ANOMALOUS RESONANCE SPIN FLIP OF A PARTICLE IN A MAGNETIC FIELD

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It is shown that if a particle rotates in a constant magnetic field, a weak high frequency radial or azimuthal magnetic field with a frequency equal to the anomalous part of the spin precession frequency should induce resonance spin flip.

IN some experimental investigations [1] the magnetic moments of particles were measured by a high frequency method, in which resonant flip of a spin precessing in the main magnetic field is observed if the frequency of the perturbation is equal to the precession frequency. It will be shown below that such a resonance is possible also in the case when the perturbation frequency is equal not to the entire precession frequency, but only to its anomalous part, which in turn is equal to

$$\Omega_0 = \gamma\omega E/mc^2, \quad \gamma = (g-2)/2 = \Delta\mu/\mu,$$

where μ is the magnetic moment of the particle, g the gyromagnetic ratio, ω the rotation frequency, and E the total energy. It is essential here that the particle must rotate in the magnetic field along an orbit of finite radius ρ, so that it is possible to produce a high frequency field directed either along the radius or along the particle rotation.

Resonance is ensured by the radial or azimuthal component of the perturbing high-frequency magnetic field, and only by its average along the orbit. The resonance is connected with the fact that the projection of the spin on the plane of rotation precesses around the orbit with frequency Ω_0; if the angle between the high-frequency field lying in the orbit plane and the tangent to the orbit also varies in time with frequency Ω_0, then we can expect a change in the orientation of the spin relative to the magnetic field.

This resonance, obviously, makes it possible to increase by approximately 700 times the accuracy with which the magnetic moments of the electron and muon are measured by the high-frequency method, since the anomalous part of the magnetic moment is measured directly. The idea of measuring the g-factor of the electron with the aid of the possible anomalous resonance was advanced by M. Vishnevskiĭ (a private communication from whom served as the impetus for the present calculation). In practice it is more convenient, apparently, to use for the resonant spin flip only the radial component H_r of the high-frequency field. This case is considered in the present paper.

Let the particle rotate in the main field H_0 along an orbit of radius ρ. Turning on the radial high-frequency field leads to the appearance of additional fields, in accordance with Maxwell's equations. Taking into account the fact that the high frequency, close to the frequency Ω_0 of the spin precession, is small compared with the rotational frequency and that the deviations of the particle from the equilibrium are small, so that $\Omega^2z^2/c^2 \ll 1$ and $\Omega^2r^2/c^2 \ll 1$, with $r = R-\rho$, where z is the vertical deviation and R the radial coordinate of the particle, we obtain the following approximate expressions for the fields:

$$H_z = H_0(1 - \delta z\rho^{-1}\cos\Omega_1 t), \quad H_r = H_0\delta\cos\Omega_1 t,$$
$$E_x = H_0\Omega_1 c^{-1} z\delta\sin\Omega_1 t, \quad H_x = E_r = E_z = 0, \tag{1}$$

where $\delta \sim H_r/H_z \ll 1$, and x is the coordinate along the circular orbit. It turns out that it is necessary to take into account in the equation of spin motion the terms $\sim \delta^2$; since z is of order δ and r of order δ^2, it is sufficient to retain in (1) the terms linear in r.

The equations of motion of the spin have the following general form [2]:

$$DS^i/D\tau = (e/mc)(1+\gamma)F^i_kS^k + \gamma(e/mc)F^n_mS^mu_nu^i, \tag{2}$$

where S^i is the four-vector of the spin and $\tau = tmc^2/E$ the proper time,

$$u^i = dx^i/cd\tau, \quad x^1 = x, \quad x^2 = r, \quad x^3 = z, \quad x^0 = ct,$$

F^i_k —the electromagnetic field tensor

$$DS^i/D\tau = dS^i/d\tau + \Gamma^i_{kl}u^kS^l$$

—covariant derivative, Γ^i_{kl} —Christoffel symbols (see, for example, [3]).

The equations of motion for the particle have the form

$$\dot{u}^1 + \frac{2\dot{r}}{\rho(1+r/\rho)} u^1 = \frac{\omega}{1+r/\rho}\Big\{\frac{\Omega}{c}\frac{mc^2}{E}\delta z u^0 \sin\Omega\tau + \Big(1-\delta\frac{z}{\rho}\cos\Omega\tau\Big)u^2 - \delta u^3\cos\Omega\tau\Big\},$$

$$\dot{u}^2 - \frac{c}{\rho}\Big(1+\frac{r}{\rho}\Big)(u^1)^2 = -\omega\Big(1+\frac{r}{\rho}\Big)\Big(1-\delta\frac{z}{\rho}\cos\Omega\tau\Big)u^1,$$

$$\dot{u}^3 = \omega\delta\Big(1+\frac{r}{\rho}\Big)u^1\cos\Omega\tau,$$

$$\dot{u}^0 = \omega\frac{\Omega}{c}\frac{mc^2}{E}\delta z\Big(1+\frac{r}{\rho}\Big)u^1\sin\Omega\tau,$$

$$\Omega = \Omega_1 E/mc^2, \qquad \omega = eH_0/mc, \qquad \dot{u}^i = du^i/d\tau. \tag{3}$$

From this we obtain, accurate to terms $\sim\delta^2$, the forced oscillations of the particle, which vanish when $\delta = 0$:

$$r = -\delta^2\omega^2\Omega^{-2}\rho, \qquad z = \delta\omega^2\Omega^{-2}\rho\cos\Omega\tau;$$

$$u^1 = p/mc\,(1+\delta^2\omega^2\Omega^{-2}\cos^2\Omega\tau),$$

$$u^0 = E/mc^2\,(1-\tfrac{1}{2}\delta^2\omega^2\Omega^{-2}(pc/E)^2\sin^2\Omega\tau),$$

$$u^2 = 0, \qquad u^3 = \delta(\omega p/\Omega mc)\sin\Omega\tau. \tag{4}$$

Here E is the equilibrium energy, that is, the particle energy when $\delta = 0$.

Substituting (1) and (4) in (2), we obtain after rather prolonged manipulations the following equations for the spin:

$$\dot{S}^1 - (E/mc^2)\Omega_0 S^2 = A_{11}\delta^2 S^1\sin 2\Omega\tau + (A_{12}^1\delta^2 + A_{12}^2\delta^2\cos 2\Omega\tau)S^2 + A_{13}\delta S^3\cos\Omega\tau,$$

$$\dot{S}^2 + (mc^2/E)\Omega_0 S^1 = A_{21}\delta^2(1-\cos 2\Omega\tau)S^1,$$

$$\dot{S}^3 = A_{31}\delta S^1\cos\Omega\tau + A_{32}\delta S^2\sin\Omega\tau + A_{33}\delta^2 S^3\sin 2\Omega\tau, \tag{5}$$

where

$$\Omega_0 = \gamma\omega E/mc^2, \quad A_{11} = -\gamma(\omega^2/2\Omega)(pc/E^2)[1-\gamma(E/mc^2)^2],$$

$$A_{12}^1 = \frac{1}{2}\gamma\omega^3\Omega^{-2}[2+(E/mc^2)^2],$$

$$A_{12}^2 = \frac{1}{2}\gamma\omega^3\Omega^{-2}[3(E/mc^2)^2 - 2],$$

$$A_{13} = -\omega[1+\gamma(E/mc^2)^2], \qquad A_{21} = \gamma\omega^3/2\Omega^2,$$

$$A_{31} = \omega(1+\gamma), \qquad A_{32} = \gamma(p/mc)^2\omega^2/\Omega,$$

$$A_{33} = -\gamma(p/mc)^2\omega^2/2\Omega.$$

In these equations we have used the connection between the spin components and the four-dimensional velocity: $u^i S_i = 0$.

We have thus obtained a system of equations (5) with periodic coefficients of period $T = 2\pi/\Omega$. Such a system is solved in accordance with the well-known scheme[4]. It is first necessary to find the fundamental system of three linearly independent solutions, determined by the initial conditions:

$$S_k^i(0) = \delta_k^i \equiv \begin{cases}1, & i=k\\ 0, & i\neq k\end{cases}, \tag{6}$$

i —number of spin projection, k —number of the solution. It is sufficient to find the values of $S_k^i(T)$ —solutions at the instant of time τ, differing from the initial time by one period T.

The development of any solution in time can be represented in matrix form:

$$\begin{pmatrix} S^1(\tau)\\ S^2(\tau)\\ S^3(\tau)\end{pmatrix} = \begin{pmatrix} S_1^1(\tau), & S_2^1(\tau), & S_3^1(\tau)\\ S_1^2(\tau), & S_2^2(\tau), & S_3^2(\tau)\\ S_1^3(\tau), & S_2^3(\tau), & S_3^3(\tau)\end{pmatrix}\begin{pmatrix} S^1(0)\\ S^2(0)\\ S^3(0)\end{pmatrix}. \tag{7}$$

According to the Floquet theorem (see, for example,[4]) the general solution of (5) can be written in the form

$$S^k(\tau) = \sum_{n=1}^{3} e^{i\alpha_n\tau 2\pi/T}\varphi^{k(n)}(\tau)N^k, \qquad N^k = \begin{cases}E/mc^2, & n=1\\ 1, & n\neq 1\end{cases}, \tag{8}$$

where $\varphi^{k(n)}(\tau) = \varphi^{k(n)}(\tau+T)$ is some periodic function, and α_n is the quasi-frequency (analogous to the quasi-momentum of the electron in a periodic field), which in this case has the meaning of the number of slow oscillations of the spin during the period T.

Introducing $\lambda_n = \exp(2\pi i\alpha_n)$ we have the following characteristic equation for the determination of the three roots λ:

$$|S_k^i(T) - \lambda\delta_k^i| = 0. \tag{9}$$

This equation is an obvious consequence of the Floquet theorem and of Eq. (7). After finding λ_n, we obtain for each λ_n its own function $\varphi^{k(n)}(0)$ from the following equations:

$$\sum_{k=1}^{3}(S_k^i(T) - \lambda_n\delta_k^i)\varphi^{k(n)}(0) = 0, \quad i = 1,2,3, \quad n = 1,2,3. \tag{10}$$

Using perturbation theory accurate to δ^2, we have obtained for the case of exact resonance $\Omega = \Omega_0$

$$S_1^1(T) = 1 - \delta^2\pi^2 J/\Omega_0^2, \qquad S_2^1(T) = \delta^2 M\pi/\Omega_0,$$

$$S_3^1(T) = \delta A_{13}\pi mc^2/\Omega_0 E, \qquad S_1^2(T) = \delta^2(K + 3J/2\Omega_0)\pi/\Omega_0,$$

$$S_2^2(T) = 1, \quad S_3^2 = 0, \quad S_1^3(T) = \delta(A_{31}E/mc^2 - A_{32})\pi/\Omega_0,$$

$$S_2^3 = 0, \quad S_3^3 = 1 - \delta^2\pi^2 J/\Omega_0^2, \tag{11}$$

where

$$J = \frac{\omega^2}{2}\frac{1+\gamma(E/mc^2)^2}{(E/mc^2)^2},$$

$$M = \frac{1}{2}\Big[A_{11} + \frac{mc^2}{E}A_{12}^2 + 2\frac{mc^3}{E}A_{12}^1 - 3\frac{E}{mc^2}A_{21}\Big],$$

$$K = \frac{1}{2}\times\Big[A_{11} + \frac{A_{13}}{4\Omega_0}\frac{mc^2}{E}\Big(5\frac{E}{mc^2}A_{31} + 3A_{32}\Big) + \frac{mc^2}{E}(A_{12}^2 - 2A_{12}^1) + \frac{E}{mc^2}A_{21}\Big].$$

Equation (9) assumes the form

$$(\lambda - 1)\,[\lambda^2 - 2\,(1 - \delta^2 J\pi^2 / \Omega_0^2)\,\lambda + 1] = 0. \quad (12)$$

This yields (under the condition $\delta^2 J\pi^2/\Omega_0^2 < 1$, that is, $\pi^2\delta^2/2\gamma^2(E/mc^2)^4 < 1$)

$$\lambda_1 = 1, \quad \lambda_{2,3} = 1 \pm i\delta\pi\sqrt{2J}\,/\,\Omega_0. \quad (13)$$

When $\lambda = 1$ the value of $\varphi^{2(1)}(0)$ is arbitrary,

$$\varphi^{1(1)}(0) \sim \delta^2\varphi^{2(1)}(0), \quad \varphi^{3(1)}(0) \sim \delta\varphi^{2(1)}(0);$$

and when $\lambda = \lambda_{2,3}$

$$\varphi^3(0) = \mp i\varphi^1(0), \quad \varphi^2(0) \sim \delta\varphi^1(0).$$

It is meaningful to consider the orientation of the spin at discrete instants of time with interval $\Delta t = T$; numbering them by the index p, which has the meaning of the number of spin revolutions around the momentum, we obtain

$$S_p^1 = \frac{E}{mc^2} a \cos p\,\beta, \quad S_p^2 = b, \quad S_p^3 = a \sin p\,\beta; \quad a^2 + b^2 = 1,$$

$$\beta = \delta T\sqrt{J/2} \approx \delta T\omega/(2E/mc^2) = \pi\delta/\gamma\,(E/mc^2)^2. \quad (14)$$

Here S_p^2 is constant only because the spin is considered at the discrete instants of time indicated above; in the interval between them, S_2 and S_1 obviously rotate with frequency Ω_0.

Formulas (14) show that under the influence of the high frequency field H_r, the spin rotates slowly with frequency $\sim\delta$ in a vertical plane when the frequencies are equal. When the frequencies Ω and Ω_0 are not equal, we obtain the usual resonance curve with half-width $\Delta\Omega \sim \delta\omega$.

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