ON THE DETERMINATION OF THE ELECTRON FORM FACTORS

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The differential cross sections for the scattering of high energy electrons with charge and magnetic moment distributions are computed taking into account initial and final longitudinal polarizations. A simple method for determining the electron form factors is presented for this case as well as for the case where only the initial (final) polarizations are fixed.

IN connection with the preparation of an electron scattering experiment with colliding beams of high energy, [1,2] it is of interest to take the anomalous magnetic moment and the possible structure of the electron into account phenomenologically. Baĭer [3] has computed the cross section for unpolarized electrons in first Born approximation, taking into account the charge and magnetic moment distribution of the electron. An analogous calculation for high energies ($E_{c.m.} \sim 1$ BeV) was carried out by Avakov. [4] He proposed a method for determining the form factors from the experimental cross sections based on these calculations. The results of Avakov have been generalized recently by Épshteĭn [5] under the assumption that the electron has an electric dipole moment.

The study of the scattering of polarized particles opens up new possibilities for the determination of the form factors. [6,7] Below we present the results of the corresponding calculations, which were based on the covariant method of direct calculation of the matrix elements of polarized particles. [8] The scattering cross sections (in Born approximation) with account of the initial and final longitudinal polarizations of both electrons in the center of mass system (c.m.s.) are, at high energies ($E/m \gg 1$) of the form

$$d\sigma_{\varepsilon_1\varepsilon_1'}^{\varepsilon_2\varepsilon_2'}/d\Omega = (e^4/2E^2)\,|M_{\varepsilon_1\varepsilon_1'}^{\varepsilon_2\varepsilon_2'}|^2 \quad (\hbar = c = 1,\quad e^2 = 1/137), \tag{1}$$

where $\epsilon_1, \epsilon_2, \epsilon_1', \epsilon_2' = \pm 1$ indicate the sign of the spin projection on the direction of motion of one of the particles and

$$M_{++}^{++} = M_{--}^{--} = {}^1/_2\,(\mu a_1^2/\nu + \beta x^2 l_2^2), \tag{2}$$

$$M_{++}^{--} = M_{--}^{++} = {}^1/_2\,(\nu a_2^2/\mu + \alpha x^2 l_1^2), \tag{3}$$

$$M_{+-}^{+-} = M_{-+}^{-+} = a_1^2/\nu + a_2^2/\mu, \tag{4}$$

$$M_{+-}^{-+} = M_{-+}^{+-} = -{}^1/_2\,x^2\,(\alpha l_1^2 + \beta l_2^2), \tag{5}$$

$$M_{++}^{+-} = M_{++}^{-+} = M_{+-}^{++} = M_{-+}^{++} = M_{--}^{+-} = M_{--}^{-+} = M_{+-}^{--} = M_{-+}^{--} = (ix/\sqrt{\mu\nu})\,(a_1 l_1 \mu - a_2 l_2 \nu). \tag{6}$$

Here $a_1 \equiv a(q_1^2)$, $a_2 \equiv a(q_2^2)$ are the charge form factors, $l_1 \equiv l(q_1^2)$, $l_2 \equiv l(q_2^2)$ are the magnetic moment form factors, $q_1 = p_2, - p_1$, $q_2 = p_1 - p_2$ are the momentum transfers for the direct and exchange graphs, $\mu = 2\cos^2(\vartheta/2)$, $\nu = 2\sin^2(\vartheta/2)$ (ϑ is the scattering angle in the c.m.s.), $\alpha = \mu + 2$, $\beta = \nu + 2$, $x = E/m$, E is the energy of the electron in the c.m.s., and m is the rest mass of the electron.

It follows from these expressions that the form factors can be determined by the scattering of polarized particles using the method of Avakov, [4] which in this case [see, e.g., (2)] gives four quadratic equations for the computation of the quantities $a(q'^2)$, $a(q''^2)$, $l(q'^2)$, and $l(q''^2)$ from the four experimental cross sections.

After summation over the final polarizations the formula for the differential cross section takes, according to (2) to (6), the form

$$d\sigma_{\pm}/d\Omega = \tfrac{1}{2} e^4 E^{-2}\,(|M_0|^2 \mp |M_p|^2); \tag{7}$$

$$\begin{aligned} |M_0|^2 = {} & \tfrac{1}{4} a_1^4 (1+\tau^2)/\eta^2 + \tfrac{1}{4} a_2^4 (\eta^2 + \tau^2) + a_1^2 a_2^2 \tau^2/2\eta \\ & + 4a_1^2 l_1^2 x^2/\eta + 4a_2^2 l_2^2 x^2 \eta + a_1^2 l_2^2 x^2 (\eta + \tau)/\eta\tau \\ & + a_2^2 l_1^2 x^2 \eta (1+\tau)/\tau + 2 l_1^4 x^4 (1+\tau)^2/\tau^2 \\ & + 2 l_2^4 x^4 (\eta+\tau)^2/\tau^2 + 2 l_1^2 l_2^2 x^4 (1+\tau)(\eta+\tau)/\tau^2 \\ & - 8 a_1 a_2 l_1 l_2 x^2, \end{aligned} \tag{8}$$

$$\begin{aligned} |M_p|^2 = {} & \tfrac{1}{4} a_1^4 (1+\tau)/\eta + \tfrac{1}{4} a_2^4 (\eta+\tau) + a_1^2 a_2^2 \tau^2/2\eta \\ & - a_1^2 l_2^2 x^2 (\eta+\tau)/\eta\tau - a_2^2 l_1^2 x^2 \eta (1+\tau)/\tau \\ & + 2 l_1^2 l_2^2 x^4 (1+\tau)(\eta+\tau)/\tau^2, \end{aligned} \tag{9}$$

where $\eta = \nu/\mu = \tan^2(\vartheta/2)$ and $\tau = 1 + \mu$.

Evidently, the difference of the cross sections (7),

$$\frac{1}{2}(d\sigma_-/d\Omega - d\sigma_+/d\Omega) = \frac{1}{2}\alpha^2 E^{-2} |M_p|^2, \qquad (10)$$

which determined the polarization correction $|M_p|^2$ [formula (9)], also leads to a simple scheme for the determination of the electron form factors (six equations of second order with six unknowns). Here σ_+ denotes the cross section for particles polarized in the same direction and σ_- for particles polarized in opposite directions with respect to a fixed axis in the direction of motion of one of the particles (in the c.m.s.).

The expression

$$d\sigma_0 / d\Omega = \frac{1}{2}\alpha^2 E^{-2} | M_0 |^2, \qquad (11)$$

where $|M_0|^2$ is given by (8), gives the cross section for the scattering of unpolarized electrons. [We note that (2) to (6) give the matrix elements in the high energy limit. The exact expressions for the matrix elements lead, instead of (11), to a formula which coincides with the result of Baĭer [3].]

It is easily seen that the determination of the form factors from the cross sections for unpolarized particles [formula (11)] is a rather complicated task, since it involves equations of fourth order with respect to the form factors. The simple method of Avakov [4] for the determination of the form factors from the cross sections for unpolarized electrons is based on the illegitimate neglect of the term proportional to $a_1 l_1 a_2 l_2$ [or $|\varphi(q^2) f(q^2) \varphi(p^2) f(p^2)|^{1/2}$ in the notation of Avakov [4]]. This term has not been taken into account by Épshteĭn [5] either. However, it is easy to see that it is of the same order of magnitude as the other terms in (11) (cf. the formula of Baĭer [3]).

Thus we can give a relatively simple interpretation in terms of form factors of electron-electron scattering experiments in which the initial and final, or only the initial (final), polarizations of the interacting particles are fixed.

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