

'UNIVERSAL' INSTABILITY OF AN INHOMOGENEOUS PLASMA IN A MAGNETIC FIELD

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It is shown that a rarefied low pressure ($p \ll H^2/8\pi$) plasma confined by a magnetic field is "universally" unstable with respect to local short-wave disturbances which do not distort the magnetic field, for any ratio between the space gradients of density and temperature. An analog of such a "universal" instability in the hydrodynamical approximation is an instability due to the finite magnitude of the thermal conductivity along the magnetic field lines of force.

1. INTRODUCTION

AN investigation of the stability of equilibrium of a plasma in a magnetic field, based on the magneto-hydrodynamic model of the plasma, leads, as is well known, to many different instabilities, depending on the configuration of the magnetic field (such instabilities are conveniently designated as "configuration" instabilities). One of the important problems of stability theory has been to find the optimal stable configurations of a plasma in a magnetic field. However, a description of a low-density plasma within the framework of the simple magneto-hydrodynamic model does not exhaust all the possible types of motion. There exist short-wave perturbations (with wavelengths considerably shorter than the characteristic mean free path of the particles), which can be considered in a consistent fashion only within the framework of kinetic theory, based on the Boltzmann equation with self-consistent electromagnetic field. It turns out that an account of several types of such short-wave perturbations leads to local instability (independent of the configuration of the magnetic field). In this sense, such an instability is universal.

A consistent development of the theory of local instability of the plasma on the basis of kinetic theory is difficult. It is therefore reasonable to consider two limiting models:

a) When the magneto-hydrodynamic description of the plasma is no longer valid, but the mean free path is not too large, so that the plasma can be regarded as two interpenetrating fluids (electronic and ionic). For this case we use the so-called two-fluid hydrodynamics.

b) When the collisions become so rare that they can be completely disregarded. For this case we use the kinetic equations without the collision integral.

It is known that a plasma sufficiently far from thermodynamic equilibrium can become unstable against self excitation of various types of plasma oscillations. By now the instabilities of a "homogeneous, infinitely extended" plasma, the non-equilibrium nature of which is manifest in the non-Maxwellian particle velocity distribution, have been well investigated. An important circumstance in this case is that the instability occurs not for all non-Maxwellian velocity particle distributions. In other words, the plasma has a certain "stability margin" near thermodynamic equilibrium.

As applied to the problem of "magnetic thermal insulation" great interest is attached to an investigation of the stability of plasma states for which the non-equilibrium nature manifests itself in the presence of spatial gradients of the density and of the temperature (even if the local particle velocity distribution is as close to Maxwellian as desired). The main problem here is as follows: does the inhomogeneous plasma have a certain "stability margin" or not, that is, does the instability begin for as small temperature gradients and density as desired, or is it necessary that the spatial gradient exceed a certain critical value if the instability is to appear?

If we follow the analogy with the instability of a homogeneous non-Maxwellian plasma, for example "sausage" instability, it may turn out that the presence of such a critical gradient is natural. Indeed, for the occurrence of "sausage" instability it is necessary that the average electron velocity relative to the ions exceed the phase velocities of propagation of the corresponding waves. (Some such "threshold" instabilities are considered in [1,2].)

In an inhomogeneous plasma, however, the situation also changes qualitatively. To verify this, let us turn to Fig. 1. Let the magnetic field \mathbf{H} be

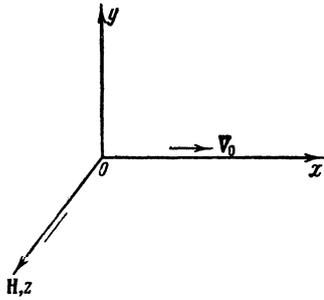


FIG. 1

directed along the z axis (we are considering local instabilities, so that we can assume that \mathbf{H} has the same direction "everywhere") and let it depend on the spatial coordinate x ; accordingly, the ion and electron distribution functions in the unperturbed plasma have the form $f_{i,e}(\mathbf{v}, x)$. We now consider a small perturbation propagating in such a plasma, along the z axis, $\exp[i(\omega t - kz)]$.

In a homogeneous plasma, the main source of the imaginary part in the dispersion equation is the term of type $(eE_z/m) \partial f / \partial z |_{v_z = -\omega/k}$. In a Maxwellian plasma, it is responsible for the damping of the wave (the so-called "Landau damping"). Naturally, terms with such a structure are retained also in a weakly inhomogeneous plasma. However, additional terms appear, resulting from the term $\mathbf{v} \cdot \nabla f$ in the kinetic equation. For example, at low frequencies ($\omega \ll \Omega_i = eH/Mc$), when the motion of the ions and electrons transverse to \mathbf{H} has the form of a drift, the term $\mathbf{v} \cdot \nabla f$ yields for the "electric" drift a contribution $c(E_y/H) \partial f / \partial x$. This term, naturally, can make an additional contribution to the imaginary part, owing to the corresponding half-residue $c(E_y/H) \times \partial f / \partial x |_{v_z = -\omega/k}$. When $E_y \gg E_z$ [which occurs, for example, for vortex-free perturbations ($\text{curl } \mathbf{E} \equiv 0$) with a spatial dependence $\exp i(k_y y + k_z z)$ with $k_y \gg k_z$], this addition may exceed the "Landau damping" even at small gradients, and the plasma will be unstable.

To verify this, let us estimate the work performed by the electric field of the wave on the plasma particles ($\omega/k_z \gg v_i, v_e \gg \omega/k_z$):

$$\begin{aligned} \overline{j_z E_z} &= e v_z f_1 E_z \Big|_{v_z = -\omega/k_z} \\ &\sim e \frac{\omega}{k_z} E_z \left(-\frac{e}{m} E_z \frac{\partial f_0}{\partial v_z} + c \frac{E_y}{H} \frac{\partial f_0}{\partial x} \right) \Big|_{v_z = -\omega/k_z} \\ &= \frac{e^2 E_z^2 f_0 \omega^2}{T k_z^2} \left[1 - c \frac{k_y T}{e H \omega} \left(\frac{n'}{n} - \frac{T'}{2T} \right) \right] \Big|_{v_z = -\omega/k_z}, \end{aligned} \quad (1.1)$$

where

$$f_0 = \frac{n(x) \sqrt{m}}{\sqrt{2\pi T(x)}} \exp\left(-\frac{mv_z^2}{2T(x)}\right).$$

As can be seen from (1.1), for sufficiently low frequencies the transfer of energy from the particles to the wave, connected with the inhomogeneity (second term), actually exceeds the Landau damping.

Under the assumptions made ($v_i \ll \omega/k_z$), we can neglect the motion of the ions along z and the continuity equation for these assumes the form

$$i\omega n_1 + v_x dn_0/dx = 0, \quad v_x = cE_y/H. \quad (2.1)$$

On the other hand, the electrons moving along the force lines with velocity $v_e \gg \omega/k_z$, have time to attain a Boltzmann redistribution

$$n_1 = n_0 e\varphi/T. \quad (3.1)$$

From (2.1) and (3.1), using $E_y = -ik_y \varphi$, we obtain the frequency

$$\omega = k_y c T n_0' / e H n_0. \quad (4.1)$$

In the derivation of (4.1) we have assumed that for ions one can employ the drift approximation (that is, the Larmor radius is assumed to be much smaller than the wavelength).

Substituting the obtained frequency (4.1) into (1.1) we obtain the following instability criterion¹⁾:

$$d \ln T / d \ln n < 0. \quad (5.1)$$

Thus, in the "zero Larmor radius" approximation, the region that remains stable is (see [3])

$$0 < d \ln T / d \ln n < 2. \quad (6.1)$$

If the finite Larmor radius is taken into account, the frequency ω decreases, so that the role of the inhomogeneous term in (1.1) increases and the gap (6.1) may close.

The reason for the decrease in the frequency when the wavelength becomes shorter than the Larmor radius is the decrease in the effective electric field averaged over the particle orbit. Therefore the average drift velocity of the particles v_x in equation (2.1) decreases. Thus, at sufficiently large $k_y r_H$, the velocity v_x will be of order $\sim cE_y / H k_y r_H$. Indeed, a rigorous analysis with the aid of the kinetic equation shows that the gap (6.1) closes, so that the plasma turns out to be universally unstable.

It is curious that the two-fluid hydrodynamics analogs of the kinetic instabilities obtained as a result of the half-residues

$$\sim c(E_y/H) \partial f / \partial x \Big|_{v_z = -\omega/k_z}$$

¹⁾If an analogous estimate is made for the case $\omega \ll k_z v_i$, we can obtain the limit of the stability region for positive $d \ln T / d \ln n$, namely $d \ln T / d \ln n = 2$.

are instabilities that arise when account is taken of the finite heat conductivity along the force lines of the magnetic field (Sec. 3).

2. INVESTIGATION OF THE STABILITY OF A DILUTE INHOMOGENEOUS PLASMA IN A MAGNETIC FIELD, NEGLECTING COLLISIONS

1. Let us consider the dispersion equation. In the derivation of the dispersion equation the following is assumed:

- 1) $p \ll H^2/8\pi$ —the plasma pressure is small compared with the magnetic pressure;
- 2) the plasma is quasi neutral, that is, $n_i = n_e$;
- 3) The electric fields of the perturbations are potential, that is, $\text{curl } \mathbf{E} = 0$ (this is true if $\omega/k_y z \ll H/\sqrt{4\pi\rho}$).

Rosenbluth et al [4] introduced a dispersion equation for the investigation of the stabilization of magnetohydrodynamic instability arising in an inhomogeneous plasma supported by the magnetic field against the force of gravity, with a finite Larmor radius. We use the method employed in that reference to obtain the dispersion equation of interest to us, without imposing the limitation $kr_i \ll 1$.

Unlike [4], we take into account also the inhomogeneity of the temperature. Therefore, the distribution function of the unperturbed plasma assumes the form

$$f_j = [1 + (x + v_y/\Omega_j) d/dx] f_{0j}, \tag{1.2}$$

where

$$f_{0j} = n_0 \left(\frac{m_j}{2\pi T} \right)^{3/2} \exp \left(- \frac{m_j v^2}{2T} \right)$$

is the Maxwellian distribution function (we retain the notation of [4]), whereas the function used in [4] is

$$f_j = \left[1 + \left(x + \frac{v_y}{\Omega_j} \right) \frac{1}{n_0} \frac{dn_0}{dx} \right] f_{0j}. \tag{2.2}$$

In addition, we leave out immediately the force of gravity, which is of no interest to us, and choose the perturbations in the form $\exp i(-\omega t + k_z z + k_y y)$. In [4] they were chosen as $\exp i(-\omega t + k_y y)$.

Leaving out the derivation, which differs only in inessential details from that of [4] (as can be seen, incidentally, even from a comparison of the dispersion equations), we obtain in place of the Rosenbluth—Krall—Rostoker dispersion equation²⁾

²⁾In [4] and accordingly in our paper, several inessential terms have been left out from the dispersion equation; these have for the cases under consideration an order of magnitude r_H/L , where L is the “characteristic dimension” of the spatial inhomogeneity. For similar reasons we have also left out terms containing the second derivatives of x with respect to f_{0j} .

$$\begin{aligned} \frac{2n}{T} - \sum_j \sum_{l=-\infty}^{\infty} \left(\frac{\omega}{T} - \frac{k_y}{m_j \Omega_j} \frac{1}{n_0} \frac{dn_0}{dx} \right) \\ \times \int_0^{\infty} \exp \left(- \frac{m_j v_{\perp}^2}{2T} \right) \frac{m_j v_{\perp} dv_{\perp}}{T(\omega - l\Omega_j)} J_l^2 \left(\frac{k_y v_{\perp}}{\Omega_j} \right) = 0 \end{aligned} \tag{3.2}$$

the equation

$$\begin{aligned} \frac{2n}{T} - \sum_j \sum_l \left(\frac{\omega}{T} - \frac{k_y}{m_j \Omega_j} \frac{d}{dx} \right) \int_{-\infty}^{\infty} \int_0^{\infty} \frac{f_j(v_z) dv_z}{\omega - k_z v_z - l\Omega_j} \\ \times \exp \left(- \frac{m_j v_{\perp}^2}{2T} \right) \frac{m_j v_{\perp} dv_{\perp}}{T} J_l^2 \left(\frac{k_y v_{\perp}}{\Omega_j} \right) = 0. \end{aligned} \tag{4.2}$$

When $\omega \ll \Omega_j$ and $k_z v_{zj} \ll \Omega_j$, precisely the case which we are considering, it is sufficient to retain in the sum over l only the term with $l = 0$, so that the dispersion equation (4.2) becomes

$$\frac{2n}{T} - \sum_j \left\{ \left[\frac{\omega}{T} - \frac{k_y}{m_j \Omega_j} \frac{d}{dx} \right] A(\theta_j) \int_{-\infty}^{\infty} \frac{f_j dv}{\omega - k_z v} \right\} = 0, \tag{5.2}$$

where

$$A(\theta) = \int_0^{\infty} e^{-t} J_0^2(\theta t^{1/2}) dt \equiv e^{-\theta^2/2} I_0(\theta^2/2),$$

$$\theta_j = k_y (2T/m_j)^{1/2} / \Omega_j.$$

As $\theta_j \rightarrow 0$ (which corresponds to the approximation in which the Larmor radius of the particle is equal to zero) $A(\theta) \rightarrow 1$ and (5.2) goes over into the dispersion equation obtained in [3].

Introducing the dimensionless frequency

$$z_j = \omega/k_z (2T/m_j)^{1/2}, \tag{6.2}$$

we rewrite (5.2) in the form

$$\sum_j \{ 1 - \beta A z_j + i V \bar{\pi} \omega(z_j) [z_j + \gamma - \beta z_j^2] A \} = 0, \tag{7.2}$$

where

$$\omega(z) = \frac{i}{\pi} \int_{-\infty}^{\infty} \frac{e^{-t}}{z-t} dt,$$

$$\gamma(0) = - \frac{1}{2} \frac{\theta}{k_z} \left[\frac{dn}{dx} \frac{1}{n} - \frac{1}{2} \frac{dT}{dx} \frac{1}{T} (1 + \delta) \right],$$

$$\delta(0) = \theta^2 \left[1 - \frac{I_1(\theta^2/2)}{I_0(\theta^2/2)} \right], \quad \beta(0) = \frac{1}{2} \frac{\theta}{k_z} \frac{dT}{dx} \frac{1}{T}. \tag{8.2}$$

The Larmor radius of the electron is assumed to be negligibly small ($k_y r_{He} = \theta_e \ll 1$), so that we can put $\delta = 0$ and $A = 1$ in the equation for the electrons.

2. Let us consider the case of “low” frequencies

$$\omega \ll k_z (2T/M)^{1/2}. \tag{9.2}$$

Using the expansion of $w(z)$ for small z :

$$\omega(z) = \sum_{k=0}^{\infty} \frac{(iz)^k}{\Gamma(1+k/2)}, \tag{10.2}$$

we reduce (7.2) to the form

$$2/A + i(z + \gamma) \sqrt{\pi} - (\beta + 2\gamma)z = 0 \quad (z = z_i). \quad (11.2)$$

This equation has the following solutions for z :

$$z = -\frac{\gamma + 4\gamma/\pi A}{1 + 4\alpha^2/\pi} + i\frac{2}{\sqrt{\pi}} \frac{1/A - \gamma\alpha}{1 + 4\alpha^2/\pi}, \quad \alpha = -\frac{1}{2}(\beta + 2\gamma). \quad (12.2)$$

The condition for instability ($\text{Im } \omega > 0$) assumes the form

$$\left[2 - \frac{d \ln T}{d \ln n} (1 + \delta)\right] \left[1 - \frac{d \ln T}{d \ln n} \left(1 + \frac{1}{2} \delta\right)\right] - \frac{2}{A} \left(\frac{2k_z}{\theta_i d \ln n / dx}\right)^2 > 0. \quad (13.2)$$

The second term in this inequality can be neglected, and, as can be readily seen, instabilities arise when

$$d \ln T / d \ln n > 2/(1 + \delta). \quad (14.2)$$

Taking into account the "finite Larmor radius," the boundary shifts from $d \ln T / d \ln n = 2$ (see Fig. 2, for $\theta_i \rightarrow 0$) to $d \ln T / d \ln n = 1$, since $\delta \rightarrow 1$ when $\theta_i \gg 1$.

3. Let us consider the case of "intermediate" frequencies, when ³⁾

$$k_z (2T/M)^{1/2} \ll \omega \ll k_z (2T/m)^{1/2}. \quad (15.2)$$

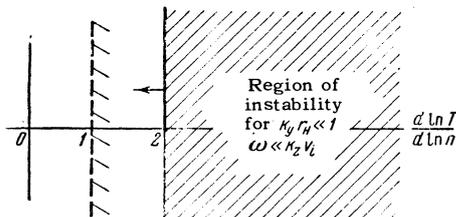


FIG. 2

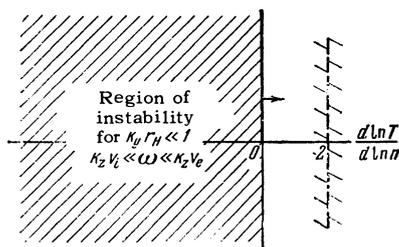


FIG. 3

³⁾Recently B. Kadomtsev and A. Timofeev investigated the instability increments corresponding to this case under the special assumption $d \ln T / d \ln n = 0$ [$n(x) \neq \text{const}$, $T(x) = \text{const}$] and found that the neutral equilibrium at $k_y r_H \ll 1$ becomes unstable if the finite Larmor radius is taken into account.

Using in (7.2) the asymptotic expansion of $w(z)$ with large z for the ions:

$$\omega(z) = \frac{i}{\pi} \sum_{k=0}^n \frac{\Gamma(k+1/2)}{z^{2k+1}} + R_n, \quad z \gg 1 \quad (16.2)$$

and the series (10.2) in powers of z for the electrons, we obtain

$$\frac{2}{A} - 1 + i\sqrt{\pi} \sqrt{\frac{m}{M}} (z - \gamma_0) \frac{1}{A} - \left(\gamma - \frac{1}{2}\right) \frac{1}{z} = 0. \quad (17.2)$$

The solution has the form

$$z = \frac{A}{2-A} \left(\gamma - \frac{\beta}{2}\right) - i\sqrt{\pi} \sqrt{\frac{m}{M}} \frac{A}{(2-A)^2} \left(\gamma - \frac{\beta}{2}\right)^2 \left(1 - \frac{2-A}{A} \frac{\gamma_0}{\gamma - \beta/2}\right). \quad (18.2)$$

From the expression for the imaginary part of the frequency we obtain the instability condition

$$-\frac{\frac{1}{2}(2/A - 1 - \delta) d \ln T / d \ln n + 2(A-1)/A}{1 - \frac{1}{2} \delta d \ln T / d \ln n} > 0. \quad (19.2)$$

In the limit of small Larmor ion radii we obtain from this the following instability criteria

$$d \ln T / d \ln n < 0, \quad (20.2)$$

$$d \ln T / d \ln n > 2/\delta, \quad (20'.2)$$

the first of which was derived previously. On the other hand, if we take into account the finite Larmor radii of the ions, the instability boundary corresponding to criterion (20.2) shifts to $d \ln T / d \ln n = 2$. Simultaneously, the instability boundary approaches the same value from the side of the large $d \ln T / d \ln n$. Thus, the instability arises for any $d \ln T / d \ln n$.

4. At "high" frequencies

$$\omega \gg k_z v_e, \quad \text{i.e.} \quad z_i \gg \sqrt{M/m} \quad (21.2)$$

we can use the asymptotic expansion (16.2) and reduce (7.2) to the form

$$1 - A - \frac{A(\gamma - \beta/2) - (\gamma_0 - \beta/2)}{z} - \frac{M}{m} \frac{1}{2z^2} + \frac{M}{m} \frac{1}{2} \frac{\gamma_0 - 3\beta/2}{z^3} = 0, \quad (22.2)$$

where $\gamma_0 = \gamma$ when $\theta_i = 0$. If

$$k_0 (2T/M)^{1/2} \theta_i \gg \omega, \quad k_0 \sim (nT)' / nT \quad (23.2)$$

the first and the third terms are small and we obtain

$$z = \left[2 \frac{M}{m} \frac{\gamma_0 - 3\beta/2}{A(\gamma - \beta/2) - (\gamma_0 - \beta/2)}\right]^{1/2} = \left[-\frac{M}{m} \frac{(d \ln T / d \ln n) + 1}{(1-A) + \frac{1}{2} \delta A (d \ln T / d \ln n)}\right]^{1/2}. \quad (24.2)$$

The solution (24.2) is applicable only for $\theta_i \ll 1$, such that (21.2) is satisfied. In this case it has the simple form

$$\omega = \pm i \sqrt{\frac{M}{m} \frac{k_z}{k_y}} \Omega_i, \quad (25.2)$$

so that the instability occurs for any value of the parameter $d \ln T / d \ln n$ (from which it is required, in accordance with the conditions of applicability of (23.2), that it be not too close to the value -1).

3. INVESTIGATION OF STABILITY OF AN INHOMOGENEOUS PLASMA IN THE MODEL OF TWO-FLUID HYDRODYNAMICS

In most modern magnetic traps with strong magnetic field (of the “Stellarator,” “Tokomak” type, etc.) conditions such that the collisions can be completely neglected in the investigation of the stability of an inhomogeneous plasma have not yet been attained in practice. At typical temperatures (\sim several times 10 eV) the mean free path (at a density 10^{14} cm^{-3}) which is the order of several dozen centimeters (whereas the dimensions of the traps are on the order of several meters). On the other hand, magnetohydrodynamics is no longer applicable here, for generally speaking the electron and ion gases do not have time to enter into equilibrium with each other during a time on the order of the time required for the development of the instability. It is natural to use in this case the two-fluid hydrodynamics of a plasma situated in a strong magnetic field^[5].

As in the preceding section, we are considering instability of plasma with respect to vortex-free perturbations, $\text{curl } \mathbf{E} = 0$, retaining the notation of the preceding sections: \mathbf{H} is directed along the z axis, T_0 and n_0 are the unperturbed temperature and density. Inasmuch as the temperatures of the ions and electrons may differ in the perturbation, we introduce T_e and T_i —the corrections to the electron and ion temperatures. Effects connected with the finite nature of the Larmor radius of the ions are left out; this means that we deal with $k_y \ll 1/r_H$. In such a plasma, furthermore, we can disregard the heat conductivity transverse to the magnetic field.

We do not write out the general dispersion equation in the two-fluid model, and confine ourselves only to the case of greatest interest, that of “intermediate” frequencies $v_i \ll \omega/k_z \ll v_e$. At such frequencies the ions do not have time to move appreciably along the force lines (along the z axis), so that their motion along z can be disregarded. For electrons, however, such frequencies

are too low and the inertia of the electrons can be neglected. The system of linearized equations for the perturbed quantities has the form

$$-ik_z(n_0 T_e + n T_0) - en_0 E_z - 0.71 n_0 ik_z T_e = 0, \quad (1.3)$$

$$-\nabla_{\perp} p_e - e \mathbf{E}_{0\perp} n - e \mathbf{E}_{\perp} n_0 - (e/c) n_0 [\mathbf{vH}]$$

$$- (e/c) n [\mathbf{v}_0 \mathbf{H}] = 0, \quad (2.3)^*$$

$$i\omega n - ick_y E_{0x} n/H + cE_y n'_0/H = 0, \quad (3.3)$$

$$\begin{aligned} \frac{3}{2} n_0 \{i\omega T_e + ik_y v_0 T_e + v_x T'_0\} - T_0 \{i\omega n + ik_y v_0 n + v_x n'_0\} \\ = -k_z^2 \kappa T_e + \frac{5}{2} (cT'_0/eH) ik_y (n T'_0 - T_e n'_0). \end{aligned} \quad (4.3)$$

Here (1.3) is the equation of motion of the electrons along z neglecting the inertia of the electrons, and the last term in (1.3) corresponds to the so-called “thermal force” which arises in the presence of a temperature gradient; (2.3) is the equation of motion of the electrons transverse to the force lines; (3.3) is the continuity equation, which has the same form for both electrons and ions; (4.3) is the heat-balance equation; v_0 is the unperturbed velocity of the electrons; k is the coefficient of electronic heat conductivity along H ; E_0 is the unperturbed electric field (directed along the x axis).

From the system of homogeneous linear equations (1.3)–(4.3) we obtain, as a condition for the solvability, the dispersion equation

$$\begin{aligned} 3\omega_0^2 + \omega_0 \{3(1+s)k_y v_T - 3k_y v_n - 2ik_z^2 \chi\} \\ - 3(1+s)k_y^2 v_n v_T = 0, \end{aligned} \quad (5.3)$$

where

$$\omega_0 = \omega + k_y (2v_n + v_T), \quad s = 0.71,$$

$$v_n = -cT_0 n'_0 / eH n_0, \quad v_T = -cT_0 T'_0 / eHT, \quad \chi = \kappa/n_0.$$

For the solution of this equation

$$\begin{aligned} \omega_0 = -\frac{1}{2} \{(1+s)k_y v_T - k_y v_n - \frac{2}{3} ik_z^2 \chi\} \\ \pm \left[\frac{1}{4} \{(1+s)k_y v_T - k_y v_n - \frac{2}{3} ik_z^2 \chi\}^2 \right. \\ \left. + k_y^2 v_n v_T (1+s) \right]^{1/2} \end{aligned} \quad (6.3)$$

we can obtain the instability condition (for $k_y v_n, T \gg k_z^2 \chi$):

$$d \ln T / d \ln n < 0. \quad (7.3)$$

The maximum increment is of the order of

$$\text{Im } \omega \sim k_z^2 \chi. \quad (8.3)$$

* $[\mathbf{vH}] = \mathbf{v} \times \mathbf{H}$.

We note that if we leave out from the dispersion equation (5.3) the heat-conduction coefficient χ , then the instability disappears. Condition (7.3) coincides with the instability condition (20'.2) obtained for the analogous limiting case.

Thus, an inhomogeneous rarefied plasma is "universally" unstable. However, since the instability develops at short wavelengths, it should lead not to a rapid escape of macroscopic plasmoids from the magnetic traps, but to slow turbulent "diffusion." This problem calls for a special study.

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