\rho} + \varepsilon_{\nu}(\alpha)\,\varepsilon_{\tau\beta}\varepsilon_{\lambda\rho} - 4\varepsilon_{\lambda}(\alpha)\varepsilon_{\tau\lambda}\varepsilon_{\beta\rho})\,h^{\tau\beta}h^{\lambda\rho} + O(\varkappa^{3/2}), \quad (11)$$

where $\epsilon^{\mu}(\alpha)$ and $\epsilon_{\nu}(\alpha)$ are determined by the relations

$$\varepsilon^{\mu}(\alpha)\varepsilon^{\nu}(\alpha) = \varepsilon^{\mu\nu} \quad \text{и} \quad \varepsilon_{\nu}(\alpha)\varepsilon_{\mu}(\alpha) = \varepsilon_{\mu\nu}.$$

The Ricci rotation coefficients

$$\Delta_{\mu,\nu\sigma} = \frac{1}{2}\left(\frac{\partial g_{\mu\sigma}}{\partial x^{\nu}} - \frac{\partial g_{\mu\nu}}{\partial x^{\sigma}} - \frac{\partial g_{\sigma\nu}}{\partial x^{\mu}}\right) + h_{\nu}(\tau)\frac{\partial h_{\sigma}(\tau)}{\partial x^{\mu}} \quad (12)$$

are found by means of the expansions (9) and (11):

$$\Delta_{\mu,\nu\sigma} = \frac{\sqrt{\varkappa}}{2}\Big(-\frac{\varepsilon_{\mu\sigma}}{2}\frac{\partial h^{\alpha\beta}}{\partial x^{\nu}}\varepsilon_{\alpha\beta} + \frac{\varepsilon_{\mu\nu}}{2}\frac{\partial h^{\alpha\beta}}{\partial x^{\sigma}}\varepsilon_{\alpha\beta} + \frac{\partial h^{\alpha\beta}}{\partial x^{\nu}}\varepsilon_{\alpha\mu}\varepsilon_{\beta\sigma} - \frac{\partial h^{\alpha\beta}}{\partial x^{\sigma}}\varepsilon_{\alpha\mu}\varepsilon_{\beta\nu}\Big) + \frac{\varkappa}{2}\frac{\partial h^{\alpha\beta}}{\partial x^{\nu}}h^{\lambda\rho}\varepsilon_{\beta\rho}\Big(\varepsilon_{\alpha\mu}\varepsilon_{\lambda\sigma} + \varepsilon_{\alpha\sigma}\varepsilon_{\lambda\mu} - \frac{1}{2}\varepsilon_{\alpha\lambda}\varepsilon_{\mu\sigma}\Big) - \frac{\varkappa}{2}\frac{\partial h^{\alpha\beta}}{\partial x^{\sigma}}h^{\lambda\rho}\varepsilon_{\beta\rho}\Big(\varepsilon_{\alpha\mu}\varepsilon_{\lambda\nu} + \varepsilon_{\alpha\nu}\varepsilon_{\lambda\mu} - \frac{1}{2}\varepsilon_{\alpha\lambda}\varepsilon_{\mu\nu}\Big) - \frac{\varkappa}{8}\frac{\partial h^{\alpha\beta}}{\partial x^{\mu}}h^{\lambda\rho}\varepsilon_{\beta\rho}(\varepsilon_{\alpha\nu}\varepsilon_{\lambda\sigma} - \varepsilon_{\alpha\sigma}\varepsilon_{\lambda\nu}) + f(h^{\alpha\beta}\varepsilon_{\alpha\beta})\,\varkappa + O(\varkappa^{3/2}). \quad (13)$$

Here we do not write down explicitly terms of order κ, which contain $h^{\alpha\beta}\epsilon_{\alpha\beta}$, since these terms are not needed.

In what follows we shall use the notation of flat space, taking $x_0 = ix_0'$. Then all the indices can be written as subscripts $[\gamma(\alpha)\epsilon_{\tau}(\alpha) = \gamma_{\tau}]$. The zero order of the expansion (5) is identical with the ordinary Lagrangian of the spinor field in flat space.

For a complete description of the interaction of the gravitational and spinor fields, it is necessary to take the Lagrangian density of the sum of the spinor field in curved space and the gravitational field:

$$\mathscr{L}_{\text{int}} = \mathscr{L}_1 + \mathscr{L}_2 + \mathscr{L}_{\text{grav}}, \quad (14)$$

where $\mathscr{L}_1$ is the first order term in the expansion of (5) in a power series of $\sqrt{\kappa}$ and is of the form [2)]

$$\mathscr{L}_1 = \frac{\sqrt{\varkappa}}{4}\Big(h_{\mu\nu} + \frac{1}{2}\varepsilon_{\mu\nu}h\Big)\Big(-i\overline{\Psi}\gamma_{\mu}\frac{\partial\Psi}{\partial x^{\nu}} + i\frac{\partial\overline{\Psi}}{\partial x_{\nu}}\gamma_{\mu}\Psi\Big) + \frac{\sqrt{\varkappa}}{2}mh\overline{\Psi}\Psi. \quad (15)$$

The difference of this expression from (1) is occasioned by the series expansion of the factor $\sqrt{-g}$ [item a) of Sec. 1]. Use of the tetrad formalism does not give additional terms in the first-order expansion.

The density $\mathscr{L}_2$—the expansion of (5) of order κ—has the form

$$\mathscr{L}_2 = \frac{\varkappa}{16}(h_{\mu\alpha}h_{\nu\alpha} + \varepsilon_{\mu\nu}h_{\alpha\beta}h_{\alpha\beta})\Big(-i\overline{\Psi}\gamma_{\mu}\frac{\partial\Psi}{\partial x_{\nu}} + i\frac{\partial\overline{\Psi}}{\partial x_{\nu}}\gamma_{\mu}\Psi\Big) + \frac{\varkappa m}{4}h_{\mu\nu}h_{\mu\nu}\overline{\Psi}\Psi - \frac{i\varkappa}{16}\overline{\Psi}\Big(\gamma_{\mu}h_{\nu\rho}\frac{\partial h_{\nu\rho}}{\partial x_{\mu}} - \gamma_{\nu}h_{\mu\rho}\frac{\partial h_{\nu\rho}}{\partial x_{\mu}} - \gamma_{\nu}\gamma_{\sigma}\gamma_{\mu}h_{\sigma\rho}\frac{\partial h_{\nu\rho}}{\partial x_{\mu}}\Big)\Psi + \varkappa f(h). \quad (16)$$

The second line is due to the more accurate account of the interaction of the spinor and gravita-

[2)]The gravitational vertex contains in the general case the mass, in contrast with the conclusion reached in [8].

tional fields obtained on the basis of the tetrad formalism [item c) of Sec. 1].

$\mathscr{L}_{\rm grav}$ is the first order expansion of the Lagrangian density of the gravitational field

$$\mathscr{L}' = -\varkappa^{-1}\sqrt{-g}\,g^{\mu\nu}\left(\Gamma^{\alpha}_{\mu\beta}\Gamma^{\beta}_{\nu\alpha} - \Gamma^{\alpha}_{\mu\nu}\Gamma^{\beta}_{\alpha\beta}\right) \qquad (17)$$

and was calculated by Gupta[11] [item b) of Sec. 1]:

$$\mathscr{L}_{\rm grav} = -\frac{\sqrt{\varkappa}}{4} h_{\mu\nu}\frac{\partial h_{\alpha\beta}}{\partial x_\mu}\frac{\partial h_{\alpha\beta}}{\partial x_\nu} + \frac{\sqrt{\varkappa}}{8} h_{\mu\nu}\frac{\partial h}{\partial x_\mu}\frac{\partial h}{\partial x_\nu} - \frac{\sqrt{\varkappa}}{2} h_{\mu\nu}\frac{\partial h_{\mu\alpha}}{\partial x_\beta}\frac{\partial h_{\nu\beta}}{\partial x_\alpha} - \frac{\sqrt{\varkappa}}{4} h_{\mu\nu}\frac{\partial h}{\partial x_\alpha}\frac{\partial h_{\mu\nu}}{\partial x_\alpha} + \frac{\sqrt{\varkappa}}{2} h_{\mu\nu}\frac{\partial h_{\mu\beta}}{\partial x_\alpha}\frac{\partial h_{\nu\beta}}{\partial x_\alpha}. \qquad (18)$$

3. TWO-GRAVITON ANNIHILATION OF A PAIR OF SPINOR PARTICLES

By making use of the results of Sec. 2, we calculate by perturbation theory the effective differential cross section of the annihilation of a pair of spinor particles into two gravitons. The calculation is carried out in the center of mass system.

The S-matrix of the process of order κ has the form

$$S = i\int T\,[\mathscr{L}_2(x)]\,d^4x - \frac{1}{2}\iint T\,[\mathscr{L}_1(x_1)\,\mathscr{L}_1(x_2) + \mathscr{L}_1(x_2)\,\mathscr{L}_1(x_1) + \mathscr{L}_1(x_1)\,\mathscr{L}_{\rm rp}(x_2) + \mathscr{L}_{\rm rp}(x_1)\,\mathscr{L}_1(x_2)]\,d^4x_1\,d^4x_2. \qquad (19)$$

Contributions of the terms of the S matrix to the matrix element can be graphically represented by means of the Feynmann diagrams a—e. Part of the matrix element described in diagrams b and c (corresponding to the Hamiltonian interaction (1)), was calculated in our previous paper.[1]

The total matrix element of the process is found in the form

$$F = \frac{i\varkappa}{32k_0^3(2\pi)^2(p_1k_1)(p_1k_2)}\,\bar{u}_{\nu}(p_1)\,\{-\hat{k}_1 h^{(1)}_{\alpha\beta}h^{(2)}_{\alpha\beta}(p_1k_1)^2(p_1k_2) - 2\hat{k}_1k_0^2h^{(1)}_{\alpha}h^{(2)}_{\alpha}(p_1k_2) + 2k_0^2(p_1k_1)(p_1k_2)\,\hat{h}^{(2)}_{\alpha}h^{(1)}_{\alpha} + 4\hat{h}^{(1)}_{\alpha}h^{(2)}_{\alpha}(p_1k_1)(p_1k_2)\,k_0^2 - 4k_0^4\hat{h}^{(2)}h^{(1)} + k_0^2\hat{h}^{(2)}_{\alpha}\hat{h}^{(1)}_{\alpha}\hat{k}_1(p_1k_1)(p_1k_2) + 2k_0^4\hat{h}^{(2)}\hat{h}^{(1)}\hat{k}_1 + 4k_0^2m\,(p_1k_1)(p_1k_2)\,h^{(1)}_{\alpha\beta}h^{(2)}_{\alpha\beta}\}\,u_{\nu'}(p_2), \qquad (20)$$

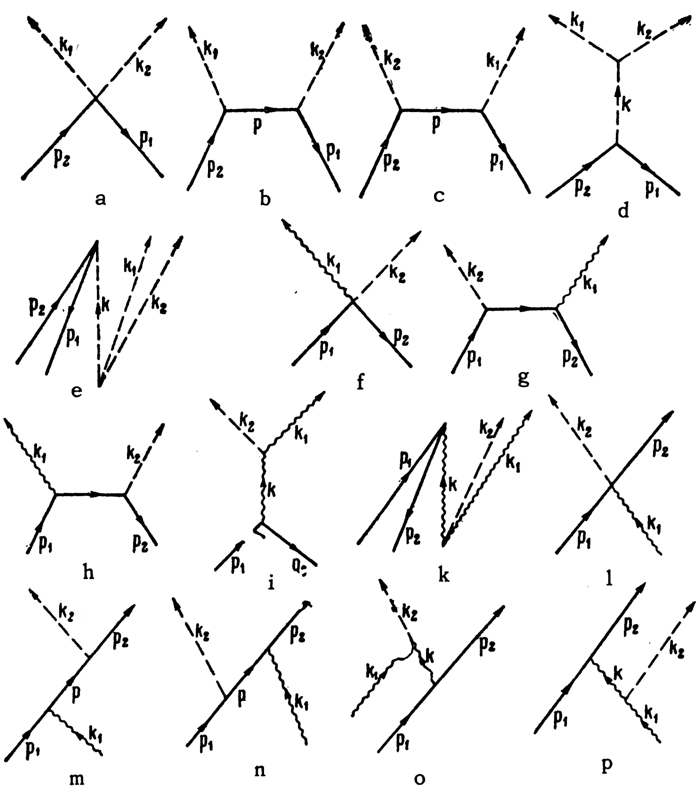
where the following notation is introduced:

$$h^{(i)} = h^{(i)}_{\alpha\beta}p_1^{\alpha}p_1^{\beta}; \quad \hat{h}^{(i)} = h^{(i)}_{\alpha\beta}\gamma_\alpha p_1^{\beta};$$

$\bar{u}_{\bar{\nu}}(p_1)$ and $\bar{u}_{\nu'}(p_2)$ are factors corresponding to the antiparticle and the particle in the initial state.

After tedious calculations, which correspond to taking the trace of the matrix element, summation over the polarizations of finite gravitons and gathering of like terms, we get

$$d\sigma = \frac{\varkappa^2}{128(4\pi)^2pk_0}\Big[p^2m^2\cos^2\theta + p^4\sin^2\theta + 2p^4\sin^2\theta\cos^2\theta + \frac{1}{2}(m^2 - p^2\sin^2\theta)^2 + \frac{3p^2m^4\cos^2\theta + m^2p^2(m^2 - p^2\sin^2\theta)}{m^2 + p^2\sin^2\theta} - \frac{p^8\sin^8\theta}{(m^2 + p^2\sin^2\theta)^2}\Big]\,d\Omega, \qquad (21)$$



—— Fermion line ---- Graviton line ~~~ Photon line

where p is the momentum of the fermion, θ is the angle between the direction of motion of the initial particles and the momentum of the graviton.

In the classical case, when $k_0^2 \sim m^2 \gg p^2$, this formula takes the form

$$d\sigma_{\rm cl} = \frac{m^2\varkappa^2}{64(8\pi)^2}\,\frac{c}{v}\,d\Omega, \qquad (22)$$

that is, the radiation of gravitons is equally probable in all directions.

In the ultrarelativistic case, in which $p^2 \sim k_0^2 \gg m^2$, we have

$$d\sigma_{\rm ur} = \frac{\varkappa^2k_0^2}{128(8\pi)^2}(3\sin^2 2\theta + 2\sin^4\theta)\,d\Omega. \qquad (23)$$

There will be no radiation of gravitons in the directions of propagation of the initial particles.

From (21), it is easy to obtain a similar formula for the annihilation of two four-component neutrinos ($m = 0$) into two gravitons.[6]

4. LAGRANGIAN DENSITY OF INTERACTION OF SPINOR, GRAVITATIONAL AND ELECTROMAGNETIC FIELDS

In Eq. (5), in the presence of an electromagnetic field, an added contribution appears that is brought about by the replacement of the ordinary derivatives of the wave functions by the expressions

$$\frac{\partial \Psi}{\partial x^\mu} \rightarrow \frac{\partial \Psi}{\partial x^\mu} - ieA_\mu \Psi; \quad \frac{\partial \overline{\Psi}}{\partial x^\mu} \rightarrow \frac{\partial \overline{\Psi}}{\partial x^\mu} + ieA_\mu \overline{\Psi}.$$

This contribution has the form

$$\sqrt{-g}\, e\overline{\Psi}\gamma(\alpha)\, h^\mu(\alpha)\, \Psi A_\mu. \tag{24}$$

Expanding this expression in powers in $\sqrt{\kappa}$, we get

$$eA_\mu \overline{\Psi}\gamma_\mu \Psi - \tfrac{1}{4} e\sqrt{\varkappa}\, \overline{\Psi}\, (\gamma_\mu A_\mu h + 2\gamma_\alpha h^{\alpha\mu} A_\mu)\, \Psi + O(\varkappa). \tag{25}$$

The Lagrangian density of the electromagnetic field in a gravitational field has the form

$$\mathcal{L} = -\tfrac{1}{4}\sqrt{-g}\, g^{\mu\lambda} g^{\nu\rho} F_{\lambda\rho} F_{\mu\nu} = -\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} + \tfrac{1}{2}\sqrt{\varkappa}\, (\varepsilon^{\mu\lambda} h^{\nu\rho} - \tfrac{1}{4}\varepsilon^{\nu\rho}\varepsilon^{\mu\lambda} h)\, F_{\lambda\rho} F_{\mu\nu} + O(\varkappa). \tag{26}$$

We write down the expansion in powers of $\sqrt{\kappa}$ of the Lagrangian interaction density of the three fields, neglecting terms of order higher than $e\sqrt{\kappa}$. By taking into account (25), (26), and (14), we get

$$\begin{aligned}\mathcal{L}_{\text{int}} = {} & e\overline{\Psi}\gamma_\mu \Psi A_\mu - \tfrac{1}{4} e\sqrt{\varkappa}\, \overline{\Psi}\gamma_\mu \Psi A_\mu h - \tfrac{1}{2} e\sqrt{\varkappa}\, \overline{\Psi}\gamma_\alpha \Psi A_\mu h_{\alpha\mu} \\ & + \tfrac{1}{2}\sqrt{\varkappa} F_{\mu\rho} F_{\mu\nu} h_{\nu\rho} - \tfrac{1}{8}\sqrt{\varkappa} F_{\mu\nu} F_{\mu\nu} h + \tfrac{1}{2}\sqrt{\varkappa}\, m\overline{\Psi}\Psi h \\ & + \tfrac{1}{4}\sqrt{\varkappa}\, (h_{\mu\nu} + \tfrac{1}{2}\varepsilon_{\mu\nu} h) \\ & \times \left(-i\overline{\Psi}\gamma_\mu \frac{\partial \Psi}{\partial x_\nu} + i\frac{\partial \overline{\Psi}}{\partial x_\nu}\gamma_\mu \Psi\right) + \sqrt{\varkappa}\, f(3h) + O(\varkappa),\end{aligned} \tag{27}$$

where $\sqrt{\kappa}\, f(3h)$ are terms containing the product of three gravition functions.

5. PHOTON-GRAVITON PAIR ANNIHILATION OF SPINOR PARTICLES

In addition to the process of two-graviton pair annihilation of spinor particles, a transformation of this pair into a photon and a graviton is possible. In calculation of the cross section of two-graviton annihilation, averaging was carried out over the spins of the initial particles. However, if one does not do this and assumes that the initial particles are arbitrary, then it is easy to see that such a process is possible only for antiparallel polarization of the colliding particles. For identical polarization, the particles can annihilate with production of a photon and a graviton. Let us construct the matrix element of such a process, using the Lagrangian interaction density (27). In the general case, the matrix element of the process can be represented by means of the diagrams f—k. If the calculation is in the center-of-mass system, the contributions of the diagrams f, i, and k mutually cancel one another.

The matrix element of the process has the form

$$F = -\frac{e\sqrt{\varkappa i}}{2(4\pi)^2 k_0}\frac{1}{(p_1 k_1)(p_1 k_2)}\, \bar{u}_\nu(\mathbf{p}_2)\, [2k_0^2(\hat{h}e + \hat{e}h) + (h_\mu e_\mu)\, \hat{k}_1 v]\, u_{\nu'}(\mathbf{p}_1), \tag{28}$$

where

$$e = e_\mu p_1^\mu; \quad \hat{h} = h_{\mu\upsilon} p_1^\mu \gamma_\upsilon; \quad (h_\mu e_\mu) = h_{\mu\upsilon} p_1^\upsilon e_\mu;$$

$v = (p_1 k_2) - (p_1 k_1)$ is the square of the matrix element, and averaging over the spins of the initial particles and summing over the spins of the final particles, we obtain the differential effective cross section of the process of photon-graviton annihilation:

$$d\sigma = \frac{e^2\varkappa}{4(8\pi)^2}\frac{p^3 \sin^2\theta}{k_0(k_0^2 - p^2\cos^2\theta)}\left[1 + \sin^2\theta + \frac{\sin^2\theta\,(k_0^2 - 2p^2\sin^2\theta)}{k_0^2 - p^2\cos^2\theta}\right] d\Omega. \tag{29}$$

In the classical case $p^2 \ll k_0^2 \sim m^2$, we have

$$d\sigma_{\text{cl}} = \frac{e^2\varkappa}{4(8\pi)^2}\left(\frac{v}{c}\right)^3 \sin^2\theta\,(1 + 2\sin^2\theta)\, d\Omega. \tag{30}$$

Because of the factor $\sin^2\theta$, the maximum of radiation of gravitons and photons will be observed at $\theta = \pi/2$. There will be no radiation in the directions of the momenta of the initial particles.

In the ultrarelativistic case ($p^2 \sim k_0^2 \gg m^2$) we have

$$d\sigma_{\text{ur}} = \frac{e^2\varkappa}{4(8\pi)^2}(1 + \cos^2\theta)\, d\Omega, \tag{31}$$

i.e., the cross section of the process ceases to depend on the energy of the particles, while the radiation of gravitons will be strongest in the directions of the momenta of the original particles. In the ultrarelativistic case, the spins of the particle and antiparticle will be oriented in a single direction, which gives evidence of the necessity in such a case of using Eq. (31) and not (23).

There is interest in making a comparison of the total cross sections of the processes of two-graviton and photon-graviton annihilation of two spinor particles.

1. Classical case. The total cross section of two-graviton annihilation has the form

$$\sigma_{cl} = r_g^2 c/2048\pi v, \qquad r_g = \varkappa m, \tag{32}$$

in contrast with the result of our previous paper, where $\sigma_{cl} \sim r_g^2 c v^{-1}(E/mc^2)$ (E is the nonrelativistic energy).

Since, in calculation of the number of acts of annihilation, the cross section of the process is multiplied by the velocity v, the number of annihilating particles does not depend on v and is proportional to m^2.

The total cross section of photon-graviton annihilation has the form

$$\sigma'_{cl} = \frac{13 r_e r_g}{480\pi}\left(\frac{v}{c}\right)^3, \quad r_e = \frac{e^2}{mc^2}. \tag{33}$$

2. Ultrarelativistic case. The total cross section of the two-graviton annihilation

$$\sigma_{ur} = \frac{\varkappa^2 k_0^2}{768\pi} = \frac{r_g^2}{768\pi}\left(\frac{k_0}{mc^2}\right)^2 \tag{34}$$

increases in proportion to k_0^2. The cross section of the two-graviton annihilation becomes of the order of the cross section of two-photon annihilation for $k_{0cr} \sim \sqrt{r_e/r_g}\, mc^2$. For electrons,[3)] $k_{0cr} = 10^{21} mc^2$. The total cross section of the photon-graviton annihilation is

$$\sigma'_{ur} = r_e r_g/48\pi = 1{,}26 \cdot 10^{-68} \text{ cm}^2 \tag{35}$$

depends neither on the energy nor on the mass of the colliding particles. The cross section of two-photon annihilation becomes of the order of the cross section of photon-graviton annihilation for the same energies as in the previous case:

$$k'_{0cr} \sim k_{0cr} \sim \sqrt{r_e/r_g}\, mc^2. \tag{36}$$

6. THE COMPTON EFFECT OF A PHOTON ON A SPINOR PARTICLE WITH REPLACEMENT BY A GRAVITON

The matrix element of the process of a Compton effect of a photon on a spinor particle with replacement by a graviton can be represented graphically in the form of diagrams l–p. We shall consider the case in which $E_1 \gg k_{01}$, $E_1 \gg k_{02}$ (E_1 is the initial energy of the fermion, k_{01} is the energy of the photon, k_{02} is the energy of the graviton), then the matrix element has the form (the principal contribution is made by diagrams m and n)

$$F = \frac{ie\sqrt{\varkappa}}{(4\pi)^2\sqrt{k_{01}k_{02}}}\, \bar{u}_\nu^+ (p_2) \frac{(p_1k_2)\,\hat{h}\,(p_1^\alpha e_\alpha) - (p_1k_1)\,\hat{e}}{(p_1k_1)\,(p_1k_2)}\, u_{\nu'}^- (p_1). \tag{37}$$

The differential effective cross section of the process has the form

$$d\sigma = \frac{(2\pi)^2 k_{02}^2 E_2}{E_2 - p_1\cos\theta} \sum |F(p_1, p_2)|^2 \, d\Omega. \tag{38}$$

After squaring Eq. (37), averaging over the initial and summing over the final spins, we obtain

$$d\sigma = \frac{e^2\varkappa p^6 \sin^4\theta \sin^2\varphi}{8(4\pi)^2 E_1 (E_2 - p_1\cos\theta)} \frac{k_{02}}{k_{01}} \frac{[p_1(k_2 - k_1)]^2}{(p_1k_1)^2 (p_1k_2)^2}\, d\Omega, \tag{39}$$

where φ is the angle between $\mathbf{k}_1$ and $\mathbf{p}_1$, θ is the angle between $\mathbf{k}_2$ and $\mathbf{p}_1$.

In the relativistic case, where $E \sim E_1 \sim E_2 \sim p_1 \sim p_2$, setting $k_1 \sim k_2 \sim k$, we get

$$\sigma \sim e^2 \varkappa (E/k)^2, \tag{40}$$

however, it must be noted that a difficulty appears for k that is similar to the case of pure electrodynamics (the infrared catastrophe), associated with the inapplicability of perturbation theory methods in this case.

Equations (39) and (40) also hold for the reverse process—the Compton effect of a graviton by a fermion with transformation into a photon. If it is assumed that a significant quantity of gravitational waves (gravitons) exists in outer space, then the effect of transformation of gravitons into photons described above must take place upon interaction of fluxes of gravitons with matter.[4)]

In particular, this effect must lead to additional radiation of photons from outer space by objects having velocity components normal to the radius vector from the earth to the given object (owing to $\sin^4\theta$ in the numerator). If this effect could be measured, then it, together with the Doppler (radial) red shift, would permit us to obtain a complete estimate of the velocity and direction of motion of cosmic objects.

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[3)] In our previous paper, an error was made: it was stated that $k_0 \approx 10^{12}$ mc^2.

[4)] In the general case, the electrons and protons of matter possess different energies. For the special case of a neutral isothermal plasma, in which the electrons and protons have the same energy, the presence of the charge e in (35) leads to the absence of the effect discussed.

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