THE THRESHOLD OF MAGNETIC ENERGY ABSORPTION IN A UNIAXIAL ANTIFERROMAGNETIC

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The absorption coefficient Γ of an alternating magnetic field polarized along the preferred axis of a uniaxial antiferromagnet is calculated. It is shown that for frequencies ω close to $\omega_0 = 2gH_0$ (where g is the gyromagnetic ratio and H_0 is the external magnetic field) $\Gamma \sim \sqrt{\omega - \omega_0}$ $(\omega > \omega_0)$. In the Appendix the Hamiltonian of the antiferromagnetic is derived under some very general assumptions.

As is known, the spin-wave spectrum of a uniaxial antiferromagnetic placed in a magnetic field $\mathbf{H}_0$ parallel to the preferred axis consists of two branches, the dispersion law of which, neglecting small dipole-dipole interactions has the following form:

$$\varepsilon_{1,2} = \sqrt{\varepsilon_0^2 + \Theta_c^2 (ak)^2} \mp \mu H_0. \qquad (1)$$

Here ϵ_0 is the activation energy of spin waves at $H_0 = 0$ resulting from the presence of an anisotropy energy; Θ_c is a quantity having the dimensions of energy of the order of the Curie temperature; a is the lattice constant; $\mathbf{k}$ is the wave vector; $\mu = g\hbar$, where g is the gyromagnetic ratio. The equidistant character of the spectrum of (1) shows that at a frequency $\omega = 2gH_0 = (\epsilon_2 - \epsilon_1)/\hbar$ a resonant absorption of magnetic energy should be observed. However, a consideration of the dipole-dipole interaction between the magnetic moments leads to a dependence of the difference $\epsilon_2 - \epsilon_1$ on the magnitude and direction of the wave vector. Hence the absorption is possible at frequencies greater than $\hbar^{-1}\min(\epsilon_2 - \epsilon_1) = \omega_0$. As it turns out, $\omega_0 = 2gH_0$. The present paper is concerned with the theory of a similar absorption.

Consider a uniaxial antiferromagnetic placed in a steady and homogeneous magnetic field $\mathbf{H}_0$ parallel to its preferred axis, and the magnitude of the field is such that the magnetic moments are antiparallel in the ground state, i.e.,

$$\mu H_0 < \varepsilon_0. \qquad (2)$$

As is shown in the appendix [see Eq. (A4)], the dispersion law for the spin waves, taking into account the dipole interactions, has the form

$$\varepsilon_{1,2}^2 = A_k^2 - B_k^2 + (\mu H_0)^2 \mp 2\sqrt{(A_k^2 - B_k^2)(\mu H_0)^2 + |C_k|^2 (A_k - B_k)^2}, \qquad (3)$$

where

$$A_k = \mu M_0 (\gamma + \beta + \alpha k^2 + 2\pi \sin^2 \theta_k),$$

$$B_k = \mu M_0 (\gamma + \alpha_{12} k^2 + 2\pi \sin^2 \theta_k),$$

$$C_k = 2\pi\mu M_0 \sin^2 \theta_k \exp(-2i\varphi_k), \qquad (4)$$

and the other symbols are defined in the appendix. Neglecting dipole interactions corresponds to the elimination of all quantities containing angles as factors. To obtain the dispersion law (1) it is also necessary to introduce the following symbols:

$$\varepsilon_0 = \mu M_0 \sqrt{2\gamma\beta}, \qquad \Theta_c = \mu M_0 \sqrt{2\gamma(\alpha - \alpha_{12})/a^2} \qquad (5)$$

and to neglect terms $\beta + (\alpha + \alpha_{12})k^2$ in comparison to γ. Note that, according to [1], the condition for the existence of antiferromagnetism is $\alpha > \alpha_{12}$.

It is known that for a number of uniaxial antiferromagnetics $\epsilon_0 \sim 10°$K. This means that antiferromagnetic resonance, i.e., excitation of spin waves with wave vector zero ($\hbar\omega = \epsilon_0 \pm \mu H_0$), in magnetic fields $\lesssim 10^3 - 10^4$ Oe can be observed only in the submillimeter and infrared region.

Using the dispersion law (3) we show that $\min(\epsilon_2 - \epsilon_1) = 2\mu H_0$. For this, we consider the law of conservation of energy for the absorption of a photon of frequency ω:

$$\varepsilon_1 + \hbar\omega = \varepsilon_2. \qquad (6)$$

We take the momentum of the photon equal to zero; this is justifiable because of the large magnitude of the velocity of light ($c \gg v_s$; $v_s = \hbar^{-1}\partial\epsilon/\partial k$ is the velocity of the spin wave). Squaring Eq. (6) and substituting the accurate values for ϵ_1 and ϵ_2, we obtain

$$(\hbar^2\omega^2 - 4\mu^2 H_0^2)[4(A^2 - B^2) - \hbar^2\omega^2] = 16|C|^2(A - B)^2.$$

From this it is clear that in order to fulfill the condition (6) it is necessary that the frequency satisfy one of two conditions:

$$4\mu^2 H_0^2 < \hbar^2\omega^2 < 4\,(A^2 - B^2), \tag{7a}$$

$$4\mu^2 H_0^2 > \hbar^2\omega^2 > 4\,(A^2 - B^2). \tag{7b}$$

The latter inequality (7b) contradicts the condition (2), proving our assertion that absorption of the type considered here begins at frequencies equal to $2\mu H_0/\hbar$. Note that the right inequality in (7a) does not limit the upper frequency, since as the modulus of the wave vector k tends toward infinity, so does $A^2 - B^2$.

In order to calculate the absorption coefficient Γ it is necessary to determine the probability W of absorption of a photon with energy $\hbar\omega$ by a spin wave of the first kind and its conversion into a spin wave of the second kind, as well as the probability of the reverse process. We shall make use of perturbation theory.

The operator for the interaction of an electromagnetic field $\mathbf{h}e^{-i\omega t}$ (h is its amplitude) with the magnetic moments has the form

$$\hat{\mathcal{H}}_{int} = -\mathbf{h}e^{-i\omega t}\int (\mathbf{M}_1 + \mathbf{M}_2)\,dv. \tag{8}$$

It is easily seen that the matrix element of the transition corresponding to the process considered differs from zero only in case the alternating magnetic field is polarized along the axis of the antiferromagnetic (chosen as the z axis), i.e.,

$$\hat{\mathcal{H}}_{int} = -he^{-i\omega t}\int (M_{1z} + M_{2z})\,dv. \tag{9}$$

As is shown in the appendix, in the representation of second quantization

$$\hat{\mathcal{H}}_{int} = -\frac{\pi\mu M_0}{H_0} he^{-i\omega t} \sum_k{}' \sin^2\theta_k \sqrt{a^2k^2 + 2\beta/\gamma}\, a^+_{k2} a_{k1}$$

$$+ \text{comp. conj.} \tag{10}$$

and we are limited to the case of relatively high steady magnetic field ($H_0 \gg M_0(T/\Theta_C)$). Since the transition probability equals $2\pi\hbar^{-1}|\langle i|\hat{\mathcal{H}}_{int}|f\rangle|^2\,\delta(\epsilon_i - \epsilon_f)$, the probability of the absorption of a photon as a result of the conversion of a spin wave of the first kind into a spin wave of the second kind equals

$$W_{12} = 2\pi\hbar^{-1}\,|\langle N_{1k};\ N_{2k}|\hat{\mathcal{H}}_{int}|N_{1k}-1;\ N_{2k}$$

$$+1\rangle|^2\,\delta(\varepsilon_{1k} + \hbar\omega - \varepsilon_{2k}). \tag{11}$$

From this, considering the possibility of reverse processes, we obtain that the amount of absorption in units of magnetic energy equals

$$Q = \frac{V}{(2\pi)^3}\hbar\omega\int \{W_{12} - W_{21}\}\,\delta(\varepsilon_1 + \hbar\omega - \varepsilon_2)\,d\tau_k \tag{12}$$

or

$$Q = \pi(\mu M_0)^2\omega V\left(\frac{h}{H_0}\right)^2\int_0^\infty \left(a^2k^2 + \frac{2\beta}{\gamma}\right)(N_1 - N_2)\,k^2dk$$

$$\times \int_0^{\pi/2} \sin^5\theta\,\delta(\varepsilon_1 + \hbar\omega - \varepsilon_2)\,d\theta. \tag{13}$$

Here

$$N_{1,2} = (e^{\varepsilon_{1,2}/T} - 1)^{-1} \tag{14}$$

is the equilibrium distribution function of the spin waves. It is most convenient to remove the δ-function during the integration over the angle θ.

In the case considered ($H_0 \gg M_0T/\Theta_C$) the argument of the δ-function has the form

$$\varepsilon_1 + \hbar\omega - \varepsilon_2 = \hbar\omega - 2\mu H_0$$

$$-\left(\frac{\pi}{\gamma}\right)^2 \sin^4\theta\,\frac{(\varepsilon_0^2 + \Theta_c^2a^2k^2)^2}{\mu H_0(\varepsilon_0^2 + \Theta_c^2a^2k^2 - \mu^2H_0^2)}.$$

Hence the integral over angle in Eq. (13) (we shall call it J) equals

$$J = \left(\frac{\gamma}{\pi}\right)^3 \frac{(\mu H_0)^{3/2}(\varepsilon_0^2 + \Theta_c^2a^2k^2 - \mu^2H_0^2)^{3/2}(\hbar\omega - 2\mu H_0)^{1/2}}{4(\varepsilon_0^2 + \Theta_c^2a^2k^2)^{5/2}}$$

$$\times \{\varepsilon_0^2 + \Theta_c^2a^2k^2 - (\gamma/\pi)(\mu H_0)^{1/2}(\hbar\omega - 2\mu H_0)^{1/2}$$

$$\times [\varepsilon_0^2 + \Theta_c^2a^2k^2 - \mu^2H_0^2]^{1/2}\}^{-1/2}. \tag{15}$$

The limits of integration over the modulus of the wave vector are determined from the condition $1 > \sin^4\theta > 0$. As we have already said, the activation energy in uniaxial antiferromagnets is relatively large; hence the region of greatest interest is that of relatively small fields $\mu H_0 \ll \epsilon_0$. In this case the limiting conditions for k will be

$$\infty > k > 0$$
$$\text{for } 0 < \gamma(\mu H_0)^{1/2}(\hbar\omega - 2\mu H_0)^{1/2}/\pi < \varepsilon_0,$$

$$\infty > k > \left[(\gamma/\pi)^2\mu H_0(\hbar\omega - 2\mu H_0) - \varepsilon_0^2\right]^{1/2}/a\Theta_c$$
$$\text{for } \gamma(\mu H_0)^{1/2}(\hbar\omega - 2\mu H_0)^{1/2}/\pi > \varepsilon_0. \tag{16}$$

From Eqs. (13) to (16) we have:

$$Q = \frac{\omega}{4\pi^2}\frac{V}{a^3}\left(\frac{h}{H_0}\right)^2\frac{\Theta_c}{\mu H_0}(\mu H_0)^{3/2}(\hbar\omega - 2\mu H_0)^{1/2}(1 - e^{-\hbar\omega/T})$$

$$\times \int_A^\infty \frac{a^2k^2(\varepsilon_0^2 + \Theta_c^2a^2k^2 - \mu^2H_0^2)^{3/2}\,d(ak)\,e^{\varepsilon_2/T}}{(e^{\varepsilon_1/T} - 1)(e^{\varepsilon_2/T} - 1)(\varepsilon_0^2 + \Theta_c^2a^2k^2)^{3/2}}$$

$$\times [\varepsilon_0^2 + \Theta_c^2a^2k^2 - (\gamma/\pi)(\mu H_0)^{1/2}(\hbar\omega - 2\mu H_0)^{1/2}$$

$$\times (\varepsilon_0^2 + \Theta_c^2a^2k^2 - \mu^2H_0^2)^{1/2}]^{-1/2}, \tag{17}$$

where

$$A = \begin{cases} 0 & \text{for } 0 < \gamma (\mu H_0)^{1/2} (\hbar\omega - 2\mu H_0)^{1/2} / \pi < \varepsilon_0 \\ \Theta_c^{-1} [(\gamma / \pi)^2 \mu H_0 (\hbar\omega - 2\mu H_0) - \varepsilon_0^2]^{1/2} & \text{for } \gamma (\mu H_0)^{1/2} (\hbar\omega - 2\mu H_0)^{1/2} / \pi > \varepsilon_0 \end{cases};$$

here $\epsilon_{1,2}$ are given by Eq. (1).

Since $\mu H_0 \ll \epsilon_0$, we can set

$$\varepsilon_1 \approx \varepsilon_2 = \varepsilon = \sqrt{\varepsilon_0^2 + \Theta_c^2 a^2 k^2}.$$

If we now introduce a new variable of integration $x = \Theta_c ak/T$, and also symbolize

$$\gamma (\mu H_0)^{1/2} (\hbar\omega - 2\mu H_0)^{1/2} = z\pi T, \qquad \varepsilon_0/T = \eta, \tag{18}$$

then the expression (17) can be written in the following fashion:

$$Q = \frac{\omega}{4\pi} \frac{V}{a^3} \left(\frac{h}{H_0}\right)^2 \mu H_0 \left(\frac{T}{\Theta_c}\right)^3 (1 - e^{-\hbar\omega/T}) F(z, \eta); \tag{19}$$

$$F(z, \eta) = z \int_{A(z,\eta)}^{\infty} \frac{x^2 \exp(\sqrt{x^2 + \eta^2})\, dx}{[\exp(\sqrt{x^2 + \eta^2}) - 1]^2 (x^2 + \eta^2)^{1/4} [(x^2 + \eta^2)^{1/2} - z]^{1/2}}, \tag{20}$$

$$A(z, \eta) = \begin{cases} 0, & 0 < z < \eta \\ \sqrt{z^2 - \eta^2}, & z > \eta \end{cases}. \tag{21}$$

The particular points $F(z, \eta)$ are, firstly, $z = 0$ ($\omega = \omega_0$), and secondly, $z = \eta$ ($\hbar\omega' = 2\mu H_0 + \pi^2\epsilon_0^2/\gamma^2\mu H_0$). That is, at these frequencies the function $F(z, \eta)$ and the absorption coefficient have a break.

The function $F(z, \eta)$ characterizes the shape of the absorption line. We shall find approximate values for F in some special cases. First, let us consider the case of relatively high temperature ($T \gg \epsilon_0$, i.e., $\eta \ll 1$):

a) close to the absorption threshold, i.e., for $z \ll \eta \ll 1$

$$F(z, \eta) \approx z \int_0^{\infty} \frac{x \exp(\sqrt{x^2 + \eta^2})\, dx}{(\exp(\sqrt{x^2 + \eta^2}) - 1)^2} \approx z \ln \frac{1}{\eta}; \tag{22}$$

b) for $\eta \ll z \ll 1$

$$F(z, \eta) \approx z \int_z^{\infty} \frac{x^{3/2} e^x dx}{\sqrt{x - z}\, (e^x - 1)^2} \approx z \ln \frac{1}{z}; \tag{23}$$

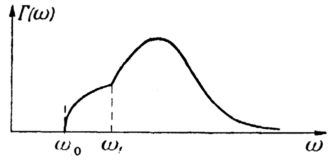
c) for $\eta \ll 1 \ll z$

$$F(z, \eta) \approx z \int_z^{\infty} \frac{x^{3/2} e^{-x}\, dx}{\sqrt{x - z}} \approx \sqrt{\pi} z^{5/2} e^{-z}. \tag{24}$$

The function $F(z, \eta)$ is shown schematically in the figure.

Let us now consider the case of low temperature ($T \ll \epsilon_0$, i.e., $\eta \gg 1$). The function $F(z, \eta)$ here has the form

$$F(z, \eta) = z \int_{A(z,\eta)}^{\infty} \frac{\exp(-\sqrt{x^2 + \eta^2})\, x^2 dx}{(x^2 + \eta^2)^{1/4} [(x^2 + \eta^2)^{1/2} - z]^{1/2}}. \tag{25}$$



We consider the limiting cases. We have:

a) close to the absorption threshold, i.e., for $z \ll 1 \ll \eta$

$$F(z, \eta) \approx z \int_z^{\infty} \frac{\exp(-\sqrt{x^2 + \eta^2})\, x^2 dx}{\sqrt{x^2 + \eta^2}} \approx \sqrt{\frac{\pi}{2}}\, z \eta^{1/2} e^{-\eta}. \tag{26}$$

This dependence continues also at higher frequencies ($1 \ll z \ll \eta$);

b) on the "tail" of the absorption line, i.e., for $z \gg \eta \gg 1$

$$F(z, \eta) \approx z \int_z^{\infty} \frac{e^{-x} x^{3/2} dx}{\sqrt{x - z}} \approx \sqrt{\pi} z^{5/2} e^{-z}. \tag{27}$$

Equations (19) through (27) are the solutions of the problem. The absorption coefficient Γ is defined as the ratio of the power loss Q to the magnetic field energy ($\Gamma = 8\pi Q/h^2 V$).

A characteristic peculiarity of the frequency dependence of the absorption coefficient should be noted: $\Gamma(\omega) \sim (\omega - \omega_0)^{1/2}$ close to the threshold ($\omega_0 = 2gH_0$) and dies out exponentially with rising frequency [see Eq. (24) and Eq. (27)]. In the most interesting case ($T \gg \epsilon_0$) the maximum in absorption is reached at $z \sim 1$, i.e., at

$$\hbar\omega = 2\mu H_0 \{1 + \xi (\pi M_0/H_0)^2 (T/\Theta_c)^2\},$$

where ξ is of order unity. In other words, the lower the temperature, the closer is the maximum to $2\mu H_0$, i.e., the finer the "line."

The quantity[1] $d\Gamma/d\omega$, according to these equations goes to infinity ($d\Gamma/d\omega = A/(\omega - \omega_0)^{1/2}$ for $\omega \gtrsim \omega_0$). To estimate the magnitude of $d\Gamma/d\omega$ at $\omega = \omega_0$ we can use the following formula: $d\Gamma/d\omega \approx A(\tau)^{1/2}$, where τ is the lifetime of the spin wave; in order of magnitude $1/\tau$ agrees with the line width of the antiferromagnetic resonance.

In conclusion, we take this opportunity to thank A. S. Borovik-Romanov, I. M. Lifshitz, and V. M. Tsukernik for helpful discussions.

[1]In some experiments it is $d\Gamma/d\omega$ (or, more precisely, $d\Gamma/dH_0$), and not Γ, that is measured.

APPENDIX

We shall write the Hamiltonian of a uniaxial antiferromagnetic in the following form:

$$\mathcal{H} = \int dV \Big\{ \gamma \mathbf{M}_1\mathbf{M}_2 - \mathbf{H}(\mathbf{M}_1+\mathbf{M}_2) + \frac{1}{2}\lambda (M_{1x}^2 + M_{1y}^2 + M_{2x}^2 + M_{2y}^2) - \delta M_{1z} M_{2z} + \frac{1}{2} \alpha \left(\frac{\partial M_{1i}}{\partial x^k}\right)^2 + \frac{1}{2}\alpha\left(\frac{\partial M_{2i}}{\partial x^k}\right)^2 + \alpha_{12}\frac{\partial M_{1i}}{\partial x^k}\frac{\partial M_{2i}}{\partial x^k}\Big\}. \qquad \text{(A1)}$$

Here $\mathbf{M}_1$ and $\mathbf{M}_2$ are the magnetic moments corresponding to the two sublattices; γ, α, α_{12} are constants associated with the exchange interaction, the term in γ describing isotropic exchange interaction in the system and those in α and α_{12} the anisotropic. It can be shown that $\alpha > \alpha_{12}$ and that α, $\alpha_{12} \sim \gamma a^2$; $\gamma \sim \Theta_C/\mu M_0 \gg 1$ (μ is of the order of the Bohr magneton, M_0 is the equilibrium value of the magnetic moment $\sim\mu/a^3$). Further, λ and δ are magnetic anisotropy constants, and the terms corresponding to them are associated with small relativistic interactions (λ, $\delta \sim \gamma(v/c)^2$, where v is the velocity of the electrons and c is the velocity of light). The magnetic field $\mathbf{H}$ is made up of the external, steady homogeneous field $\mathbf{H}_0$ and the internal field of the spin waves, which is described by the magnetostatic equations:

$$\operatorname{curl} \mathbf{H}_s = 0, \qquad \operatorname{div} \mathbf{H}_s = -4\pi \operatorname{div}(\mathbf{M}_1 + \mathbf{M}_2).$$

In the Hamiltonian of the system (A1) we express $\mathbf{M}_1$ and $\mathbf{M}_2$ in the usual way by means of Holstein-Primakoff operators:

$$M_i^- = (2\mu M_0)^{1/2}\,[1 - (\mu/2M_0)\,a_i^+a_i]^{1/2}\,a_i,$$
$$M_i^+ = (2\mu M_0)^{1/2}\,a_i^+\,[1 - (\mu/2M_0)\,a_i^+a_i]^{1/2},$$
$$M_{iz} = -M_0(-1)^i + \mu\, a_i^+a_i, \qquad \text{(A2)}$$

where i = 1, 2; $M^\pm = M_x \pm iM_y$, and the operators $a_i(\mathbf{r})$, $a_i^+(\mathbf{r})$ obey the usual Bose commutation rules.

We expand the operators a_i and a_i^+ in Fourier series:

$$a_i(\mathbf{r}) = \sum_{n=1}^{2}\sum_{\mathbf{k}} \{u_{\mathbf{k}n}^{(i)} a_{\mathbf{k}n}(t)\, e^{i\mathbf{kr}} + v_{\mathbf{k}n}^{*(i)} a_{\mathbf{k}n}^+(t)\, e^{-i\mathbf{kr}}\},$$
$$a_{\mathbf{k}n}(t) = a_{\mathbf{k}n} e^{-i\varepsilon_{\mathbf{k}n}t/\hbar}. \qquad \text{(A3)}$$

Utilizing the usual method for diagonalizing the Hamiltonian, in which only quadratic terms are retained, we find the energy of the two possible branches of oscillation:

$$\varepsilon_{\mathbf{k}n}^2 = A_{\mathbf{k}}^2 - B_{\mathbf{k}}^2 + (\mu H_0)^2 \mp 2\,\{(\mu H_0)^2 (A_{\mathbf{k}}^2 - B_{\mathbf{k}}^2) + |C_{\mathbf{k}}|^2 (A_{\mathbf{k}} - B_{\mathbf{k}})^2\}^{1/2}. \qquad \text{(A4)}$$

Here $A_{\mathbf{k}}$, $B_{\mathbf{k}}$, and $C_{\mathbf{k}}$ are defined by Eqs. (4). At the same time we find the amplitudes u and v:

$$|u_{\mathbf{k}1}^1|^2 = |C|^2\,\gamma_2(1-\alpha_2)(\beta_2-\alpha_2)(1-\alpha_1)^2/VD,$$
$$|u_{\mathbf{k}2}^1|^2 = |C|^2\,\gamma_1(\alpha_1-1)(\beta_1-\alpha_1)(1-\alpha_2)^2/VD,$$
$$|v_{\mathbf{k}1}^1|^2 = \gamma_1^2\gamma_2(1-\alpha_2)(\beta_2-\alpha_2)/VD,$$
$$|v_{\mathbf{k}2}^1|^2 = \gamma_1\gamma_2^2(\alpha_1-1)(\beta_1-\alpha_1)/VD,$$
$$u_{\mathbf{k}n}^2 = -\alpha_n v_{\mathbf{k}n}^1, \qquad v_{\mathbf{k}n}^2 = -\beta_n u_{\mathbf{k}n}^1;$$
$$D = \gamma_2(1-\alpha_2)(\beta_2-\alpha_2)\,[|C|^2(1-\alpha_1)^2 - \gamma_1^2] - \gamma_1(1-\alpha_1)(\beta_1-\alpha_1)[|C|^2(1-\alpha_2)^2 - \gamma_2^2]. \qquad \text{(A5)}$$

The quantities α, β, and γ are, respectively,

$$\alpha_{\mathbf{k}n} = \frac{\varepsilon_{\mathbf{k}n} + \mu H_0 + A_{\mathbf{k}} - B_{\mathbf{k}}}{\varepsilon_{\mathbf{k}n} + \mu H_0 - A_{\mathbf{k}} + B_{\mathbf{k}}}, \qquad \beta_{\mathbf{k}n} = \frac{\varepsilon_{\mathbf{k}n} - \mu H_0 - A_{\mathbf{k}} + B_{\mathbf{k}}}{\varepsilon_{\mathbf{k}n} - \mu H_0 + A_{\mathbf{k}} - B_{\mathbf{k}}},$$
$$\gamma_{\mathbf{k}n} = \frac{(\varepsilon_{\mathbf{k}n} - \mu H_0)^2 - A_{\mathbf{k}}^2 + B_{\mathbf{k}}^2}{\varepsilon_{\mathbf{k}n} - \mu H_0 + A_{\mathbf{k}} - B_{\mathbf{k}}}. \qquad \text{(A6)}$$

We shall write out the values of the amplitudes in two limiting cases.

1. $\epsilon_0 > \mu H_0 \gg |C|(A - B)/(A + B)$. In this case the dispersion law is quite simple:

$$\varepsilon_{\mathbf{k}(1,2)} = \sqrt{A_{\mathbf{k}}^2 - B_{\mathbf{k}}^2} \mp \mu H_0,$$

and the amplitudes u and v acquire the following form (n, m = 1, 2):

$$u_{\mathbf{k}n}^n = \frac{e^{2i\chi_n}}{4V^{1/2}}\frac{C_{\mathbf{k}}}{\mu H_0}\left(\frac{A_{\mathbf{k}} - B_{\mathbf{k}}}{A_{\mathbf{k}} + B_{\mathbf{k}}}\right)^{1/4} \frac{\sqrt{A_{\mathbf{k}}^2 - B_{\mathbf{k}}^2} + (-1)^n 2\mu H_0 + A_{\mathbf{k}} - B_{\mathbf{k}}}{\sqrt{A_{\mathbf{k}}^2 - B_{\mathbf{k}}^2} + (-1)^n \mu H_0},$$

$$u_{\mathbf{k}m}^n = (-1)^m \frac{e^{2i\chi_m}}{(2V)^{1/2}}\left(\frac{A_{\mathbf{k}} + \sqrt{A_{\mathbf{k}}^2 - B_{\mathbf{k}}^2}}{\sqrt{A_{\mathbf{k}}^2 - B_{\mathbf{k}}^2}}\right)^{1/2}, \qquad n \neq m,$$

$$v_{\mathbf{k}n}^n = (-1)^{n+1}\frac{e^{2i\chi_n}}{(2V)^{1/2}}\left(\frac{A_{\mathbf{k}} - \sqrt{A_{\mathbf{k}}^2 - B_{\mathbf{k}}^2}}{\sqrt{A_{\mathbf{k}}^2 - B_{\mathbf{k}}^2}}\right)^{1/2},$$

$$v_{\mathbf{k}m}^n = -\frac{e^{2i\chi_m}}{4V^{1/2}}\frac{C_{\mathbf{k}}}{\mu H_0}\left(\frac{A_{\mathbf{k}} - B_{\mathbf{k}}}{A_{\mathbf{k}} + B_{\mathbf{k}}}\right)^{1/4} \times \frac{\sqrt{A_{\mathbf{k}}^2 - B_{\mathbf{k}}^2} + (-1)^m 2\mu H_0 - A_{\mathbf{k}} + B_{\mathbf{k}}}{\sqrt{A_{\mathbf{k}}^2 - B_{\mathbf{k}}^2} + (-1)^m \mu H_0}, \quad m \neq n. \qquad \text{(A7)}$$

For $(A^2 - B^2)^{1/2} \gg \mu H_0 \gg |C|(A - B)^{1/2}/(A + B)^{1/2}$ it is possible to simplify these expressions further:

$$u_{\mathbf{k}1}^1 \approx u_{\mathbf{k}2}^2 \approx v_{\mathbf{k}2}^1 \approx v_{\mathbf{k}1}^2 \approx 0;$$
$$|u_{\mathbf{k}2}^1| = |u_{\mathbf{k}1}^2| = \frac{1}{(2V)^{1/2}}\left(\frac{A_{\mathbf{k}} + \sqrt{A_{\mathbf{k}}^2 - B_{\mathbf{k}}^2}}{\sqrt{A_{\mathbf{k}}^2 - B_{\mathbf{k}}^2}}\right)^{1/2},$$
$$|v_{\mathbf{k}1}^1| = |v_{\mathbf{k}2}^2| = (2V)^{-1/2}\left(\frac{A_{\mathbf{k}} - \sqrt{A_{\mathbf{k}}^2 - B_{\mathbf{k}}^2}}{\sqrt{A_{\mathbf{k}}^2 - B_{\mathbf{k}}^2}}\right)^{1/2}. \qquad \text{(A8)}$$

2. Magnetic field equal to zero ($H_0 = 0$). In this case

$$u^1_{\mathbf{k}m} = \frac{e^{2i\chi_m}}{2V^{1/2}}\left(\frac{A_\mathbf{k} + (-1)^m |C_\mathbf{k}| + \varepsilon_{\mathbf{k}m}}{\varepsilon_{\mathbf{k}m}}\right)^{1/2},$$
$$u^2_{\mathbf{k}m} = (-1)^m e^{2i\varphi_\mathbf{k}} u^1_{\mathbf{k}m},$$

$$v^2_{\mathbf{k}m} = -\frac{e^{2i\chi_m}}{2V^{1/2}}\left(\frac{A_\mathbf{k} + (-1)^m |C_\mathbf{k}| - \varepsilon_{\mathbf{k}m}}{\varepsilon_{\mathbf{k}m}}\right)^{1/2}, \quad \text{(A9)}$$
$$v^1_{\mathbf{k}m} = (-1)^m e^{2i\varphi_\mathbf{k}} v^2_{\mathbf{k}m}.$$

If, in addition, $|C| \ll (A^2 - B^2)^{1/2}$, then

$$|u^m_{\mathbf{k}n}| = \frac{1}{2V^{1/2}}\left(\frac{A_\mathbf{k} + \sqrt{A^2_\mathbf{k} - B^2_\mathbf{k}}}{\sqrt{A^2_\mathbf{k} - B^2_\mathbf{k}}}\right)^{1/2},$$

$$|v^m_{\mathbf{k}n}| = \frac{1}{2V^{1/2}}\left(\frac{A_\mathbf{k} - \sqrt{A^2_\mathbf{k} - B^2_\mathbf{k}}}{\sqrt{A^2_\mathbf{k} - B^2_\mathbf{k}}}\right)^{1/2}. \quad \text{(A10)}$$

Consider Eqs. (A8) and (A10). In getting them we ignored magnetic interactions among the moments and between the moments and the external magnetic field. However, in the first case [Eq. (8)] it was assumed that the magnetic interaction between the moments played a greater role than the interactions with the external field and in the second case, the opposite. As can be seen from the equations, the limiting values of the amplitudes are therefore not in agreement.

The Hamiltonian for the interaction of the system with the alternating magnetic field [see Eq. (9)] in the second-quantization representation is written in the form

$$\hat{\mathcal{H}}_{int} = -h\mu V e^{-i\omega t}\sum_\lambda (u^{*2}_{\lambda 2} u^2_{\lambda 1} - u^{*1}_{\lambda 2} u^1_{\lambda 1} + v^{*2}_{\lambda 2} v^2_{\lambda 1} - v^{*1}_{\lambda 2} v^1_{\lambda 1})\, a^+_{\lambda 2} a_{\lambda 1} + \text{comp. conj.}$$

where u and v in the case considered are defined by Eqs. (A5). Substitution leads to Eq. (10).

[1] M. I. Kaganov and V. M. Tsukernik, JETP 34, 106 (1958), Soviet Phys. JETP 7, 73 (1958).

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