QUANTUM OSCILLATIONS OF THE TUNNEL CONTACT CURRENT OF TWO METALS IN A MAGNETIC FIELD

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The oscillations of the tunnel contact current of two normal metals separated by a thin dielectric layer in a magnetic field perpendicular to the interface between the metals are investigated theoretically. The external cross sections of the Fermi surface and effective masses of the quasiparticles can be determined on the basis of this phenomenon. The amplitude of the oscillations is sufficiently great for small electron groups; for large groups it is very small since the probability of "tunnelling" through the potential barrier is minimum for electrons on the external cross sections responsible for the oscillation phenomena. Another type of oscillations due to oscillations of the chemical potentials of the metals can be observed in pulsed magnetic fields in which the pulse duration is smaller than the relaxation time of the "tunnel diode." (As a matter of fact only a sufficiently thin dielectric layer between the metals, say $1\,\mu$, is required in order that the tunnel current be negligible and the voltage on the plates of such a film capacitor oscillate). An analysis of the conditions which make it possible to observe the oscillations on the chemical potential of metals with the aid of the given phenomenon is performed.

THE tunnel effect has been attracting much attention in recent years in connection with the investigation of pn junctions in degenerate semiconductors with film structures of the metal-dielectric-metal type (in the case of both superconducting and normal metals). It is assumed that an investigation of the volt-ampere characteristic of such a "tunnel diode" will make it possible to study the distribution of the density of electron states in solids.

It has been observed that a magnetic field exerts an appreciable influence on the tunnel effect in semiconductors [1,2]. It is therefore of interest to investigate the possibility of appearance of a non-monotonic (oscillating) dependence of the characteristics of tunnel structures on the magnetic field. Up to now this question was investigated only for semiconductors [2-4]. In the present note we present a theoretical analysis of the oscillations of the tunnel current through the contact between two normal metals in a magnetic field perpendicular to the surface of the contact, and also oscillations in a pulsed magnetic field, the latter effect being of greatest interest.

1. Let us consider the contact between two different metals I and II, separated by a layer of dielectric $0 < z < d$ (Fig. 1). The levels of the chemical potentials ζ of both metals differ by an amount Δ, where Δ/e is the constant bias applied to the contact. We henceforth assume that $\zeta \gg \Delta$, and

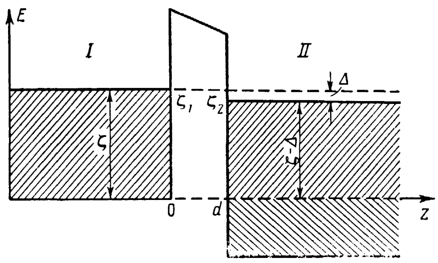

FIG. 1

also $\zeta \gg \Theta$, and $\zeta \gg \mu H$. We introduce a common reference level for the energies of both metals, after which the chemical potential of the first metal will be denoted by ζ and that of the second by $\zeta - \Delta$. Electrons with energies $E > 0$ participate in the tunnel current.

In calculation of the tunnel current we start from the formula

$$J_z = \frac{e^2H}{h^2c}\sum_{\sigma}\sum_{n_1}\int_{v_{z1}>0} dp_{z1}\frac{\partial\varepsilon_1}{\partial p_{z1}} D_1 f_1 (1-f_2) + \frac{e^2H}{h^2c}\sum_{\sigma}\sum_{n_2}\int_{v_{z2}<0} dp_{z2}\frac{\partial\varepsilon_2}{\partial p_{z2}} D_2 f_2 (1-f_1), \qquad (1)$$

where $\epsilon = \epsilon_n(p_z) + \sigma\mu_0 H$ are the quantized energy levels of the electron in the magnetic field, $\sigma = \pm 1/2$ is the projection of the electron spin, f is the Fermi distribution function, and D is the barrier penetrability coefficient (the indices 1 and 2 pertain to the metals I and II).

In writing down formula (1) we start from the assumption that the wave functions of the electrons to the left and to the right of the barrier can be represented in the form of Bloch waves, for which we then seek the probability of the "tunnelling" through the barrier. In such an approach, we are not interested in the explicit satisfaction of the conservation laws at the junction, nor in the specular or diffuse character of the reflection or transmission of the electrons. An account of these factors is important for the calculation of D [1]. The calculation of the penetrability coefficient at a tunnel junction can be found in several papers ([5,6] and others).

With the aid of the Poisson summation formula expression (1) reduces to

$$J_z = J_z^0 + J_z^{\rm osc}, \tag{2}$$

where J_z^0 is the nonoscillating part of the current, equal in order of magnitude (for $\Theta = 0$) to

$$J_z^0 \sim \frac{4\pi me}{h^3}\Delta\int_0^{\zeta} D(\varepsilon)\,d\varepsilon;$$

$$D(\varepsilon) \sim \exp\left\{-2d\frac{\sqrt{2m}}{\hbar}\sqrt{W-\varepsilon}\right\}, \tag{3}$$

$J_z^{\rm osc}$ is the oscillating part of the current. The reason for the oscillations, as in other oscillation phenomena (the de Haas—van Alphen effect, the Shubnikov—de Haas effect) is the nonmonotonic (jump-like) dependence of the number of electron states on the magnetic field.

Calculation of $J_z^{\rm osc}$ leads to the formula

$$J_z^{\rm osc} = \Theta\frac{e^2H}{h^2c}\left\{\frac{D_1^{ext}g_1}{1-e^{-\Delta/\Theta}}\,{\rm Re}\sum_{p=1}^{\infty}\frac{1-\exp(-2\pi ip\Delta/\mu_1H)}{p\,{\rm sh}\,(2\pi^2p\Theta/\mu_1H)}\right.$$
$$\times\exp\left[2\pi ip\frac{cS_{m1}(\zeta_1)}{eHh}\right]\cos\left(\pi p\frac{m_1^*}{m_0}\right)$$
$$-\frac{D_2^{ext}g_2}{1-e^{\Delta/\Theta}}\,{\rm Re}\sum_{p=1}^{\infty}\frac{1-\exp(2\pi ip\Delta/\mu_2H)}{p\,{\rm sh}\,(2\pi^2p\Theta/\mu_2H)}$$
$$\left.\times\exp\left[2\pi ip\frac{cS_{m2}(\zeta_2)}{eHh}\right]\cos\left(\pi p\frac{m_2^*}{m_0}\right)\right\}, \tag{4}$$

[1]For example, in pn junctions of degenerate semiconductors, the centers of the electron groups (on the n-side) and hole groups (on the p-side) can be located at different points of p-space, so that the law of conservation of the quasimomentum component parallel to the surface of the tunnel diode can be satisfied only when the passage of the electron through the barrier is accompanied by emission or absorption of a phonon (see the work by Keldysh[6]).

where D^{ext} is the value of the penetrability coefficient on the extremal Fermi section; $\mu = e\hbar/m^*c$; $m^* = (2\pi)^{-1}\partial S_m/\partial\zeta$ is the effective mass of the electron in the magnetic field; m_0 is the mass of the free electrons; $S_m(\zeta)$ is the extremal cross section of the Fermi surface;

$$g = \frac{\partial S_m}{\partial\zeta}\left(\frac{\partial^2\varepsilon}{\partial p_z^2}\right)_m\Big/\left(\frac{\partial^2 S}{\partial p_z^2}\right)_m$$

is a dimensionless coefficient of order of unity.

As can be seen from the expression obtained, each of the metals makes a contribution to the oscillating part of the tunnel current in the form [2]

$$\cos\left[\frac{cS_m(\zeta)}{eH\hbar}\right] - \cos\left[\frac{cS_m(\zeta-\Delta)}{eH\hbar}\right]$$
$$\approx\left(1-\cos\frac{2\pi\Delta}{\mu H}\right)\cos\left[\frac{cS_m(\zeta)}{eH\hbar}\right]$$
$$-\sin\frac{2\pi\Delta}{\mu H}\sin\left[\frac{cS_m(\zeta)}{eH\hbar}\right].$$

Consequently, the oscillations are determined not only by the extremal Fermi sections $S_m(\zeta)$, but also by the sections corresponding to the energy $\epsilon = \zeta - \Delta$. Thus, with the aid of a study of the oscillations of the tunnel current we can investigate a layer of equal-energy surfaces, and not only the Fermi surface. Such a possibility is due to the presence of the parameter Δ—the bias applied to the tunnel diode.

As can be seen from (4), the tunnel current oscillates not only as a function of 1/H, but also as a function of Δ, the oscillation period being μH in the latter case. Observation of similar oscillations makes it possible to determine directly the effective mass m^* of the electron.

The most important is the question of the amplitude of the tunnel-current oscillations. To calculate the amplitude we compare (4) with (3). In order of magnitude, we have

$$\frac{J_z^{\rm osc}}{J_z^0} \sim \begin{cases}\dfrac{\mu H}{\zeta}\dfrac{\Theta}{\Delta}\dfrac{D^{ext}}{\overline{D}}\exp(-2\pi^2\Theta/\mu H), & \Delta \gg \mu H,\\[2mm] \dfrac{\Theta}{\zeta}\dfrac{D^{ext}}{\overline{D}}\exp(-2\pi^2\Theta/\mu H), & \Delta \lesssim \mu H.\end{cases} \tag{5}$$

($\overline{D}$ is the average value of the transmission coefficient, which coincides in order of magnitude with $D^{max} \sim \exp(-2d\sqrt{2m_0}\sqrt{W-\zeta}/\hbar)$.

As can be seen from (5), the amplitude of the oscillating addition contains a small factor $\mu H/\zeta$.

[2]When the condition $\Delta \gg \theta$ is satisfied, the contribution to the oscillating part of the current is determined only by one of the metals [in formula (4) – by metal I], which makes it possible to separate experimentally the oscillations of the two metals of the contact.

In addition, there exists an additional decrease in oscillation amplitudes, connected with the fact that the electrons on the extremal section, responsible for the oscillations, have the lowest probability of tunnelling through the potential barrier. This makes it practically impossible to observe oscillations of large electron groups (with the exception of the case when the thickness of the gap is on the order of two or three lattice constants). Oscillations connected with small electron groups have sufficient amplitude provided the following condition is satisfied

$$d \lesssim a_0 w / \zeta, \tag{6}$$

where d is the thickness of the gap, $a_0 = \hbar/\sqrt{2mw}$ is a quantity on the order of the lattice constant, w is the work function, and ζ is the chemical potential of the small group.

2. We now consider the behavior of a tunnel diode in a pulsed magnetic field. As shown by Kosevich [7] oscillations in pulsed fields can be regarded in the same fashion as in the static case, provided the following conditions are satisfied

$$\omega_H T \gg \delta/\lambda; \qquad \delta \gg L \gg r, \tag{7}$$

where ω_H is the cyclotron frequency, T the duration of the magnetic-field pulse, $\delta = \sqrt{c^2 T/\sigma}$ the characteristic dimension of the inhomogeneity of the magnetic field inside the specimen, λ the de Broglie wavelength of the electron, L the length of the specimen, and r the radius of the electron orbit (it is assumed that the magnetic field **H** is parallel to the surface). When inequality (7) is satisfied, the conditions for quasiclassical quantization have the same form as in the static case.

However, the pulsed character of the field changes utterly the physical picture of the tunnel-current oscillations. Indeed, in the static case one could neglect the oscillations of the chemical potential of the metals. But if the magnetic field changes rapidly during a time too short to permit establishment of equilibrium current, then ζ^{osc} is contained directly in the expression for the voltage between the metals producing the contact. Consequently, observation of the tunnel effect in pulsed magnetic fields whose duration T satisfies the condition $T \lesssim \tau$, where τ is the relaxation time of the tunnel diode (that is, the time during which the charge imparted to one of the electrodes diffuses away), enables us to observe directly the oscillations of the chemical potential of metals. [3)]

[3)]The possibility of experimental observation of oscillations of the chemical potential of a metal in the static case was investigated by Kaganov, I. Lifshitz, and Sinel'nikov.[8]

As was shown by I. Lifshitz and Kosevich [9], the order of magnitude of the chemical-potential oscillations is

$$\zeta^{osc} \sim \frac{\pi}{\sqrt{2}} \Theta \left(\frac{\mu H}{\zeta}\right)^{1/2} \exp\left(-2\pi^2 \Theta / \mu H\right). \tag{8}$$

For $\Theta = 10°$ K and $H = 10^4$ Oe this estimate yields for small groups $\zeta^{osc} \sim 10^{-3}$–10^{-4} eV; for large groups the amplitude of the oscillations is smaller, on the order of 10^{-6} eV. If $T \ll \tau$, the amplitude of the alternating voltage on the electrodes of the "capacitor" produced by the two metals is $V^{osc} = (\zeta_1^{osc} - \zeta_2^{osc})/e$ (in the absence of an external constant bias Δ), and consequently, can be readily measured.

It is easy to find the amplitude of the oscillations even when the ratio of T and τ is arbitrary. Assuming the oscillation to be sinusoidal with frequency ω, we obtain

$$V^{osc} = \frac{\zeta^{osc}/e}{1 + i/\omega\tau} \tag{9}$$

(see the equivalent circuit on Fig. 2).

The relaxation time of the tunnel diode τ can be found as $\tau = RC$, where R is the contact resistance and C the contact capacitance (Fig. 2):

$$R = \frac{2\zeta}{e^2/a_0} \frac{1}{n v_F a_0 S} \frac{1}{\bar{D}}, \qquad C = \frac{\varepsilon S}{4\pi d} \tag{10}$$

(S is the contact area, ϵ is the dielectric constant of the gap material, n is the conduction-electron concentration, and v_F the Fermi velocity).

Putting $v_F \sim 10^8$ cm/sec, $n \sim 10^{22}$ cm^{-3}, and $S \sim 1$ cm^2, we get

$$\tau \sim \frac{\varepsilon}{2\pi} \cdot \frac{10^{-15}}{\bar{D}} \text{ sec.}$$

When $\bar{D} \sim 10^{-12}$ (this is already the value of the penetrability coefficient for a gap thickness several times the lattice constant) we obtain $\tau \sim 10^{-3}$ sec, that is, the same order as the duration of the magnetic-field pulses in apparatus in which large magnetic fields are produced. The condition $T \lesssim RC$ can always be readily satisfied by increasing the gap thickness d; then $RC \to \infty$ [4)].

Thus, it is possible to observe in pulsed magnetic fields oscillations of a new type (differing from those considered in Sec. 1), connected with

[4)]Actually an increase of R to infinity does not lead to $\tau \to \infty$, since the tunnel diode is loaded by the measuring equipment, which has a certain shunting resistance R_{sh} (see Fig. 2). Assuming $R_{sh} \sim 1 M\Omega$, we find that in order to obtain a relaxation time on the order of 10^{-3} sec we must have $C \gtrsim 10^3$ pF. Consequently, the thickness of the gap must not exceed $\sim 1\mu$, if $\varepsilon \sim 1$(at such thicknesses the tunnel current is already negligibly small in fact).

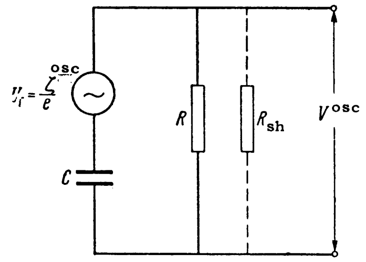

FIG. 2

the oscillations of the chemical potential of the metals. Experimental observation of this phenomenon is facilitated by the fact that (a) it is very easy to obtain very large pulsed magnetic fields (on the order of several hundred kOe); (b) the amplitude of the oscillations is relatively large; (c) a contact in which one of the metals is a single crystal can be readily prepared; (d) the mean free path does not exert an appreciable influence on the given effect, since ζ is a macroscopic characteristic of the metal. This makes it possible to observe oscillations of the chemical potential at relatively high temperatures.

Along with the foregoing advantages of this phenomenon, there are many difficulties, one of which is the problem of compensating for the induction from the alternating magnetic field, which is more significant for metals with large electron groups, in which the amplitude of the effect is smaller. It is obvious that it is best to carry out the experiments with "poor" metals such as bismuth, which have a small number of carriers and poor conductivity [for which it is also easiest to satisfy the quasi-static condition (7)].

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