ASYMPTOTIC RELATIONS BETWEEN CROSS SECTIONS FOR LARGE ANGLE SCATTERING

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As has been shown in a number of papers [1-3] it is possible to establish within the framework of the Regge pole method asymptotic relations between cross sections for various processes for small angle scattering. The hypothesis that analogous relations should exist for scattering through large angles has been advanced by Pomeranchuk. In this note such relations are obtained for certain processes.

Let us consider the following process: nucleon Compton effect, photoproduction of pions and πN scattering. We denote by

$$\langle\lambda|f_j^{\pm}|\lambda'\rangle, \quad \langle\lambda|g_j^{\pm}|\lambda'\rangle, \quad \langle\lambda|h_j^{\pm}|\lambda'\rangle$$

the respective partial amplitudes in the u channel for these processes, corresponding to transitions between states with parity $\pm(-1)^{j-1/2}$ and total spin λ and λ'. The generalized unitarity condition [4] for the partial amplitudes has the form:

$$\frac{1}{2i}(\langle\lambda|f_j|\lambda'\rangle - \langle\lambda|f^*_{j^*}|\lambda'\rangle) = \frac{k}{\omega}\langle\lambda|g_j|\lambda''\rangle\langle\lambda''|g^*_{j^*}|\lambda'\rangle,$$

$$\frac{1}{2i}(\langle\lambda|g_j|\lambda'\rangle - \langle\lambda|g^*_{j^*}|\lambda'\rangle) = \frac{k}{\omega}\langle\lambda|h_j|\lambda''\rangle\langle\lambda''|g^*_{j^*}|\lambda'\rangle,$$

$$\frac{1}{2i}(\langle\lambda|h_j|\lambda'\rangle - \langle\lambda|h^*_{j^*}|\lambda'\rangle) = \frac{k}{\omega}\langle\lambda|h_j|\lambda''\rangle\langle\lambda''|h^*_{j^*}|\lambda'\rangle. \qquad (1)$$

Assuming that there exists for one of the amplitudes a principal Regge pole (with largest Re j) with definite isotopic spin we find with the help of Eq. (1) that every one of the amplitudes should have a pole for the same j, with the residues at that pole related as follows:

$$\langle\tfrac{1}{2}|r_f|\tfrac{1}{2}\rangle\langle\tfrac{3}{2}|r_f|\tfrac{3}{2}\rangle = \langle\tfrac{1}{2}|r_f|\tfrac{3}{2}\rangle^2,$$

$$\langle\tfrac{1}{2}|r_h|\tfrac{1}{2}\rangle\langle\tfrac{1}{2}|r_f|\tfrac{1}{2}\rangle = \langle\tfrac{1}{2}|r_g|\tfrac{1}{2}\rangle^2,$$

$$\langle\tfrac{1}{2}|r_h|\tfrac{1}{2}\rangle\langle\tfrac{3}{2}|r_f|\tfrac{3}{2}\rangle = \langle\tfrac{1}{2}|r_g|\tfrac{3}{2}\rangle^2. \qquad (2)$$

Relation (2) gives rise to a relation between the amplitudes best seen when the latter are written in a factored form [5,6]:

$$A_{\gamma\gamma} = C_{\gamma\gamma}\Gamma_\mu(i\hat{f} - \sqrt{u})\Gamma_\nu\frac{s^{j-1/2}}{\cos\pi j}(1 \pm e^{-i\pi(j-1/2)}) + (\sqrt{u}\to-\sqrt{u}),$$

$$A_{\gamma\pi} = C_{\gamma\pi}\Gamma_\mu(i\hat{f} - \sqrt{u})\gamma_5\frac{s^{j-1/2}}{\cos\pi j}(1 \pm e^{-i\pi(j-1/2)}) + (\sqrt{u}\to-\sqrt{u}),$$

$$A_{\pi\pi} = C_{\pi\pi}\gamma_5(i\hat{f} - \sqrt{u})\gamma_5\frac{s^{j-1/2}}{\cos\pi j}(1 \pm e^{-i\pi(j-1/2)}) + (\sqrt{u}\to-\sqrt{u}). \qquad (3)$$

The subscripts refer to the bosons participating in the reaction. When relation (2) is taken into account all the Γ_μ in Eq. (3) turn out to be equal and $C^2_{\gamma\pi} = C_{\gamma\gamma}C_{\pi\pi}$. From here we have the following relations for the differential cross sections for scattering through large angles at high energies:

$$(d\sigma/d\Omega)^2_{\gamma\pi} = (d\sigma/d\Omega)_{\gamma\gamma}(d\sigma/d\Omega)_{\pi\pi}. \qquad (4)$$

Analogous relations exist between the correspondingly chosen polarization effects.

We note that, in contrast to the situation in the t channel, [1-3] the experimental verification of these relations does not require the scattering of unstable particles on each other.

We consider next the following processes:

$$\gamma + N \to K + \Lambda(\Sigma), \quad K + \Lambda(\Sigma) \to K + \Lambda(\Sigma),$$
$$\pi + N \to K + \Lambda(\Sigma).$$

Analogous to Eq. (4) we obtain from the unitarity condition

$$\left(\frac{d\sigma}{d\Omega}\right)^2_{\gamma K} = \left(\frac{d\sigma}{d\Omega}\right)_{\gamma\gamma}\left(\frac{d\sigma}{d\Omega}\right)_{KK}, \quad \left(\frac{d\sigma}{d\Omega}\right)^2_{\pi K} = \left(\frac{d\sigma}{d\Omega}\right)_{\pi\pi}\left(\frac{d\sigma}{d\Omega}\right)_{KK}. \qquad (5)$$

The intermediate states of all processes considered are characterized by the same quantum numbers. It is therefore natural to assume that they are all dominated by one principal Regge pole, just like in the t channel the vacuum pole dominates.

Combining Eqs. (4) and (5) we find the following relations between cross sections of processes, for the experimental verification of which no unstable targets are needed:

$$\left(\frac{d\sigma}{d\Omega}\right)^2_{\gamma K}\left(\frac{d\sigma}{d\Omega}\right)_{\pi\pi} = \left(\frac{d\sigma}{d\Omega}\right)^2_{\pi K}\left(\frac{d\sigma}{d\Omega}\right)_{\gamma\gamma},$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{\gamma K}\left(\frac{d\sigma}{d\Omega}\right)_{\pi\pi} = \left(\frac{d\sigma}{d\Omega}\right)_{\gamma\pi}\left(\frac{d\sigma}{d\Omega}\right)_{\pi K}, \qquad (6)$$

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