maximum in the angular diagram of a binary sample always appears when the projection of the hexagonal axis on the plane of rotation of the magnetic field coincides with the direction of that field. The appearance of a minimum at which $\Delta\rho/\rho$ tends to saturation with increase of the field is in this case probably due to the fact that the current is perpendicular to an open direction of the Fermi surface. Consequently one of the open directions is parallel to the hexagonal axis. Therefore the quadratic rise of the resistance at the maximum should be considered as a result of the compensation of the hole and electron volumes of the Fermi surface. The minima in the hexagonal angular dependence indicate that, apart from the open directions parallel to the hexagonal axis, there should also be open directions in the hexagonal plane. However, it is possible that such directions appear only in strong fields, i.e., that they are due to so-called "magnetic breakdown." [4,5] Indeed, if we compare curves 2 and 3 in Fig. 2 we can easily see that while curve 3 even at $H \approx 20$ kOe shows a clear tendency to saturation, curve 2 in fields up to 25 kOe is practically indistinguishable from the quadratic dependence and the tendency to saturation appears in it only in fields exceeding 50 kOe.

Thus, we may assume that the Fermi surface of Re consists of two independent parts: hole and electron. From the data obtained by measurements of the Hall emf it follows that the electron surface is an open one. This open surface has open directions both along the hexagonal axis and in the hexagonal plane.

Concluding, we regard it as a pleasant duty to thank Academician P. L. Kapitza for his constant interest in this work and G. É. Karstens for determination of the crystallographic orientations of the samples.

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[1)]The effective field is the quantity $H_{eff} = H\rho(300°K)/\rho(4.2°K)$. The maximum value of $H_{eff}$ in our experiments was $2.5 \times 10^8$ Oe.

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## *ON THE THEORY OF GREEN'S FUNCTIONS OF VECTOR FIELDS*

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A vector field with mass m and spin 1 is described by the equations

$$(\Box - m^2) A_n^{in} = 0, \quad (1a)$$

$$(g^{nn}\, \partial A_n^{in} / \partial x^n) = 0. \quad (1b)$$

The Lorentz condition (1b) excludes the quanta with zero spin. The quantization is usually carried out in terms of the three-dimensional operators $a_n^{\pm}(\mathbf{k})$, which are related to the four-dimensional operators $a_n(\pm k)$ in the usual way. [1] However, the whole procedure can also be carried through directly in terms of the covariant quantities $a_n(\pm k)$. In place of the usual expression for the commutator function we obtain then

$$D_{mn}^{\pm}(k) = \pm \frac{1}{(2\pi)^3 i}\left(g^{mn} - \frac{k_m k_n}{k^2}\right) \theta(\pm k_0)\, \delta(k^2 - m^2). \quad (2)$$

Because of the factor $\delta(k^2 - m^2)$, this expression is not essentially different from the usual one. However, it is somewhat more general. In particular, it is adapted to the quantization of a field with vanishing mass.

The next step consists in defining the chronological product, i.e., the manner in which to go off the mass shell. The Green's function of different fields is, by definition, constructed in the following way: [1] the Green's function for an arbitrary field is obtained from the Green's function for a scalar field by applying the same operator as the one used to obtain the corresponding commutator function. This operator is usually taken in the x representation. But since the quantization is carried out in the momentum representation and the whole S matrix formalism is adapted to the momentum space, it is natural to consider this operator in k space. We have then instead of the usual expression for $D^c_{mn}$

$$\widetilde{D}^c_{nm}(k) = (2\pi)^{-4}\frac{g^{mn} - k_m k_n k^{-2}}{m^2 - k^2 - i\varepsilon}. \qquad (3)$$

The usual Green's function and the function (3) are obtained from (2) by going off the mass shell in a different fashion. In the first case $k^2$ in the denominator remains on the mass shell, in the second case it does not.

The operators under the chronological pairing operator do not satisfy free field equations. As we go off the mass shell we drop Eq. (1a). We still have some freedom with respect to (1b): either we retain the Lorentz condition for virtual particles or we do not require the exclusion of virtual zero spin quanta. It is easily seen that the usual expression corresponds to the second choice and expression (3) to the first:

$$(k^n \widetilde{D}^c_{mn}(k)) = 0. \qquad (4)$$

$D^c_{mn}$ does not satisfy this relation.

It can be shown that taking (3) as the unperturbed transverse expression leads to a total Green's function including all radiative corrections which is also transverse. By virtue of the relation

$$A_m(x) = A^{in}_m(x) - \int D^R_{mn}(x - x')\, j^n(x')\, dx' \qquad (5)$$

we obtain therefore

$$(g^{mm}\partial A_m(x)/\partial x^m) = 0 \qquad (6)$$

and, by the equation of motion for $A_m(x)$, are led to the conservation law for the current $j_m$:

$$(g^{mm}\,\partial j_m/\partial x^m) = 0. \qquad (7)$$

The choice of the Green's function in the form (3), therefore, automatically leads to the conservation of the corresponding current. It is therefore natural to choose the Green's function in gauge invariant theories of the type of that of Sakurai[2] or Salam[3] in the form (3). Thus gauge invariance implies the absence of real as well as virtual quanta of zero spin. The renormalizability of the theory becomes obvious.

Let us show now that our procedure gives the correct results in electrodynamics, which in the present case is treated in complete analogy with other vector fields with conserved current. As is known, the quantization of the electromagnetic field in the usual formalism meets with serious difficulties. The Lorentz condition cannot be imposed on the field operators and an indefinite metric has to be introduced. Our method is free from these complications. We start from Eqs. (1) with $m = 0$. As noted above, the quantization is carried out in the same way as for a field with nonvanishing mass. As a result we obtain (2) with $m = 0$.

Going off the mass shell in the manner described above, we find for the Green's function the well known transverse expression

$$D^{0c}_{mn}(k) = -(2\pi)^{-4}\left(g^{mn} - \frac{k_m k_n}{k^2}\right)\frac{1}{k^2 + i\varepsilon}. \qquad (8)$$

At no place did we have to use the artifice of an indefinite metric.

The Green's function (3) has a pole at $k^2 = 0$. This pole might at first glance lead to a violation of unitarity. However, for interactions with a conserved current $j_n = \bar{\psi}\gamma^n\psi$, as well as in electrodynamics, this violation does not affect observable quantities, since the part of the Green's function proportional to $k_m k_n$ can be transformed away by a contact transformation.[4] Moreover, as shown by Jouvet,[5] it is necessary in this case that the total Green's function have a pole at $k^2 = 0$ in order that unitarity be preserved in the observable quantities. Our results are in complete accordance with those of Jouvet. In our scheme, in contrast to Jouvet's paper, the polarization tensor is transverse in all orders of perturbation theory.

In theories of the Yang-Mills type,[6] in which the current is also conserved, the presence of the pole at $k^2 = 0$ may lead to a violation of unitarity, since the longitudinal part cannot be removed in this case. This question is currently under investigation. In this connection, it is of interest to mention the work of Goldstone et al.,[7] in which it is shown that theories based on "broken symmetries," to which the Yang-Mills field with nonvanishing mass belongs, require the introduction of particles with vanishing mass.

Our procedure permits us thus to construct a renormalizable theory of vector particles in which zero spin quanta are excluded at all stages. Application of this procedure to electrodynamics leads to the usual results. Here the electromagnetic field is considered on a par with fields with nonzero mass and does not require the introduction of an indefinite metric.

In conclusion I take this opportunity to express my gratitude to V. V. Medvedev, M. K. Polivanov, and A. D. Sukhanov for a useful discussion.

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## *IONIZATION LOSSES OF THE ENERGY OF FAST ELECTRONS IN THIN FILMS*

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It is known that the energy losses, due to ionization, of fast charged particles in dense media at high values of $p/\mu c$ ($p$ is the momentum and $\mu$ is the mass of the particle) remain practically constant due to the density effect. Garibyan[1] showed that if a particle passes through a sufficiently thin plate then its electric field remains the same as it was in vacuum. Therefore, in such a plate the particle ionizes in a manner as if there is no screening action of the medium, i.e., there is no density effect.

Earlier calculations[1] showed that for this to happen the plate thickness $a$ should satisfy the inequality

$$a \ll \frac{2v\Omega}{\sigma} \ln \frac{v\varkappa_0}{\sqrt{1-\beta^2}\,\Omega}, \qquad (1)$$

where $v$ is the velocity of the particle; $\beta = v/c$; $\sigma = 4\pi Ne^2/m$ (the plasma frequency); $\Omega$ is the frequency above which the dielectric constant of the medium can be given by the formula $\epsilon(\omega) = 1 - \sigma/\omega^2$; $\kappa_0$ is a quantity inversely proportional to the distance beyond which the macroscopic approach is applicable.

In plates of this thickness one should expect a logarithmic increase of the ionization losses with increase of the particle energy. The presence of such an increase of the ionization losses may be used to measure the energies of very fast particles in those cases when other methods are not practicable.

This method can obviously be used to determine the energies of particles in monoenergetic beams, although in principle it can also be used for single particles. In the latter case a large number of thin plates must be used to measure the particle energy, so that the total energy losses can be measured sufficiently accurately and with minimum fluctuations. Then, as pointed out earlier,[2] the distance between the thin plates should be much smaller than the quantity in the right-hand part of the inequality (1). This condition is necessary to ensure that the field of the particle is not distorted by the polarization of the medium not only in the first plate but in all the subsequent ones.

### EXPERIMENTAL SECTION AND RESULTS

Measurements were carried out using the linear accelerator of the Physico-technical Institute of the Ukrainian Academy of Sciences. The experimental setup is shown in Fig. 1. Electron beams of energies 20.5, 40, 47.5, 88 MeV were in turn focused on a target consisting of a scintillation film of $10^{-6}$ cm thickness deposited on an aluminim substrate of $10\,\mu$ thickness. In all the measurements the current passing through the target reached $0.01\,\mu$A. The beam intensity was measured with a secondary-emission monitor. The monitor was calibrated by means of a Faraday cylinder and had a constant secondary-emission coefficient in the range of electron energies used in the present work. The scintillation film was prepared from a plastic polystyrene-based scintillator by the usual method of deposition on a substrate.

From Eq. (1) we deduced the following critical values of the thickness $a_0$ of the polystyrene film for different values of the electron energy:

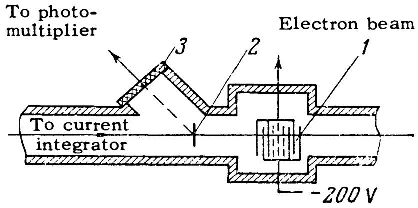


FIG. 1. Schematic representation of the experimental setup: 1) secondary-emission monitor; 2) target; 3) vacuum window.