ELECTRICAL CONDUCTIVITY AND GALVANOMAGNETIC COEFFICIENTS OF SEMIMETALS AND DEGENERATE SEMICONDUCTORS IN A STRONG ELECTRIC FIELD

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It is shown that the electrical conductivity and galvanomagnetic coefficients of semimetals and degenerate semiconductors in a strong electric field are considerably modified if the phonon system is not in equilibrium. The lack of phonon equilibrium is manifested by the "heating" of phonons (increase of the number of long-wavelength phonons in a strong electric field) and by the "mutual" drag of electrons and phonons. The first circumstance leads to a decrease of the path length of electrons scattered by phonons when the field strength is increased, and is the cause of the dependence of the electrical conductivity on the field strength in the zeroth approximation with respect to degeneracy. In a strong magnetic field the electrical conductivity increases at first with increase of the electric field, then reaches a maximum, and at sufficiently high field strengths decreases in inverse proportion to the field strength and is independent of the magnetic field strength; the current on the other hand increases monotonically and approaches saturation. The Hall conductivity decreases with increase of the electric field E and in a sufficiently strong field is $\sim E^{-2}$, whereas the Hall current has a maximum (Fig. 3). A coefficient β, representing the deviation from Ohm's law in a weak electric field [cf. Eq. (3.4)], is negative in a weak magnetic field, changes its sign with increase of the field and approaches zero in strong magnetic fields. The "mutual" drag of electrons and phonons results in a considerable increase of the electron path length. This leads to a decrease of the electric field at which the current saturates.

1. INTRODUCTION

THE electron temperature and electrical conductivity of metals in a strong electric field have been dealt with in [1-5]. In the present work we shall show that the phenomena in a strong electric field are considerably altered if the phonon system is not in equilibrium, in particular if there is considerable drag of phonons by electrons. It is difficult to establish a strong electric field in metals. For this purpose it is more convenient to study crystals of lower electrical conductivity. We investigated semimetals and semiconductors with degenerate electrons.

Electrons are "heated" by an electric field which, however, at realizable field intensities cannot eliminate the degeneracy and increases only slightly the diffuseness of the Fermi surface. Therefore the isotropic part of the electron distribution function $n^{(i)}(\epsilon - \zeta)$, where ζ is the chemical potential, remains almost a step function, i.e.,

$$\partial n^{(i)}/\partial \varepsilon = -\delta(\varepsilon - \zeta), \quad n^{(i)}(1 - n^{(i)}) = T(E)\,\delta(\varepsilon - \zeta), \quad (1.1)$$

where $T(E)$ is some characteristic energy having the meaning of the electron temperature in an electric field E. (The crystal temperature will be denoted by T.)

Mutual interaction between the electrons alters only the details of the electron distribution near the Fermi surface, making it approach the equilibrium distribution corresponding to the temperature $T(E)$.

Electrons in the crystals considered interact only with long-wavelength phonons whose momentum is of the order of or less than the electron. The fraction of these phonons is of the order of $(sp_F/T)^3$, where s is the velocity of sound, and p_F is the electron momentum at the Fermi surface. We shall consider temperatures at which this fraction is small, i.e., temperatures above 1—10°K, depending on the effective electron mass. Long-wavelength phonons obtain energy from electrons "heated" by the electric field, and transfer this energy to the main body of phonons which acts as a thermal reservoir.

The relative heating of long-wavelength phonons, and consequently the relative increase of their number $\Delta N(\mathbf{q})/N^{(0)}(\mathbf{q})$, where $N^{(0)}(\mathbf{q})$ and $N(\mathbf{q})$ are the equilibrium and non-equilibrium distribution functions of long-wavelength phonons, is proportional to the relative "heating" of electrons $[T(E) - T]/T \equiv \Delta T/T$ and the ratio of the phonon paths L_{fe} (energy transfer to the reservoir and to electrons) and L_e (energy obtained from electrons):

$$\frac{N(\mathbf{q}) - N^{(0)}(\mathbf{q})}{N^{(0)}(\mathbf{q})} = \frac{L_{fe}}{L_e}\frac{T(E) - T}{T}.$$

The quantities L_f and L_e are independent of the electric field—the former because long-wavelength phonons are scattered on the reservoir phonons, and the latter because the electron distribution is almost unchanged.

Increase of the number of long-wavelength phonons leads to an increase in the same ratio, of the probability of electrons being scattered by these phonons, and consequently decreases the path length of electrons scattered by phonons l_f. If electrons are scattered mainly by phonons, then on strong heating of phonons, when $\Delta N(\mathbf{q})/N^{(0)}(\mathbf{q}) \gg 1$, the electrical conductivity σ becomes proportional to $(L_{fe}\Delta T/L_e T)^{-1}$. In a strong electric field $\Delta T/T \sim E$ and $\sigma \sim E^{-1}$.

In the presence of a magnetic field the dependence of the electrical conductivity on the electric field is more complex. This is because in the case of strong degeneracy $\sigma(E, H) = \sigma(E, 0)/[1 + (l/R)^2]$, where R is the Larmor radius and l is the mean free path of electrons at the Fermi surface. In a strong magnetic field ($l \gg R$) when $l \approx l_f$ the electrical conductivity is $\sigma(E, H) \sim \sigma(E, 0)/l_f^2 \sim 1/l_f$. On increase of the electric field l_f decreases and therefore $\sigma(E, H)$ increases until the decreasing path length l_f reaches the value R. On further increase of the electric field $\sigma(E, H) \sim l_f$, i.e., the conductivity decreases. Thus $\sigma(E, H)$ as a function of the electric field has a maximum but the electric current $j_1(E)$ increases monotonically, approaching saturation (Figs. 1, 2). The Hall conductivity $\sigma'(E, H)$ decreases monotonically with increase of the electric field. In a strong electric field (when $l_f < R$) $\sigma' \sim l^2 \sim E^{-2}$, so that the Hall current decreases with increase of the electric field intensity (Fig. 3).

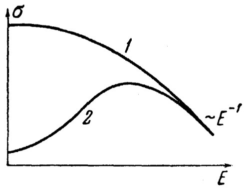

FIG. 1. Dependence of the electrical conductivity on the electric field in mutually perpendicular magnetic and electric fields: 1) $l_1/R \ll 1$; 2) $l_1/R \gg 1$. The maximum occurs at $E \approx [6L_{fd}L_e/L_f L]^{1/2}(sH/c)$; $l_1/R \approx H$.

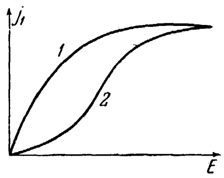

FIG. 2. Dependence of the conduction current $j_1(E)$ on the electric field: 1) $l_1/R \ll 1$; 2) $l_1/R \gg 1$; $l_1/R \sim H$.

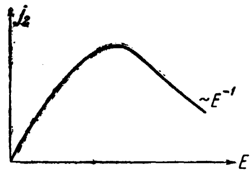

FIG. 3. Dependence of the Hall current $j_2(E)$ on the electric field in mutually perpendicular electric and magnetic fields. The maximum is reached at $E = [3L_{fd}L_e/L_f L]^{1/2}(sH/c)$.

In a weak electric field

$$\sigma(E, H) = \sigma(0, H)\,[1 + \beta(H)E^2].$$

The coefficient $\beta(H)$ is negative at $H = 0$, rises algebraically with increase of H, becomes positive and, having reached a maximum, decreases in strong fields to zero (Fig. 4).

Apart from phonon "heating" the non-equilibrium state is also manifested by another process which appears most simply in the absence of a magnetic field. It is the "mutual" drag of electrons and phonons, which occurs as follows: phonons acquire momentum from electrons and form a directed current, the drift velocity of which may approach the drift velocity of the electron current if electrons and phonons are scattered mainly on one another [6]. In the case of such parallel motion the mean free path of the electrons may increase considerably. This leads to a reduction of the electric field intensity at which the field can be regarded as strong, i.e., at which the current reaches saturation.

In the presence of a magnetic field the situation becomes more complicated because the behavior of the system then depends on two parameters.

Concluding, we note that we are not studying here the role of optical phonons, which may be considerable and which deserves separate consideration, and moreover we shall limit ourselves to non-quantizing magnetic fields.

2. SYSTEM OF EQUATIONS FOR ELECTRONS AND PHONONS AND ITS GENERAL SOLUTION

The system of transport equations for electrons and phonons has the form

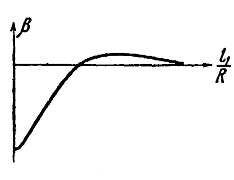

FIG. 4. Dependence of $\beta(H)$ on the ratio $l_1/R \sim H$ in mutually perpendicular electric and magnetic fields.

$$e\left(\mathbf{E} + \frac{1}{c}[\mathbf{vH}]\right)\frac{\partial n_{\mathbf{p}}}{\partial \mathbf{p}} = \sum_{\mathbf{q}} W_{\mathbf{q}} \{[n_{\mathbf{p}+\mathbf{q}}(1-n_{\mathbf{p}}) \times (N_{\mathbf{q}}+1) - n_{\mathbf{p}}(1-n_{\mathbf{p}+\mathbf{q}})N_{\mathbf{q}}]\,\delta(\varepsilon_{\mathbf{p}+\mathbf{q}}-\varepsilon_{\mathbf{p}}-\hbar\omega_{\mathbf{q}}) + [n_{\mathbf{p}-\mathbf{q}}(1-n_{\mathbf{p}})N_{\mathbf{q}} - n_{\mathbf{p}}(1-n_{\mathbf{p}-\mathbf{q}})(N_{\mathbf{q}}+1)]\,\delta(\varepsilon_{\mathbf{p}-\mathbf{q}} - \varepsilon_{\mathbf{p}}+\hbar\omega_{\mathbf{q}})\} - v\,(n_{\mathbf{p}} - n_{\mathbf{p}}^{(i)})/l_d; \qquad (2.1)^*$$

$$\sum_{\mathbf{p}} W_{\mathbf{q}}\,[n_{\mathbf{p}+\mathbf{q}}(1-n_{\mathbf{p}})(N_{\mathbf{q}}+1) - n_{\mathbf{p}}(1-n_{\mathbf{p}+\mathbf{q}})N_{\mathbf{q}}] \times \delta(\varepsilon_{\mathbf{p}+\mathbf{q}}-\varepsilon_{\mathbf{p}}-\hbar\omega_{\mathbf{q}}) - s\,(N_{\mathbf{q}}-N_{\mathbf{q}}^{(0)})/L_f - s\,(N_{\mathbf{q}}-N_{\mathbf{q}}^{(i)})/L_{fd} = 0. \qquad (2.2)$$

Here $W_{\mathbf{q}}$ is a quantity which determines the probability of electrons being scattered by phonons and of phonons by electrons: for acoustic-mode phonons it is proportional to the absolute value of the phonon momentum q.

We shall separate the distribution functions of electrons and phonons into an isotropic part and a small correction

$$n_{\mathbf{p}} = n_{\mathbf{p}}^{(i)} + n_{\mathbf{p}}' = n_{\mathbf{p}}^{(i)} + (\mathbf{f}(\varepsilon), \mathbf{p}/p), \qquad (2.3)$$
$$N_{\mathbf{q}} = N_{\mathbf{q}}^{(i)} + N_{\mathbf{q}}' = N_{\mathbf{q}}^{(i)} + (\mathbf{F}(\omega), \mathbf{q}/q)$$

[the functions $n_{\mathbf{p}}^{(i)}$ and $N_{\mathbf{q}}^{(i)}$ occur in Eqs. (2.1) and (2.2); from now on we shall omit the superscript (i)]; further, we shall assume that

$$n_{\mathbf{p}}' \ll n_{\mathbf{p}},\quad N_{\mathbf{q}}' \ll N_{\mathbf{q}}.$$

The vectors $\mathbf{f}(\epsilon)$ and $\mathbf{F}(\omega)$ are parallel to the current vector when the energy spectrum is isotropic, as is assumed here. The relationships (2.3) represent the first step of the expansion of the distribution functions as series of Legendre polynomials in terms of the angles $(\mathbf{p}, \mathbf{f})$, $(\mathbf{q}, \mathbf{F})$, in which all the polynomials beginning with the second are rejected. Linearizing Eqs. (2.1) and (2.2) with respect to $n_{\mathbf{p}}'$ and $N_{\mathbf{p}}'$ and expanding the collision operators in terms of the small quantity $\hbar\omega/T$, we obtain, as in [3,7]

$$\frac{\partial n_{\mathbf{p}}}{\partial \varepsilon} + \frac{l_f(E)}{l_f}\frac{n_{\mathbf{p}}(1-n_{\mathbf{p}})}{T} = \frac{e l_f(E)}{3s^2p^2}(\mathbf{E}, \mathbf{f}), \qquad (2.4)$$

$$\mathbf{f} + \frac{el(E)}{cp}[\mathbf{Hf}] + \frac{s^2 l(E)}{4p^3 T l_f}\frac{\partial n_{\mathbf{p}}}{\partial \varepsilon}\int_0^{2p}\mathbf{F}(\omega)\,q^4 dq = -e\mathbf{E}\,l(E)\,\partial n_{\mathbf{p}}/\partial\varepsilon; \qquad (2.5)$$

$$N_{\mathbf{q}} = N_{\mathbf{q}}^{(0)}\left\{1 + \frac{L_f}{L_e}\frac{v_F^2}{T(E)\,p_F^2}\int_{\varepsilon(q/2)}^{\infty} n_{\mathbf{p}}(1-n_{\mathbf{p}})\frac{p^2}{v^2}d\varepsilon\right\} \times \left\{1 - \frac{L_f}{L_e}\frac{v_F^2}{p_F^2}\int_{\varepsilon(q/2)}^{\infty}\frac{\partial n_{\mathbf{p}}}{\partial\varepsilon}\frac{p^2}{v^2}d\varepsilon\right\}^{-1}, \qquad (2.6)$$

$$\mathbf{F}(\omega) = \frac{L_{fd}}{L_e}N_{\mathbf{q}}\frac{v_F^2}{s p_F^2} \times \int_{\varepsilon(q/2)}^{\infty}\mathbf{f}(\varepsilon)\frac{p}{v^2}d\varepsilon\left\{1 - \frac{L_{fd}}{L_e}\frac{v_F^2}{p_F^2}\int_{\varepsilon(q/2)}^{\infty}\frac{\partial n_{\mathbf{p}}}{\partial\varepsilon}\frac{p^2}{v^2}d\varepsilon\right\}^{-1}. \qquad (2.7)$$

Here l and L are respectively the path lengths of electrons and phonons, with subscripts indicating the scattering mechanism (f—phonons, d—defects, e—electrons, no subscript—total path lengths), and

$$l_f^{-1}(E) = \frac{1}{4\pi^2\hbar^3p^2v^2}\int_0^{2p} W_{\mathbf{q}} N_{\mathbf{q}} q^3 dq, \qquad (2.8)$$

l_f is the electron path in the case of scattering by phonons in the absence of a field, s is the velocity of sound, and the subscript F denotes the Fermi surface.

We find

$$l_f \approx \tfrac{1}{2}a\,(\hbar^2 v/E_0 p a^2)^2\,(Ms^2/T),\quad l_d = \text{const},$$

$$L_e \approx (\hbar^2 v/E_0 p a^2)^2\,(Ms/q),\quad L_f \approx a\,(Ms^2/T)\,\Theta^4/(sq)^k\,T^{4-k},$$

and L_d = const (in the case of scattering on walls; in semimetals the scattering of phonons by defects can hardly be important).

Here M is the mass of a unit cell, E_0 is the deformation potential constant, a is the lattice constant, Θ is the Debye temperature, k = 2 for cubic crystals and k = 3 for trigonal ones. The path length of phonons scattered by electrons is equal to the path length of electrons when energy is transferred to phonons in the absence of an electric field, i.e., $2l_f N_{\mathbf{q}}^{(0)}$.

Using the step-like nature of $n_{\mathbf{p}}$, we obtain, integrating Eq. (2.4) with respect to ϵ:

$$\frac{(l_f(E))_F}{(l_f)_F}\,\frac{T(E)}{T} - 1 = \frac{e}{3s^2}\int\frac{l_f(E)}{p^2}(\mathbf{E}, \mathbf{f}(\varepsilon))\,d\varepsilon. \qquad (2.9)$$

Calculation of the quantities in Eqs. (2.6)—(2.8) by means of Eq. (1.1) gives

$$N_{\mathbf{q}} = N_{\mathbf{q}}^{(0)}\left(1 + \frac{L_{fe}}{L_e}\frac{\Delta T}{T}\right). \qquad (2.10)$$

$$\mathbf{F}(\omega) = \frac{L}{L_e}\frac{N_{\mathbf{q}}}{s}\frac{v_F^2}{p_F^2}\int_{\varepsilon(q/2)}^{\infty}\mathbf{f}(\varepsilon)\frac{p}{v^2}d\varepsilon, \qquad (2.11)$$

$$l_f(E) = l_f\left[1 + \frac{1}{4p^4}\int_0^{2p}\frac{L_{fe}}{L_e}q^3dq\,\frac{\Delta T}{T}\right]^{-1}. \qquad (2.12)$$

The expression (2.10) for $N_{\mathbf{q}}$ applies to $q \leq 2p_F$, i.e., to long-wavelength phonons interacting with electrons. For the remaining phonons the function $N_{\mathbf{q}}^{(i)}$ is unchanged.

Having determined $\mathbf{f}(\epsilon)$ from Eq. (2.5) and sub-

*[vH] = v × H.

stituted it into Eq. (2.11), we obtain an integral equation for $\mathbf{F}(\omega)$:

$$\mathbf{F}(\omega) = \frac{L}{L_e}\frac{N_q}{sp_F}\frac{el_F(E)}{1 + l_F^2(E)/R^2}\left\{\mathbf{E} + \frac{l_F(E)}{R}[\mathbf{Eh}] + \frac{l_F^2(E)}{R^2}\mathbf{h}(\mathbf{E},\mathbf{h}) + \frac{s^2}{4ep_F^3T(l_f)_F}\int_0^{2p_F}\left[\mathbf{F} + \frac{l_F(E)}{R}[\mathbf{Fh}] + \frac{l_F^2(E)}{R^2}\mathbf{h}(\mathbf{F},\mathbf{h})\right]q^4dq\right\}. \quad (2.13)$$

Here R is the Larmor radius, $R = cp_F/EH$; $\mathbf{h}$ is a unit vector along the magnetic field direction. Hence we see that $\mathbf{F}L_e/LN_q$ is independent of q and therefore the equation is easily solved. Substituting the solution into Eq. (2.5) we obtain

$$\mathbf{f} = -el(E)\left[(1-\nu(E))^2 + \frac{l^2(E)}{R^2}\right]^{-1}\left\{(1-\nu(E))\,\mathbf{E} + \frac{l(E)}{R}[\mathbf{Eh}] + \frac{1}{1-\nu(E)}\frac{l^2(E)}{R^2}\mathbf{h}(\mathbf{E},\mathbf{h})\right\}\frac{\partial n_{\mathbf{p}}}{\partial\varepsilon}. \quad (2.14)$$

Here and later all the quantities depending on the electron momentum refer to the Fermi surface. The quantity $\nu(E)$ is

$$\nu(E) = \frac{l(E)}{l_f}\frac{1}{4p^4}\int_0^{2p}\frac{L}{L_e}\left(1 + \frac{L_{fe}}{L_e}\right)\frac{\Delta T}{T}q^3dq.$$

It represents the "mutual" drag of electrons and phonons, which, as expected, is important when $l(E)L/l_fL_e \approx 1$, i.e., when electrons and phonons are scattered mainly on one another. The mutual drag appears to increase the electron path, replacing $l(E)$ by $l_1(E) = l(E)/[1-\nu(E)]$. The general relationships contain the following value of l_1 in the absence of electron heating

$$l_1 = \frac{l}{1-\nu} = l\left(1 - \frac{L}{L_e}\frac{l}{l_f}\right)^{-1}. \quad (2.15)$$

In the limiting case of considerable mutual drag we have

$$l_1 = l_d\left(1 + \frac{l_d}{l_f}\frac{L_e}{L_{fd}}\right)^{-1}. \quad (2.16)$$

3. DETERMINATION OF THE ELECTRON TEMPERATURE AND ELECTRICAL CONDUCTIVITY

We shall consider a crystal in mutually perpendicular electric and magnetic fields when the current is $\mathbf{j} = \sigma\mathbf{E} + \sigma'\mathbf{E}\times\mathbf{h}$.

Substituting Eq. (2.14) into Eq. (2.9) we obtain an equation for determining the electron temperature:

$$\frac{l_f(E)}{l_f}\frac{T(E)}{T} - 1 = \frac{e^2E^2l(E)\,l_f(E)\,(1-\nu(E))}{s^2p^2[(1-\nu(E))^2 + l^2(E)/R^2]}. \quad (3.1)$$

In view of the complexity of this equation we shall consider only the limiting cases in which one phonon scattering mechanism predominates. Since the phonon path L depends on q in step-like fashion, an allowance for this dependence produces only numerical multipliers ~ 1 in the integrals of q in (3.1). The concept of the path length L_f is purely qualitative since the corresponding collision integral cannot be written in the form $-s(N_q - N_q^{(0)})/L_f$. Therefore in the use of this quantity an allowance for numerical coefficients of the order of 1 has no meaning and we are correct in neglecting the dependence of L on q. The electron temperature equation representing interpolation between the limiting cases can then be written in the form

$$\left[\left(1 + \frac{L_{fe}}{L_e}\frac{L}{L_{fd}}\frac{l_1}{l_f}\frac{\Delta T}{T}\right)^2 + \frac{l_1^2}{R^2}\right]\frac{\Delta T}{T} = \frac{1}{3}\left(\frac{eEl_f}{sp}\right)^2\frac{L_f}{L_{fe}}\frac{l_1}{l_f}\left(1 + \frac{L_{fe}}{L_e}\frac{L}{L_{fd}}\frac{l_1}{l_f}\frac{\Delta T}{T}\right). \quad (3.2)$$

This cubic equation has only one root with physical meaning. Its solution is given simply in terms of physical parameters in the following limiting cases:

1) Weak electric field $(L_{fe}Ll_1/L_eL_{fd}l_f)(\Delta T/T) \ll 1$, or, as given by the solution

$$eEl_1/sp \ll [(L_eL_{fd}/LL_f)\,(1 + l_1^2/R^2)]^{1/2}.$$

Then

$$\frac{\Delta T}{T} = \frac{1}{3}\frac{l_f}{l_1}\frac{L_f}{L_{fe}}\left(1 + \frac{l_1^2}{R^2}\right)^{-1}\left(\frac{eEl_1}{sp}\right)^2. \quad (3.3)$$

When $L_f \ll L_e$ this expression reduces to the one obtained in [3,4] for the case when the phonons are in equilibrium. Strong heating not only of electrons but also of phonons may also occur when scattering of electrons by defects is the dominant process. However, the electrical conductivity then depends weakly on the field.

A small deviation from Ohm's law may be due to two causes: as a result of an increase of the diffuseness of the Fermi surface by electron heating, and as a result of the reduction of the electron path length because of phonon heating.

If the second cause is important, then in the expression

$$\sigma(E, H) = \sigma(0, H)\,[1 + \beta(H)\,E^2] \quad (3.4)$$

the coefficient is

$$\beta(H) = \frac{1}{3}\frac{L}{L_e}\frac{L_f}{L_{fd}}\left(\frac{l_1^2}{R^2} - 1\right)\left(1 + \frac{l_1^2}{R^2}\right)^{-2}\left(\frac{el_1}{sp}\right)^2. \quad (3.5)$$

It vanishes when $l_1/R = 1$, changes sign, reaches a maximum at $l_1/R = \sqrt{3}$, and in strong magnetic fields approaches zero (Fig. 4).

The Hall conductivity

$$\sigma'(E, H) = \sigma'(0, H)\,[1 + \beta'(H)\,E^2], \tag{3.6}$$

$$\beta' = -\frac{2}{3}\frac{L}{L_e}\frac{L_f}{L_{fd}}\left(1 + \frac{l_1^2}{R^2}\right)^{-2}\left(\frac{el_1}{sp}\right)^2 \tag{3.7}$$

decreases monotonically with increase of the magnetic field.

2) Intermediate electric and strong magnetic fields:

$$1 + \frac{L_{fe}}{L_e}\frac{L}{L_{fd}}\frac{l_1}{l_f}\frac{\Delta T}{T} \ll \frac{l_1}{R}, \text{ or } \frac{l_1}{R}\left[1 - \frac{L_f L}{3L_{fd}L_e}\left(\frac{cE}{sH}\right)^2\right] \gg 1.$$

Then

$$\frac{\Delta T}{T} = \frac{1}{3}\frac{l_f}{l_1}\frac{L_f}{L_{fe}}\left(\frac{cE}{sH}\right)^2\left[1 - \frac{1}{3}\frac{L_f}{L_{fd}}\frac{L}{L_e}\left(\frac{cE}{sH}\right)^2\right]^{-1}, \tag{3.8}$$

$$\sigma(E, H) \approx \sigma(0,0)\frac{R^2}{l_1^2}\left[1 - \frac{1}{3}\frac{L_f}{L_{fd}}\frac{L}{L_e}\left(\frac{cE}{sH}\right)^2\right]^{-1}, \tag{3.9}$$

$$\sigma'(E, H) \approx \sigma(0, 0)\,R/l_1. \tag{3.10}$$

When l_1/R is sufficiently large, i.e., in sufficiently strong magnetic fields, $\sigma(E, H)$ may rise steeply when $(L_f L/3L_{fd}L_e)(cE/sH)^2$ approaches unity reaching values $(l_1/R)\sigma(0, H) \approx (R/l_1)\sigma(0, 0)$, and heating may be strong.

3) Strong electric field:

$$\frac{L_{fe}}{L_e}\frac{L}{L_{fd}}\frac{l_1}{l_f}\frac{\Delta T}{T} \gg 1, \text{ or } \frac{l_1^2}{R^2}\left[\frac{1}{3}\frac{L_f}{L_{fd}}\frac{L}{L_e}\left(\frac{cE}{sH}\right)^2 - 1\right] \gg 1.$$

Then

$$\frac{\Delta T}{T} = \frac{L_e}{L_{fe}}\frac{L_{fd}}{L}\frac{l_f}{R}\left[\frac{1}{3}\frac{L_f}{L_{fd}}\frac{L}{L_e}\left(\frac{cE}{sH}\right)^2 - 1\right]^{1/2} \gg 1, \tag{3.11}$$

$$\sigma(E, H) = 3\sigma(0,0)\frac{R}{l_1}\frac{L_{fd}}{L_f}\frac{L_e}{L}\left(\frac{sH}{cE}\right)^2\left[\frac{L_f}{3L_{fd}}\frac{L}{L_e}\left(\frac{cE}{sH}\right)^2 - 1\right]^{1/2}, \tag{3.12}$$

$$\sigma'(E, H) = 3\sigma(0,0)\frac{R}{l_1}\frac{L_{fd}}{L_f}\frac{L_e}{L}\left(\frac{sH}{cE}\right)^2. \tag{3.13}$$

The quantity $\sigma(E, H)$ as a function of the electric field has in this region a maximum equal to $\sigma_{max}(E, H) \approx \sigma(0, 0)R/l_1$ (Fig. 1).

If, apart from the conditions listed under 3), the following inequality is satisfied

$$\frac{L_f}{L_{fd}}\frac{L}{L_e}\left(\frac{cE}{sH}\right)^2 \gg 1,$$

then

$$\frac{\Delta T}{T} = [L_e L_{fd} L_f / 3LL_{fe}^2]^{1/2}\, eEl_f / sp \gg 1. \tag{3.14}$$

In this case the number of phonons in the isotropic part of the distribution function increases and the path length of electrons scattered by phonons decreases in a ratio proportional to the electric field, both these quantities being independent of the magnetic field.

For the electrical conductivity we have

$$\sigma(E) = \sigma(0)\,[3L_{fd}L_e/L_f L]^{1/2} sp/eEl_1. \tag{3.15}$$

Thus, the electrical conductivity is independent of the magnetic field and inversely proportional to the electric field, while the current approaches saturation (Figs. 1, 2). Since $\sigma'(E, H) \sim E^{-2}$, the Hall current $j_2(E)$ is inversely proportional to E in this region.

In the case of considerable "mutual" drag the asymmetric part of the phonon distribution function $F(\omega)$ in a strong electric field becomes comparable with the symmetric part $N(q)$, if the scattering of phonons by defects or by the surface is unimportant compared with the phonon-phonon scattering. Therefore our results for this case have only qualitative meaning.

Concluding, let us consider briefly the case of an arbitrary angle ϑ between the electric and magnetic fields. Then

$$\mathbf{j} = \sigma\mathbf{E} + \sigma'[\mathbf{Eh}] + \sigma''\mathbf{h}(\mathbf{E}, \mathbf{h}).$$

It is obvious that $\sigma(E, 0) = \sigma'(E, H) + \sigma''(E, H)$. The maximum of the electrical conductivity $\sigma(E, H)$ is independent of the angle ϑ and its position is displaced toward weaker electric fields on reduction of ϑ [$\sigma(E, H)$ has a maximum at $E = [6L_{fd}L_e/L_f L]^{1/2}(sH/c)(1 + \cos^2\vartheta)^{-1/2}$]. The expression for the coefficient β acquires, compared with Eq. (3.1), an additional multiplier $1 + (l_1^2/R^2)\cos^2\vartheta$, and when $\cos\vartheta > \sqrt{3}/3$ the maximum of β in the positive region disappears. The function $\sigma''(E, H)$ decreases monotonically with increase of the electric field; in the region of strong electric fields $\sigma''(E, H) \sim E^{-3}$.

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