RADIATIVE CORRECTIONS TO THE β DECAY OF THE NEUTRON

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Submitted to JETP editor December 10, 1962

J. Exptl. Theoret. Phys. (U.S.S.R.) **44**, 1628-1631 (May, 1963)

It is shown that, if terms of order $\alpha m/M$ are neglected, the radiative corrections to the β decay of the neutron only affect the value of the decay constants, but do not change the correlation or polarization properties of the decay. The reason for this lies in the γ_5 invariance of the theory in the approximation under consideration, which forbids the S, T, and P variants.

1. INTRODUCTION

THE decay of the neutron is the simplest process which permits, in principle, a verification of the universal theory of weak interactions and the conserved vector current hypothesis of Feynman and Gell-Mann. The effect of weak magnetism on the decay of the neutron has been studied by Bilen'kiĭ et al.[1] These authors noted that the weak magnetism (together with the kinematic corrections for the recoil) is the only source of corrections, if one restricts oneself to effects whose contribution does not exceed the fraction m/M of the main effect (m and M are the masses of the electron and the nucleon). In this connection it is important to investigate whether the radiative corrections also contribute significantly in this order.

The radiative corrections to the β decay of the neutron have been discussed in several papers.[2-4] However, in all these calculations an average over the spins of the particles was taken and only formulas for the radiative corrections to the decay rate were given. For our purpose we would need formulas in which the spin appears explicitly. Such formulas would allow us to obtain the corrections to the effective interaction Hamiltonian itself.

It is known from the old papers[2-4] (see also [5]) that the radiative corrections, first of all, make the decay constants infinite. Therefore, we can consider the magnitude of these constants only with an accuracy up to order α. However, the divergent integrals do not necessarily enter in all observable effects.

Let us consider, for example, the polarization of the electron. If we neglect the electron mass, the longitudinal polarization in the vector as well as pseudovector variants is exactly equal to -1. A change in the polarization can in this case only come from admixtures of the remaining three variants S, T, and P, for which the sign of the polarization is reversed. Owing to the finite mass of the electron, the magnitude of the polarization is actually $-v/c$ (velocity) instead of -1, but is still the same in both variants V and A. Therefore, the polarization of the electron can only be altered through an admixture of the variants S, T, and P.

If we regard the quantities g_V and g_A as experimental constants, we are, of course, only interested in the question whether the effects of the S, T, and P interactions appear in the terms of order α or only in the terms of order $\alpha m/M$. The results described below show that these other types of interaction have no effects of order α so that the renormalization of the decay constants is the only effect of the radiative corrections in this order. In view of this circumstance, the conserved vector current hypothesis can be tested, in principle, with an accuracy of order $\alpha m/M \sim 5 \times 10^{-6}$.

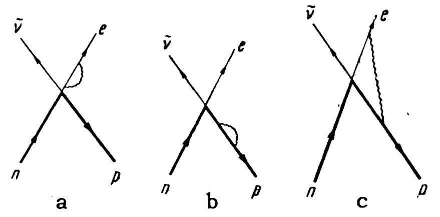
2. EFFECTIVE HAMILTONIAN WITH RADIATIVE CORRECTIONS

As mentioned above, we shall only consider radiative corrections of order α and neglect effects of order $\alpha m/M$. We can therefore calculate the radiative corrections with the help of the following uncorrected β decay Hamiltonian:[1)]

$$H_0 = G2^{-1/2} \langle e \mid \gamma_\mu (1 + \gamma_5) \mid \nu \rangle \langle p \mid \gamma_\mu (1 + a\gamma_5) \mid n \rangle. \quad (1)$$

The graphs describing the radiative corrections in lowest order are shown in the figure. The matrix

1)The effect of the weak magnetism gives a contribution of order $\alpha m/M$.



elements for the graphs a and b, corresponding to the renormalization of the external lines, are proportional to the Hamiltonian (1). Therefore, only the matrix element for the graph c has to be considered:[2)]

$$M_3 = (2^{-1/2}Ge^2/16\pi^2)\{4pPI^0\bar{u}_e\gamma_\mu(1+\gamma_5)v_\nu\bar{u}_p\gamma_\mu(1+a\gamma_5)u_n + 2I_\alpha[\bar{u}_e\hat{P}\gamma_\alpha\gamma_\mu(1+\gamma_5)v_\nu\bar{u}_p\gamma_\mu(1+a\gamma_5)u_n - \bar{u}_e\gamma_\mu \times(1+\gamma_5)v_\nu\bar{u}_p\hat{p}\gamma_\alpha\gamma_\mu(1+a\gamma_5)u_n] - I_{\alpha\beta}\bar{u}_e\gamma_\sigma\gamma_\alpha\gamma_\mu \times(1+\gamma_5)v_\nu\bar{u}_p\gamma_\sigma\gamma_\beta\gamma_\mu(1+a\gamma_5)u_n\}, \qquad (2)$$

where p and P are the four-momenta of the electron and the proton, and

$$I^0 = \frac{1}{\pi^2 i}\int d^4k\frac{1}{[k^2+2kp][k^2-2kP][k^2+\lambda^2]},$$

$$I_\alpha = \frac{1}{\pi^2 i}\int d^4k\frac{k_\alpha}{[k^2+2kp][k^2-2kP][k^2+\lambda^2]} = I^1P_\alpha + I^2p_\alpha,$$

$$I_{\alpha\beta} = \frac{1}{\pi^2 i}\int d^4k\frac{k_\alpha k_\beta}{[k^2+2kp][k^2-2kP][k^2+\lambda^2]} = I^3\delta_{\alpha\beta} + I^4P_\alpha P_\beta + I^5p_\alpha p_\beta + I^6(p_\alpha P_\beta + p_\beta P_\alpha). \qquad (3)$$

The first term in (2) is proportional to the Hamiltonian (1), and the second and third are equal to

$$-2I^1M^2\bar{u}_e\gamma_\mu(1+\gamma_5)v_\nu\cdot\bar{u}_p\gamma_\mu(1+a\gamma_5)u_n - 2I^1\bar{u}_e\gamma_\mu(1+\gamma_5)v_\nu\;\bar{u}_p\hat{p}\hat{P}\gamma_\mu(1+a\gamma_5)u_n + 2I^2\bar{u}_e\hat{P}\hat{p}\gamma_\mu \times(1+\gamma_5)v_\nu\cdot\bar{u}_p\gamma_\mu(1+a\gamma_5)u_n + 2I^2m^2\bar{u}_e\gamma_\mu(1+\gamma_5) \times v_\nu\cdot\bar{u}_p\gamma_\mu(1+a\gamma_5)u_n; \qquad (4)$$

and

$$-2(1+a)I_{\alpha\alpha}\bar{u}_e\gamma_\mu(1+\gamma_5)v_\nu\cdot\bar{u}_p\gamma_\mu(1+\gamma_5)u_n$$
$$-2(1-a)I^3\bar{u}_e\gamma_\mu(1+\gamma_5)v_\nu\cdot\bar{u}_p\gamma_\mu(1-\gamma_5)u_n$$
$$-2(1-a)I^4\bar{u}_e\hat{P}(1+\gamma_5)v_\nu\cdot\bar{u}_p\hat{P}(1-\gamma_5)u_n$$
$$-2(1-a)I^5\bar{u}_e\hat{p}(1+\gamma_5)v_\nu\cdot\bar{u}_p\hat{p}(1-\gamma_5)u_n$$
$$-2(1-a)I^6\bar{u}_e\hat{p}(1+\gamma_5)v_\nu\cdot\bar{u}_p\hat{P}(1-\gamma_5)u_n$$
$$-2(1-a)I^6\bar{u}_e\hat{P}(1+\gamma_5)v_\nu\cdot\bar{u}_p\hat{p}(1-\gamma_5)u_n \qquad (5)$$

respectively.

The first term in (4) is proportional to the Hamiltonian (1), the first three terms in (5) give contributions of order α only to the V and A variants (in the nonrelativistic approximation), and the remaining terms are of order $\alpha m/M$. Thus, in the order α, the radiative corrections lead only to a renormalization of the constants g_V and g_A but not to the appearance of other variants, i.e., in this approximation the effective Hamiltonian with radiative correction also has the form (1). Therefore, the radiative corrections do not change the correlation or polarization properties of the decay in this order, so that the latter are completely determined by the weak magnetism.

[2)]We note that all terms in (2) are invariant under the transformation $(u_e, u_\nu) \to \gamma_5(u_e, u_\nu)$.

3. CONNECTION WITH γ_5 INVARIANCE

The physical meaning of our result becomes clear if we consider the change in the effective Hamiltonian under multiplication of the lepton spinors u_e and u_ν by γ_5, i.e., under the substitution

$$u_e \to \gamma_5 u_e, \qquad u_\nu \to \gamma_5 u_\nu,$$
$$\bar{u}_e \to -\bar{u}_e\gamma_5, \qquad \bar{u}_\nu \to -\bar{u}_\nu\gamma_5. \qquad (6)$$

If the electron mass is set equal to zero, the weak interaction Lagrangian is known to be invariant under the transformation (6) (in the general case the sign of the mass has to be reversed, $m \to -m$). The electromagnetic interaction is also invariant under (6). The variants S, T, and P, on the other hand, change sign under this transformation. From this it follows immediately that for $m \to 0$ only those terms remain in the effective Hamiltonian which correspond to the vector and pseudovector variants. The other terms must be proportional to the mass of the electron and appear only in the order $\alpha m/M$.

4. APPLICATION OF THE RESULTS TO THE DECAY $\pi \to e + \nu$

In the universal V-A theory of weak interactions the matrix element for the decay

$$\pi \to e + \nu \qquad (7)$$

is proportional to the mass of the electron m_e, so that the ratio of the decay rates

$$R = w(\pi \to e + \nu)/w(\pi \to \mu + \nu)$$

is very small ($\sim 10^{-4}$). A measurement of this ratio is one of the experimental tests of the V-A theory. Here the question arises: can the radiative corrections give a contribution of order α? If so, the radiative corrections change the ratio R.

We have made an explicit calculation and obtained the following result: the radiative corrections give a contribution of order $m_e\alpha/m_\pi$; for

$m_e = 0$ the matrix element for the decay (7) vanishes even if radiative corrections are included. Thus the radiative corrections change the ratio R only by a few percent. Evidently, this fact is also a consequence of the γ_5 invariance of the electromagnetic and weak interactions.

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[2] Behrends, Finkelstein, and Sirlin, Phys. Rev. **101**, 866 (1956).

[3] S. M. Berman, Phys. Rev. **112**, 267 (1958).

[4] T. Kinoshita and A. Sirlin, Phys. Rev. **113**, 1652 (1959).

[5] Ya. A. Smorodinskiĭ and Ho Tso-hsiu, JETP **38**, 1007 (1960), Soviet Phys. JETP **11**, 724 (1960).

Translated by R. Lipperheide
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