THE RESISTANCE OF THIN PLATES AND WIRES IN A MAGNETIC FIELD

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The resistance of plates and wires that are thin compared with the mean free path, l, is calculated for the entire range of magnetic fields. In strong magnetic fields ($r \ll l$, where r is the Larmor orbit radius) the static "skin-effect" previously pointed out[1] is taken into account.

1. INTRODUCTION

A large amount of work has been devoted to calculating the resistance of metals in a constant magnetic field **H**. The great interest in this question is associated with the possibility of additional checks on the basic assumptions of the modern electronic theory of metals, and of clarifying the features of the dispersion law for conduction electrons close to the bounding Fermi surface. The study of thin plates and wires can naturally throw additional light on this problem.

In the existing works, however, the range of fields H for which $r > d$ (r is the Larmor orbit radius, d is the thickness of the specimen; for $d \sim 10^{-3}$ cm this corresponds to $H < 10^4$ Oe) has been little studied. Also, the region of "strong" fields, $r < d$, has been treated without taking into account the inhomogeneity of the Hall field and the "screening" of the static current thereby caused. As shown in [1], this usually leads to incorrect results precisely in the case that allows the subsequent theory to be developed—the case of a specimen with a "good" surface (imperfections small compared with r), when the surface can be taken into account with an appropriate boundary condition. If the specimen surface is "bad" (imperfections of order r) the development of the subsequent theory is, on the one hand, extremely complex, and on the other hand is unnecessary, since all the results of interest to us can be obtained from simple physical considerations.

In the present paper the resistance of plates and wires is found in the entire range of magnetic fields for $d \ll l$ (l is the free path of electrons). It is assumed that the linear dimensions of the wire cross section are of equal order in all directions; the shape of the cross section is arbitrary. The direction of the magnetic field is everywhere taken as the z axis, and the direction of the total current **J** is taken as the ν axis (for the wire the ν axis coincides of course with its axis).

2. STRONG MAGNETIC FIELD; $r \ll d$

A. If the magnetic field is very strong, $r \ll d$, and is not directed along the wire axis (or parallel to the plate surface) all the electrons playing an essential part collide with the surfaces, if $d \ll l$, so that d assumes the role of free path, and in all other respects everything proceeds just as in the bulk metal where $d \gg l$.

The resistance ρ of bulk metal for a specimen with a "poor" [1)] surface has been found in a paper by I. Lifshitz, the present author, and Kaganov [2], $\rho_\infty^b = \rho(d, l)$, and for a "good" surface in a paper by the present author [1], $\rho_\infty^g = \rho(d, r, l)$ (for $d \ll l$ the condition for the applicability of the formulae of [1] is obviously satisfied). In accordance with what has been said we obtain for a thin specimen

$$\rho_d^b = \rho(r, d), \qquad \rho_d^g = \rho(d, r, d). \tag{1}$$

Using (1) and the results of the works mentioned above [1,2] we arrive at the following relations for the resistance:

<u>Specimen with Bad Surface (Specimen Shape Arbitrary)</u>

For $n_1 \neq n_2$ (n_1 and n_2 are respectively the number of electrons and "holes") we have

$$\rho_d^b(H) \sim \rho_\infty(0)\, l/d \sim d^{-1}, \tag{2}$$

and for $n_1 = n_2$

[1)]The characteristic dimension δ of the random imperfections of the specimen surface is of course small compared with d. Therefore a "good" surface implies a magnetic field such that $r \gg \delta$, and a "poor" one has $r \ll \delta$ and occurs for thin specimens only in limiting strong magnetic fields.

$$\rho_d^b(H) \sim \rho_\infty(0)\,(l/d)\,(d/r)^2 \sim \rho_\infty(0)\,ld/r^2 \sim dr^{-2} \sim H^2 d, \quad (3)$$

where $\rho_\infty(0)$ is the resistance of an infinite specimen with H= 0.

Specimen with Good Surface

For a wire with any ratio of n_1 to n_2, for a plate with $n_1 \neq n_2$ and with **H** not lying in the $\xi\nu$ plane (ξ is the normal to the surface), and for a plate with $n_1 = n_2$ and with the magnetic field lying in the same plane as the total current **J** and the normal to the surface ξ, we have

$$\rho_d^g(H) \sim \rho_\infty(0)\,l/r \sim H. \quad (4)$$

For a plate with $n_1 = n_2$ in the remaining cases, and with $n_1 \neq n_2$ when **H** lies in the $\xi\nu$ plane but is not parallel to ξ we have

$$\rho_d^g(H) \sim \rho_\infty(0)\,ld/r^2 \sim d/r^2 \sim H^2 d. \quad (5)$$

For a plate with $n_1 \neq n_2$ and $\mathbf{H} \parallel \xi$ we have

$$\rho_d^g(H) \sim \rho_\infty(0)\,l/d \sim d^{-1}. \quad (5a)$$

B. We now consider the remaining special case in which the magnetic field is parallel to the axis of a wire (and consequently parallel to the direction of the total current) or to the surface of a plate. In this case the only characteristic parameter in directions normal to the surface is r, and the surface affects matters only at such distances. Therefore, in the case of a specimen with a poor surface (in the initial approximation the formulae for bulk metal are valid) the relative increase above the resistance of a bulk specimen is of order r/d, and is positive both for $n_1 \neq n_2$ with any mutual orientations of **J** and **H**, and for $\mathbf{J} \parallel \mathbf{H}$ with any $n_1 - n_2$ (since the surface decreases the free path of the electrons colliding with it):

$$\rho_d^b(H) \sim \rho_\infty^b(H)\,(1 + \alpha r/d), \qquad \alpha \sim 1,$$

i.e., for a wire and plate with $\mathbf{H} \parallel \mathbf{J}$, $n_1 \neq n_2$

$$\rho_d^b(H) \sim \rho_\infty(0)\,(1 + \alpha r/d) \quad (6)$$

and for a plate with $n_1 = n_2$ and **H** not parallel to **J**

$$\rho_d^b(H) \sim \rho_\infty(0)\,(l/r)^2. \quad (7)$$

If the specimen surface is good, we obtain, using directly the formulae in [1]: for a wire with any $n_1 - n_2$

$$\rho_d^g(H) \sim \rho_\infty(0)\,(1 + \alpha r/d); \quad (8)$$

for a plate with $n_1 \neq n_2$ and $\mathbf{H} \parallel \nu$, or for $n_1 \neq n_2$ and any mutual orientations of **H** and ν

$$\rho_d^g(H) \sim \rho_\infty(0)\,(1 + \alpha r/d) \quad (9)$$

and for $n_1 = n_2$ and **H** not parallel to ν

$$\rho_d^g(H) \sim \rho_\infty(0)\,d/r \sim d/lr. \quad (10)$$

We point out the formulae obtained in other works for the particular case of a quadratic dispersion law. These are: the formulae for the plate with a good surface: 1) in a perpendicular field ($\mathbf{H} \parallel \xi$) [3,4] [formulae (5) and (5a) of the present paper], 2) in a parallel field with $n_1 \neq n_2$ [4-6] [formula (9) here].

It should be noted that for $r \ll d \ll l$ a curious phenomenon occurs—an oscillation of the resistance in an inclined magnetic field, which was first predicted by Sondheimer[3], and which was treated for an arbitrary dispersion law by Gurevich[7]. However, as noted in [1], Gurevich wrote down the conductivity tensor incorrectly, and—what is extremely important—did not take into account the inhomogeneity of the Hall current. The correction oscillatory in H can be found by successive approximations if the initial approximation found in [1] is used. The period of the oscillations then agrees, of course, with that found by Gurevich, but the order of magnitude of the amplitude can differ. The consideration of this question will be the subject of a separate communication.

3. WEAK MAGNETIC FIELD

We now proceed to fields corresponding to $r \gg d$. Here, of course, the quality of the surface does not play an important part. Because the smallest characteristic length is d, pronounced inhomogeneity of the electric field and current cannot occur in this depth. This allows us to write down immediately a formula for the effective conductivity and to obtain all the asymptotic relations of interest to us without solving the equation for the field normal to the surface (since it is clear at the start that $\mathbf{E}(\mathbf{r})$ is of the order of **E** at the surface), which makes the solution unusually simple compared with that we treated in [1]. Of course, because the actual form of $\mathbf{E}(\mathbf{r})$ is not found, the numerical constants in front of the corresponding expressions are not hereby determined. However, their evaluation presents no interest because they are related to the form of the collision integral, and to the way the electrons are reflected from the surface, and none of them can be used for comparing theory with experiment.

The calculation of the asymptotic relations presents no difficulties of principle; it is, however, an extremely tedious theoretical problem. Wherever possible, therefore, we shall find the form of $\rho(H)$ by using obvious physical considerations.

A. Weak Magnetic Field: $d \ll r$. Wire

In the case of a wire in a magnetic field inclined with respect to its axis, the field causes only a small "twisting" of the orbit, as a result of which the path traversed between collisions with the surface (since $d \gg l$, the mean free path does enter into the problem in the initial approximation with respect to the magnetic field) is of order $d\{1 + \alpha(d/r)^2\}$, $\alpha \sim 1$. Therefore,

$$\rho_d(H) \sim \rho_\infty(d)\left\{1 - \alpha\left(\frac{d}{r}\right)^2\right\} \sim \rho_\infty(0)\frac{l}{d}\left\{1 - \alpha\left(\frac{d}{r}\right)^2\right\}. \tag{11}$$

If the magnetic field is directed along the axis of the wire (and, consequently, parallel to the total current) electrons appear, which owing to the small radius of their orbits $r < d$ do not collide with the surface, and traverse without collision a path of order l. The relative number of such electrons corresponds to the relative number of orbits with diameters less than d when the mean diameter is of order 2r. Since only electrons on the limiting Fermi-surface play an essential role ($r \sim cp_\perp/eH$, $p_\perp$ is the projection of electron momentum on the plane $p_x p_y$, $\mathbf{H} \parallel \mathbf{z}$), the electrons of interest to us correspond to the surface of a section of radius d on a surface of radius r, i.e., their relative number is of order $(d/r)^2$. The remaining electrons contribute a conductivity of the same order as in the absence of a magnetic field.

The resulting conductivity is thus

$$\sigma_d(H) \sim \sigma_d(0) + \alpha\sigma_\infty(0)\left(\frac{d}{r}\right)^2, \qquad \alpha \sim 1$$

and the resistance is

$$\rho_d(H) \sim \rho_\infty(0)\left\{\frac{d}{l} + \alpha\left(\frac{d}{r}\right)^2\right\}^{-1} \sim \rho_d(0)\left\{1 + \alpha\frac{ld}{r^2}\right\}^{-1}, \qquad \alpha \sim 1. \tag{12}$$

B. Weak Magnetic Field Parallel to the Surface; $r \gg d$. Plate

We now turn to the calculation of the resistance of a plate in fields such that $r \gg d$. It is apparent that this region can be divided into several ranges. In order to understand what influences the variation of ρ with $\mathbf{H}$ in such fields, we start with the case when the magnetic field is parallel to the surface of a plate.

The conductivity of a plate of thickness d in a magnetic field parallel to its surface is influenced by the following groups of electrons (see Fig. 1):

a) Electrons of type a, which do not collide with the surface and contribute a conductivity the same as in the bulk metal. The relative number of such

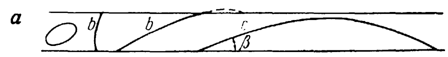

FIG. 1. The trajectories of electrons in a plate in a weak magnetic field parallel to its surface.

electrons is of order $(d/r)^2$ [see the derivation of formula (12)]. The conductivity associated with these electrons is

$$\sigma_{ik}^{(a)}(d, H) \sim (d/r)^2\sigma_{ik}(\infty, H).$$

b) Electrons of type b, which collide with both surfaces of the plate. For $r \gg d$, these consist of almost all the electrons. These electrons are only slightly "twisted" by the magnetic field, and traverse a path $1 + \alpha(d/r)^2$ times greater than in zero magnetic field with $\alpha \sim 1$. In the absence of a magnetic field, the principal role is played by the "grazing" electrons that proceed almost parallel to the plate surface, (see [8,9]) which contribute a conductivity of order $(\sigma_0 d/l)\ln(l/d)$ (σ_0 is the conductivity of the bulk metal; $\sigma_0 d/l$ does not depend on l). In a magnetic field the maximum free path of electrons of type b is of order l if $\sqrt{rd} > l$, and of order $\sqrt{rd}$ if $\sqrt{rd} < l$ (since, if $\sqrt{rd} < l$, electrons reflected from the surface at an angle smaller than $\sqrt{rd}/d$ do not collide with the second surface, and the maximum path of electrons colliding with both surfaces is of order $\sqrt{rd}$). The contribution of these electrons to the conductivity is, therefore, of order

$$\sigma_0\frac{d}{l}\ln\frac{l}{d} \text{ for } \sqrt{rd} > l, \qquad \sigma_0\frac{d}{l}\ln\frac{\sqrt{rd}}{d} \text{ for } \sqrt{rd} < l.$$

c) Electrons of type c that return to the same surface from which they set out. For $\sqrt{rd} < l$ the relative number of these electrons is of order $d/\sqrt{rd} \sim \sqrt{d/r}$ (since they consist of all electrons that avoid collisions with the second surface, i.e., those setting out at an angle $\beta \lesssim d/\sqrt{rd}$, and departure at any angle is almost equally probable), and the path traversed by these electrons is of order $\sqrt{rd}$, so that their contribution to the conductivity (which is proportional to the effective free path and the relative number of electrons) is of order

$$\sigma_0\frac{\sqrt{rd}}{l}\sqrt{\frac{d}{r}} \sim \sigma_0\frac{d}{l}$$

and does depend on the magnetic field.

For $\sqrt{rd} > l$ the majority of electrons of type c traverse a path of order l, do not have time to return to the surface without collisions in the bulk, and behave in the main just as in the absence of the magnetic field. There are, however, electrons with a small radius r' (i.e., with small p_y) such that $\sqrt{r'd} \lesssim l$. The relative number of such electrons is of order $(r'/r)^2$, i.e., of order $(r_1/r)^2$ with $r_1 \sim l^2/d$. Of the similar electrons, those that depart at an angle $\beta \lesssim d/l$ (their relative number is d/l) are "safeguarded" by the magnetic field against colliding with the second surface, and traverse a path of order l, which is greater than in zero magnetic field. It is clear that the contribution of these electrons to the conductivity is of order $\sigma_0(r_1/r)^2 d/l$.

We now determine the conductivity of a plate in various magnetic fields (but with $r \gg d$).

1) Very weak magnetic fields: $r > r_1 \sim l^2/d \gg l$. Taking into account that here we have $\sigma_{ik}(\infty, H) \sim \sigma_{ik}(\infty, 0) \sim \sigma_0$, we easily see that in the initial approximation the conductivity is, as would be expected, the same as in zero magnetic field, and is provided by electrons of type b, while the principal field-dependent contribution to the conductivity is due to electrons of type c being "safeguarded" against collisions with the second surface. As a result:

$$\sigma(H, d) \sim \sigma_0 \frac{d}{l} \ln \frac{l}{d} + \delta\sigma_0 \frac{d}{l}\left(\frac{r_1}{r}\right)^2 \sim \sigma(0, d)\left\{1 + \delta \frac{1}{\ln(l/d)}\left(\frac{r_1}{r}\right)^2\right\}, \quad \delta \sim 1, \ r_1 \sim l^2/d. \tag{13}$$

Thus even in fields such that $r \sim r_1 \sim l^2/d$ (for $d \sim 10^{-3}$ cm, $l \sim 10^{-1}$ cm, this corresponds to $H \sim 1$ Oe) a significant increase in the conductivity occurs (by a factor of about $1 + \delta \ln(l/d)$). It is clear from the method of derivation that a similar effect is only possible in a plate of length greater than l.

For a quadratic dispersion with $\mathbf{H} \parallel \mathbf{j}$ ($\mathbf{j}$ is the current density), formula (13) was first obtained in [6].

2) Weak magnetic fields: $l \ll r \ll r_1$. As before we have $\sigma_\infty(H) \sim \sigma_\infty(0) \sim \sigma_0$. The main contribution to the conductivity is due to electrons of type b, and

$$\sigma(d, H) \sim \sigma_0 \frac{d}{l} \ln \sqrt{\frac{r\alpha}{d}}, \qquad \alpha \sim 1. \tag{14}$$

3) Strong magnetic fields: $d \ll r \ll l$. Electrons of types a and b play the principal roles. Because the conductivity components due to electrons of type a are strongly anisotropic, the entire effective conductivity tensor should be written out. Since d is the smallest length it is clear at the start that $E_y(y) \sim E_y(0)$ (see also the start of the section). Calculating the contribution these electrons make to the conductivity components in a way similar to that used in [1] (only now the electric field changes in a region of order of the orbit radius for the electrons of interest to us), we obtain

$$\hat{\sigma}_d(H) \sim \sigma_\infty(0)\begin{pmatrix} k+\gamma\lambda^2 & k+\gamma\lambda^2 & k+\lambda^2 \\ k+\gamma\lambda^2 & k+\gamma^2\lambda^2 & k+\gamma\lambda^2 \\ k+\gamma\lambda^2 & k+\gamma\lambda^2 & k+\lambda^2 \end{pmatrix};$$

$$\lambda \sim \frac{r}{d}, \qquad \gamma \sim \frac{r}{l}, \qquad k \sim \frac{d}{l}\varepsilon \ln\frac{r\alpha}{d}, \qquad \varepsilon = 1 - \frac{d}{2r_{max}}, \tag{15}$$

where λ^2 takes into account the relative number of electrons of type a, γ is the effect of the magnetic field on these electrons, $(d/l)\ln\sqrt{r\alpha}/d$ is the contribution to the conductivity of electrons of type b, and ϵ is the relative number of electrons of type b (it is obvious that when the maximum orbit diameter $2r_{max}$ approximates to the thickness d of the plate, the number of electrons that collide with both surfaces diminishes; for $2r_{max} < d$ there are in general no such electrons).

From (15) we easily find:

a) In the case when $1 - d/2r_{max} \sim 1$ ($\epsilon \sim 1$) i.e., when the field H is not close to the "critical" field (for which $2r_{max} = d$), the effective resistivity tensor in the plane of the plate has the form:

$$\hat{\rho}_d(H) \sim \rho_\infty(0)\begin{pmatrix} k^{-1} & (k+\lambda^2)^{-1} \\ k^{-1} & (k+\lambda^2)^{-1}\end{pmatrix}. \tag{16}$$

For all directions of **H**, except $\mathbf{H} \parallel \mathbf{J}$, we have

$$\rho_d(H) \sim \rho_\infty(0)\frac{l}{d}\left(\ln\frac{r\alpha}{d}\right)^{-1}. \tag{17}$$

If $\mathbf{H} \parallel \mathbf{J}$, then

$$\rho_d(H) \sim \rho_\infty(0)\frac{l}{d}\left\{\ln\frac{r\alpha}{d} + \beta\frac{ld}{r^2}\right\}^{-1}, \qquad \alpha, \beta \sim 1. \tag{18}$$

b) In the case $\epsilon \ll 1$, when the field H is close to the "critical" field ($2r_{max} \approx d$), the longitudinal specific resistance tensor has the form

$$\hat{\rho}_d(H) \sim \rho_\infty(0)\begin{pmatrix} 1+k\gamma^{-2} & 1 \\ 1+k\gamma^{-2} & 1\end{pmatrix} \sim \rho_\infty(0)\begin{pmatrix} 1+k^{-1}\varepsilon & 1 \\ 1+k^{-1}\varepsilon & 1\end{pmatrix}. \tag{19}$$

Thus for **H** not parallel to **J**

$$\rho_d(H) \sim \rho_\infty(0)\left\{1 + \alpha\frac{l}{d}\left(1 - \frac{d}{2r_{max}}\right)\right\}, \tag{20}$$

and for $\mathbf{H} \parallel \mathbf{J}$

$$\rho_d(H) \sim \rho_\infty(0). \tag{21}$$

C. Weak Inclined Magnetic Field, $r \gg d$. Plate

The trajectories of electrons in an inclined field (Fig. 2) differ from the trajectories in a parallel field in a way of interest to us only in that the electrons that do not collide with the surface should have not only a small radius, but also a small velocity v_z. In order that electrons should move

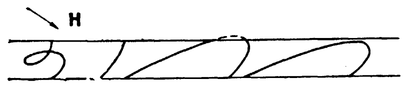

FIG. 2. The trajectories of electrons in a plate in an inclined magnetic field.

without collisions for a time τ, it is necessary that $v_z/v \sim d/l$. As a result, electrons of type a do not provide a significant contribution to the conductivity.

An exception is the particular case of the magnetic field normal to the surface, and electron orbits that are plane in real space when $v_z = 0$, i.e., v_z dependent only on p_z. The case of such special Fermi surfaces is completely analogous to the case of a quadratic dispersion law treated in [3,4], where the path along the film is of order r when $r < l$ and of order l when $r > l$; so that

$$\rho_d(H) \sim \rho_\infty(0)\frac{l}{d}\left\{\ln\left(\frac{d}{l}+\frac{d}{r}\right)\right\}^{-1}.$$

In the general case for $r \gg l$ and in an inclined field we obtain formulae (13), (14). For $r \ll l$, because electrons of group a are unimportant, formula (14) is valid up to $r \sim d$.

CONCLUSIONS

We now present, using all the formulae obtained, graphs of the function $\rho(H)$ for the entire range of magnetic fields in the various cases: Fig. 3—the wire in an inclined field; Fig. 4—the wire in a longitudinal field; Fig. 5—the plate in an inclined field and Fig. 6—the plate in a parallel field [δ is the characteristic dimension of the random "roughnesses" of the surface (see [1])]. The curves given allow the anisotropy of ρ to be seen easily.

It is curious to note: a) the rapid fall in the resistance of a plate at the field $r \sim l^2/d$ (for $d \sim 10^{-3}$ cm, $l \sim 10^{-1}$ cm, this corresponds to $H \sim 1$ Oe); b) the linear relation that is the general case in strong fields and the break at $2r_{max} = d$, for the plate, which allows the diameter of the Fermi surface to be determined; c) for $H \not\parallel J$ in Fig. 6, the slow change of resistance—the resistance increases for H not parallel to J by a factor of $\ln(l/d)$ on increasing the field by a factor of $(l/d)^2$ (curve $A_2 + B_2/\ln(H_0/H)$ and d) the rapid fall in resistance by a factor of l/d for only a severalfold increase in the field (curve $A_3 - B_3H$).

Also seen from the figures is the strong dependence of the resistance in high fields on the quality of the specimen surface (on the ratio of r to δ). We also note that the question of whether the curve going to saturation in Fig. 6 goes to $\rho_d(\infty)$ or differs from it depends, as is easily seen, on whether a direction of open trajectories coincides with the y-axis. The large number of characteristic points allows the quantities r, l, and δ to be determined for the same specimen under different conditions.

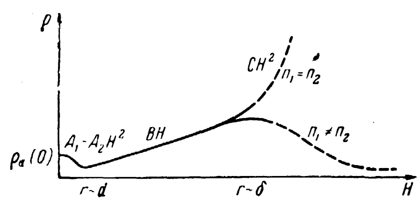

FIG. 3. The function $\rho(H)$ for a wire in an inclined field.

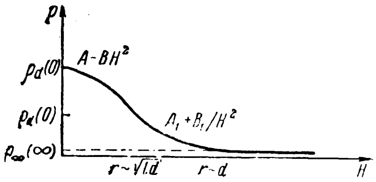

FIG. 4. The function $\rho(H)$ for a wire in a parallel field.

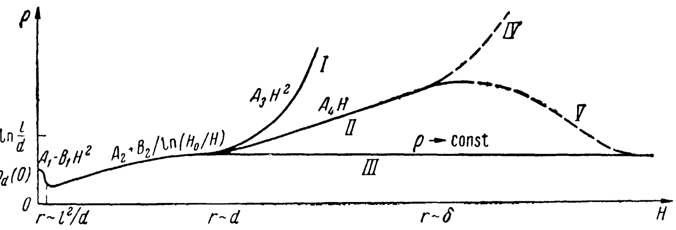

FIG. 5. The function $\rho(H)$ for a plate in an inclined field. Branch I is for $n_1 = n_2$, H not in the $\xi\nu$-plane or $H \parallel \xi$; branch II for $n_1 = n_2$, H in the $\xi\nu$-plane but $H \not\parallel \xi$, or for $n_1 \neq n_2$, H not in the $\xi\nu$-plane; branch III for $n_1 \neq n_2$, H in the $\xi\nu$-plane; branch IV, A_5H^2, for $n_1 = n_2$; branch V for $n_1 \neq n_2$.

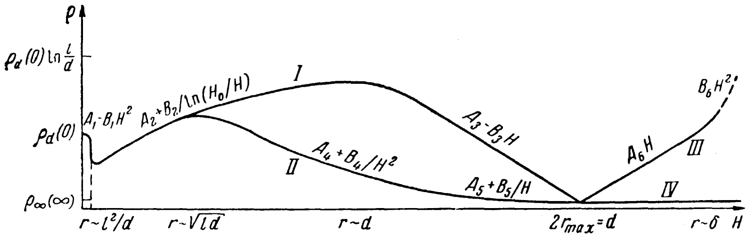

FIG. 6. The function $\rho(H)$ for a plate in a parallel field. Branch I is for $H \not\parallel \nu$; branch II for $H \parallel \nu$; branch III for $n_1 = n_2$, $H \not\parallel \nu$; branch IV for $n_1 \neq n_2$ or $n_2 = n_2$ and $H \parallel \nu$.

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