# RESONANCE EFFECTS ASSOCIATED WITH PARTICLE MOTION IN A PLANE ELECTROMAGNETIC WAVE

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We consider the motion of a particle in the field of a plane electromagnetic wave in the presence of an external magnetic field. It is shown that resonance effects that arise under certain conditions can increase the particle energy. Moreover, the energy growth does not disturb the resonance condition.

It is well known[1,2] that the average energy of a charged particle that moves in the field of a plane electromagnetic wave remains constant. If some very weak radiation effects (associated with the light pressure) are neglected it can be shown that a particle in a plane wave describes a figure-eight trajectory (plane polarization) or a circle (circular polarization). The center of this trajectory moves with a constant velocity determined by the initial conditions and the wave parameters.

However, the situation described above is changed markedly when a fixed magnetic field is applied. In this case the interaction between the wave and the particle can exhibit a resonance effect and the particle energy can grow. It is important to note, in this connection, that the resonance condition is maintained automatically, in spite of the reduction in the gyromagnetic frequency $\omega_B$ caused by the increase in particle energy (mass). The existence of this effect was pointed out in [3], where we considered briefly the simple case of propagation of a wave in vacuum in the presence of a uniform longitudinal magnetic field.

In addition to its theoretical interest, this "auto-resonance" mechanism may be of some importance in the understanding of cosmic processes in which charged particles are accelerated by radio waves and light fluxes in cosmic fields. This mechanism may also find application in acceleration of particles by intense light fluxes or in the amplifaction of radio waves in different regions and so on.

Below we consider the theory of auto-resonant particle motion in a plane wave in vacuum and in various media (isotropic and anisotropic).

## 1. EQUATION OF MOTION IN AN ISOTROPIC MEDIUM IN THE PRESENCE OF A LONGITUDINAL MAGNETIC FIELD

Let us consider the motion of a particle in the field of a plane wave **E** and **B** in the presence of a uniform magnetic field $\mathbf{B}_0$ in the same direction as the wave vector **k**. It is assumed that the motion occurs in an isotropic refractive medium in which the phase velocity is given by $\beta_{\text{ph}} = n^{-1}$, where n is the refractive index. The effect of the external magnetic field on the properties of the medium is neglected. The wave components satisfy the conditions

$$\mathbf{B} = \beta_{\text{ph}}^{-1}[\mathbf{nE}], \quad \mathbf{nE} = \mathbf{nB} = 0, \qquad (1.1)^*$$

where $\mathbf{k} = k\mathbf{n}$ and $k = \omega/\beta_{\text{ph}}c$. We now introduce the dimensionless vector $\rho = k\mathbf{r}$ so that all distances are measured in units of $\lambda\!\!\!^{-}$, the wavelength divided by $2\pi$. If small radiation corrections 1) and particle losses in the medium (Cerenkov radiation etc.) are neglected, using (1.1) we write the equation of motion of a particle with charge e and rest mass m in the form

$$\frac{d}{dt}\gamma\dot{\rho} = \frac{e}{m}\left\{\left(1 - \frac{\dot{\rho}\mathbf{n}}{\omega}\right)k\mathbf{E} + \frac{\mathbf{n}}{\beta_{\text{ph}}c}(\dot{\rho}\mathbf{E}) + \frac{B_0}{c}[\dot{\rho}\mathbf{n}]\right\}, \qquad (1.2)$$

where $\gamma mc^2$ is the total energy of the particle while the electric field **E** is proportional to $\exp[i\omega(t - \rho\cdot\mathbf{n}/\omega)]$. The first term in the curly brackets corresponds to the transverse force caused by the electric field and the Lorentz force; the second term corresponds to the longitudinal Lorentz force, which arises by virtue of the transverse velocity of the particle and the magnetic field of the wave itself. The third term is associated with the external magnetic field.

Taking the scalar product of Eq. (1.2) with **n** and integrating, using the relation $e\dot{\rho}\cdot\mathbf{E} = kmc^2\dot{\gamma}$, we obtain the following important expression:

$$\gamma\,(1 - \beta_{\text{ph}}^2\,\dot{\rho}\mathbf{n}/\omega) = \text{const} = \gamma_i\,(1 - \beta_{\text{ph}}\beta_{ni}), \qquad (1.3)$$

1) Radiation effects (wave pressure) in a magnetic field have been treated by Ollendorf[4] and, more rigorously, by Faĭnberg and Kurilko.[5]

*$[\mathbf{nE}] = \mathbf{n}\times\mathbf{E}$, $(\mathbf{nE}) = \mathbf{n}\cdot\mathbf{E}$.

where the subscript "i" denotes the initial value of a quantity and $\beta_{ni}$ is the initial velocity in the **n** direction (longitudinal velocity). In particular, when $\beta_{ph} = 1$, i.e., motion in a vacuum, Eq. (1.3) states that the difference between the energy and longitudinal momentum (measured in dimensionless units) is an integral of the motion.

Equation (1.2) can be simplified considerably if the independent variable is taken to be the particle phase with respect to the wave rather than the time

$$\psi = \omega t - \rho\mathbf{n} + \psi_i \tag{1.4}$$

and if (1.3) is used. Converting to the new variable $\psi$ and introducing the notation

$$\Omega = \frac{eB_0}{mc\gamma\ (1 - \beta_{ph}\beta_{ni})}, \qquad \eta = \frac{\Omega \mathbf{E}}{B_0\omega\beta_{ph}} = \eta_0 \sin\psi,$$

$$M = [1 + (1 - \beta_{ph}^2)\,\rho'\mathbf{n}]^{-1} = \left[1 - \frac{\dot\rho\mathbf{n}}{\omega}\right]\left[1 - \beta_{ph}^2\,\frac{\dot\rho\mathbf{n}}{\omega}\right]^{-1}, \tag{1.5}$$

we reduce (1.2) to the form

$$[M\rho']' = \eta + \mathbf{n}(\rho'\eta) + \frac{\Omega}{\omega}[\rho'\mathbf{n}], \tag{1.6}$$

where the primes denote differentiation with respect to $\psi$. We write one more expression that shows how the particle energy varies as a function of $\rho'$:

$$\gamma = \gamma_i (1 - \beta_{ph}\beta_{ni})(1 + \rho'\mathbf{n})\,[1 + (1 - \beta_{ph}^2)\,\rho'\mathbf{n}]^{-1}. \tag{1.7}$$

Before analyzing (1.6) for particular cases we note that the characteristic frequency $\Omega$ can (relativistic initial conditions and $\beta_{ph} = 1$) be much larger than the initial gyromagnetic frequency $\omega_{Bi} = eB_0/mc\gamma_i$. In particular, it is possible to choose conditions such that $\Omega = \omega$ even if $\omega \gg \omega_{Bi}$. It will be shown below that physically this effect corresponds to a resonance of the motion. This resonance can be understood qualitatively as follows. If the particle moves in the direction of **k** with velocity $\beta_n c$ the effective frequency with which the wave acts on it in the laboratory coordinate system is $\omega_\beta = \omega(1 - \beta_n)$; thus, this frequency is reduced as the longitudinal velocity increases. At the same time the particle executes transverse oscillations at a frequency $eB_0/mc\gamma$, which diminishes with increasing particle energy. It is evident that these two frequencies can remain in resonance when $\beta_n \approx 1$; thus there can be a resonance in the wave-particle interaction in which the particle energy is increased because of the forces exerted by the electric field of the wave. Because of the magnetic field associated with the wave some of the acquired momentum is converted into longitudinal motion; as has been shown above [cf. (1.3)] however, the product $\gamma(1 - \beta_n)$ remains constant. Thus, the frequency ratio $\omega_\beta/\omega_B$ is an integral of the motion, i.e., once a resonance occurs it is maintained regardless of the energy growth.

## 2. RESONANCE MOTION IN THE FIELD OF A PLANE WAVE ($\beta_{ph} = 1$)

We first consider the case of a plane-polarized wave. We choose the Ox axis of a rectangular coordinate system to be along the vector **k**; the Oy axis lies in the plane of polarization while Oz is along the magnetic field of the wave. When $\beta_{ph} = 1$ we note that the effective mass M appearing in (1.6) is equal to unity identically. Hence the vector equation (1.6) can be written in the following component form:

$$x'' = \eta y', \qquad y'' = \eta + \frac{\Omega}{\omega} z', \qquad z'' = -\frac{\Omega}{\omega} y', \tag{2.1}$$

whence

$$y'' + \left(\frac{\Omega}{\omega}\right)^2 y = \eta - \frac{\Omega}{\omega} z_i'; \tag{2.2a}$$

$$x' - x_i' = \frac{1}{2}\left[y'^2 + \left(\frac{\Omega}{\omega}\right)^2 y^2\right] - \frac{y_i'^2}{2} - \frac{\Omega}{\omega} z_i' y. \tag{2.2b}$$

Now $\eta = \eta_0 \sin\psi$ and (2.2a) shows that a resonance occurs when $\Omega = \omega$; because of the resonance the amplitude of the y oscillations increases linearly, causing $x'$ to increase, as indicated by (2.2b).

We shall be primarily interested in the case $\psi \gg 1$, in which the oscillations in the transverse direction and the particle energy have already grown appreciably. Omitting the intermediate computations we give the dimensionless energy $\gamma$, "acceleration length" $L = \eta_0 x$, and trajectory radius $R = [y^2 + z^2]^{1/2}$ as asymptotic[2)] functions of the parameter $\tau = \eta_0(\psi - \psi_i)$ (for $\beta_{ph} = 1$ and $\psi - \psi_i \gg 1$):

$$\gamma \approx \gamma_i + \gamma_i(1 - \beta_{ni})\left[\frac{\tau^2}{8} + \frac{\tau}{2}(y_i' \sin\psi_i - z_i' \cos\psi_i)\right],$$

$$L \approx \frac{\tau^3}{24} + \frac{\tau^2}{4}(y_i' \sin\psi_i - z_i' \cos\psi_i) + \frac{\beta_{ni}}{1 - \beta_{ni}}\tau,$$

$$R \approx \tau/2. \tag{2.3}$$

Before investigating (2.3) we consider the question of allowable initial conditions. We introduce the quantities $\alpha_y$ and $\alpha_z$, which represent the projections on the corresponding planes, of the angle $\alpha$ formed by the initial momentum and **k** (injection angle). The initial energy and the initial values $y_i'$ and $z_i'$ are related to $\alpha_y$ and $\alpha_z$ by the expressions

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2)Formulas of this kind can also be obtained for low-amplitude waves by solving the averaged equations in [5].

$$\gamma_i^2 = [1 - \beta_{ni}^2 (1 + \mathrm{tg}^2\, \alpha_z + \mathrm{tg}^2\, \alpha_y)]^{-1},$$

$$y_i' = \frac{\beta_{ni}}{1 - \beta_{ni}} \mathrm{tg}\, \alpha_y, \quad z_i' = \frac{\beta_{ni}}{1 - \beta_{ni}} \mathrm{tg}\, \alpha_z. \qquad (2.4)^*$$

On the other hand, the resonance condition, $\Omega = \omega$, and (1.5) yield

$$\gamma_i (1 - \beta_{ni}) = eB_0 \lambda / mc^2 = \Gamma, \qquad (2.5)$$

where the dimensionless parameter $\Gamma$ which, as will be shown below, determines both the qualitative and quantitative nature of the solution, can vary over wide limits. This quantity is of the order of $10^{-5}$ for a light wave (electrons) but is greater than unity in the centimeter region.

Eliminating $\gamma_i$ from (2.4) and (2.5) we have

$$\mathrm{tg}^2\, \alpha_z + \mathrm{tg}^2\, \alpha_y = (1 - \beta_{ni}^2)/\beta_{ni}^2 - (1 - \beta_{ni})^2/\Gamma^2\beta_{ni}^2. \quad (2.6)$$

Inasmuch as the left-hand side of this equation is always positive, we arrive at the condition:

$$\Gamma^2 > (1 - \beta_{ni})/(1 + \beta_{ni}), \qquad (2.7)$$

This expression shows that to obtain resonance motion at lower values of $\Gamma$ it is necessary to use more relativistic velocities in the x direction, i.e., ($\beta_{ni} \to 1$); this can be achieved by reducing the injection angle and/or increasing the initial energy. The maximum injection angle is determined by the conditions

$$\beta_{ni} = 1 - \Gamma^2, \qquad |\sin \alpha_{np}| = |\Gamma|. \qquad (2.8)$$

Thus, when $|\Gamma| > 1$ injection at angles up to $\pi/2$ is possible (a given injection angle has a corresponding energy). However, if $\Gamma \ll 1$, as in the case of a light wave, the injection angle is limited to values satisfying the condition

$$-\Gamma < \alpha < \Gamma. \qquad (2.9)$$

It is evident from (2.3) that a particle injected into a plane wave under resonance conditions moves in a helix of increasing radius and pitch and that its average energy increases. It is evident from (2.6) that for a given injection angle $\alpha > 0$ the resonance conditions can be satisfied for two different values of $\beta_{ni}$; correspondingly there are two initial energies $\gamma_i$ that satisfy the resonance conditions. When $\Gamma^2 \to 1$ one of these energies $\beta_{ni}$ is positive while the other is negative. Under these conditions a particle can move against the wave and then be reflected and accelerated in the opposite direction. It is also possible to find conditions for which a particle loses energy by the resonance mechanism. In principle, this effect can be used to amplify electromagnetic waves in various regions.

*tg = tan.

We now consider a circularly polarized wave. The electric field of a wave with right-handed (left-handed) circular polarization can be written in the form

$$\eta_y = \eta_0 \sin \psi, \qquad \eta_z = \mp \eta_0 \cos \psi. \qquad (2.10)$$

From the vector equation (1.6) we obtain the equations ($\beta_{ph} = 1$)

$$y''' + \left(\frac{\Omega}{\omega}\right)^2 y' = \left(1 \mp \frac{\Omega}{\omega}\right) \eta_0 \cos \psi,$$

$$z''' + \left(\frac{\Omega}{\omega}\right)^2 z' = -\left(\frac{\Omega}{\omega} \mp 1\right) \eta_0 \sin \psi. \qquad (2.11)$$

It is thus evident that a resonance again occurs when $\Omega = \omega$, but now only for the left-handed wave (if $eB_0 > 0$). Under these conditions the solution of (2.11) coincides with the solution of (2.1) if we make the substitution $\eta_0 \to 2\eta_0$, that is to say, a factor of 2 is gained in the amplitude of the useful component of the wave. The electric field has two circularly polarized components; however, the resonance effect is due only to the electric field that rotates in the same sense as the particle and this accounts for the factor of 2.

A more detailed analysis of the motion for small values of $\psi$ and for nonresonance regions (i.e., $\Omega \neq \omega$) is facilitated by the use of phase trajectories. Although an analytical investigation is not fundamentally difficult it does result in rather complicated expressions. Furthermore, the phase diagram technique can be used in a more general case (cf. Sec. 3).

We introduce the variables V and $\varphi$, which are related to $y'$ and $z'$ by the expressions:

$$y' = V \sin(\psi + \varphi), \qquad z' = V \cos(\psi + \varphi). \qquad (2.12)$$

Written in terms of these variables the equations for the phase trajectories become

$$\tfrac{1}{2} V^2 [\Omega/\omega - 1] - V\eta_0 \sin \varphi = \mathrm{const.} \qquad (2.13)$$

Off resonance there exists a singular point $\varphi = \pm\pi/2$, $V = \eta_0 |\Omega/\omega - 1|^{-1}$ about which the corresponding quantities vary. The amplitude of these deviations can be very large at small values of the "detuning." The maximum possible energy increment in the beating process is

$$\gamma_{max} - \gamma_i = \frac{2\Gamma\eta_0}{|\Omega/\omega - 1|} \left| \frac{\eta_0}{|\Omega/\omega - 1|} - V_i \right|. \quad (2.14)$$

In particular, for zero initial conditions ($V_i = 0$)

$$\gamma_{max} - \gamma_i = 2\Gamma\eta_0^2/(\Omega/\omega - 1)^2, \qquad (2.15)$$

while for the limiting conditions we must set $V_i = |1 - \Gamma^2|^{1/2}/\Gamma$.

Theoretically, the acquired energy becomes infinite at exact resonance, and we again obtain

(2.3). In this case the phase trajectories diverge and go to infinity, corresponding to a monotonic energy increase and phase velocity equal to the velocity of light. The more general case is considered in the next section.

## 3. MOTION IN AN ACCELERATED OR RETARDED WAVE ($\beta_{ph} \neq 1$)

We now consider the motion of a particle in a plane wave whose phase velocity is different from the velocity of light, i.e., $\beta_{ph} \neq 1$. Although an analytical solution cannot be found in this case, the motion can be investigated by the phase-trajectory technique. When $\beta_{ph} \neq 1$ the basic equation (1.6) for a left-hand polarized wave assumes the form

$$(Mx')' = \eta_0 y' \sin\psi + \eta_0 z' \cos\psi,$$
$$(My')' = \frac{\Omega}{\omega} z' + \eta_0 \sin\psi,$$
$$(Mz')' = -\frac{\Omega}{\omega} y' + \eta_0 \cos\psi, \qquad (3.1)$$

where M is given by (1.5) and

$$\gamma = \gamma_i (1 - \beta_{ph}\beta_{ni})(1 - M\beta_{ph}^2)/(1 - \beta_{ph}^2). \qquad (3.2)$$

Again using the variables V and $\varphi$ (2.12) we obtain the following integral of the motion, which is analogous to (2.13):

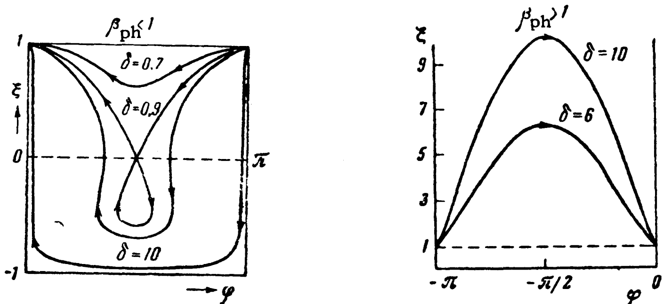
$$H = \frac{(MV)^2 - (M_i V_i)^2}{2}\left[\frac{2\Omega}{\omega(M + M_i)} - 1\right] - \eta_0 MV \sin\varphi = \text{const}. \qquad (3.3)$$

We note, incidentally, that the quantity H, expressed in terms of the canonical variables Y $= (MV)^2/2$ and $\varphi$, is the Hamiltonian of the motion.

The fundamental difference between (3.3) and (2.13) is the fact that the coefficient of the quadratic term cannot vanish identically for any choice of the parameters (with the exception of the trivial case $\beta_{ph} \to 1$). In turn, this restriction means that the resonance cannot be maintained automatically over the entire motion as was the case in Sec. 2 when $\Omega = \omega$. Thus, resonance motion is not possible in a slow or fast plane wave.

In Fig. 1 we show phase diagrams for the cases $\beta_{ph}^2 \gtrless 1$ in the dimensionless variables $\varphi$ and $\xi = M/M_i$ (if $\beta_{ni} = \beta_{ph}^{-1}$, the quantity $\xi$ corresponds to the ratio $\gamma/\gamma_i$). To avoid complicating the analysis we assume zero initial conditions, i.e., we assume that $\alpha = 0$ and consequently $V_i = 0$. The parameters shown in these curves have the following meanings:

$$\delta = \frac{2\eta_0\beta_{ph}^2(1 - \beta_{ph}\beta_{ni})}{(\beta_{ph} - \beta_{ni})\,|1 - \beta_{ph}^2|^{1/2}}, \qquad \nu = \Gamma\beta_{ph}\frac{(1 - \beta_{ni})^{1/2}}{\beta_{ph} - \beta_{ni}}. \qquad (3.4)$$




a FIG. 1 b

It is interesting to note that the phase trajectories for the cases $\beta_{ph}^2 \lessgtr 1$ diverge sharply. In the first case $\xi$ varies within the limits $\pm 1$ and the ratio $\gamma/\gamma_i$ can vary between unity and $(\beta_{ph}^2 - 2\beta_{ph}\beta_{ni} + 1)/(1 - \beta_{ph}^2)$; in the second case, the variation of $\xi$ and, consequently $\gamma/\gamma_i$, can be arbitrarily large.

## 4. RESONANCE MOTION OF A PARTICLE IN A PLANE WAVE PROPAGATING IN AN ANISOTROPIC MEDIUM

We have shown above that a vacuum is the only isotropic medium in which resonance motion is possible for a plane wave. However, many other possibilities are opened up by the use of anisotropic media, and we now consider some of these.

For simplicity we consider a uniaxial crystal characterized by a dielectric tensor with two components $e_{\parallel}$ and $e_{\perp}$:

$$D_\xi = \varepsilon_\perp E_\xi, \qquad D_\eta = \varepsilon_\perp E_\eta, \qquad D_\zeta = \varepsilon_\parallel E_\zeta, \qquad (4.1)$$

where $O\zeta$ is along the crystal axis while $O\xi$ and $O\eta$ are perpendicular to this axis. If $\theta$ is the angle between $O\zeta$ and the direction of the wave normal **n**, the refractive index n is given by: [6]

$$n^{-2}(\theta) = \frac{\cos^2\theta}{\varepsilon_\perp} + \frac{\sin^2\theta}{\varepsilon_\parallel}. \qquad (4.2)$$

We note that the electric induction vector **D** and the vector **B** are perpendicular to the wave normal **n** in contrast to the vector **E** which does not exhibit this property in an anisotropic medium. The vectors **E**, **D**, and **n** are coplanar and the following relations hold between **D**, **E**, and **B**:

$$\mathbf{B} = n\,[\mathbf{nE}], \qquad \mathbf{D} = n\,[\mathbf{Bn}]. \qquad (4.3)$$

We introduce the notion of a ray direction, characterized by a unit vector **s**, perpendicular to **E** and **B**. The vector **s** and the vector **n** form an angle $\chi$ given by

$$\cos\chi = n^{-2}(\theta)\left[\frac{\cos^2\theta}{\varepsilon_\perp^2} + \frac{\sin^2\theta}{\varepsilon_\parallel^2}\right]^{-1/2}. \qquad (4.4)$$

Following these preliminaries we now consider the motion of a charged particle in the field of a plane wave characterized by a wave normal **n**. It is assumed, as before, that there is a uniform fixed external magnetic field $\mathbf{B}_0$; in the present case this field is in the same direction as the ray, i.e., along **s**. The equation of motion is of the same form as for the isotropic medium except that **n** is replaced by **s** in the last term. As in Sec. 1, we obtain the integral of motion

$$\gamma (1 - \beta_{ph}\beta_s / \cos\chi) = \text{const} = \gamma_i (1 - \beta_{ph}\beta_{si} / \cos\chi), \quad (4.5)$$

which differs from that for the isotropic medium by the appearance of the denominator $\cos\chi$ in the second term.

We now introduce the coordinate system x, y, z with the Ox axis along the ray velocity of the wave (direction of **s**), the Oy axis along **E**, and the Oz axis along **B**. In the equation of motion we convert from the variable t to the variable $\psi = \omega t - kx\cos\chi + \psi_i$, where x is taken in the direction of the ray; using the invariant (4.5) we can write this equation in the same form as in the isotropic case (1.6). However, the coefficients M, $\Omega$, and $\eta$ are somewhat different from those in (1.5) because of the factor $\cos\chi$:

$$\Omega = \frac{eB_0}{mc\gamma_i (1 - \beta_{ph}\beta_{si} / \cos\chi)}, \qquad \eta = \frac{\Omega E}{\omega\beta_{ph}B_0} = \eta_0 \sin\psi; \quad (4.6)$$

$$M = \left(1 - \frac{\cos\chi}{\beta_{ph}}\beta_s\right)\left(1 - \frac{\beta_{ph}\beta_s}{\cos\chi}\right)^{-1}. \quad (4.7)$$

An important result follows from (4.6) and (4.7):

$$\text{for } \beta_{ph} = \cos\chi \quad \text{or} \quad n\cos\chi = 1 \quad (4.8)$$

equation (1.6) for the anisotropic medium becomes the same as the equation for a particle in the field of a plane wave propagating in vacuum with velocity c. In this case we know that the condition $\Omega = \omega$ means a resonance between the wave and the particle and that the particle energy changes continuously. We note that by definition the ray velocity of the wave is $c\beta_{ph}/\cos\chi$. Thus, simply stated Eq. (4.8) means that the ray velocity of the wave must equal the velocity of light in vacuum.

If (4.8) is not satisfied no resonance is possible. Equation (1.6) can be investigated in the phase plane in this case; we do not carry out this calculation, however, because by using the notation $\beta^*_{ph} = \beta_{ph}/\cos\chi$ the problem is reduced to the equivalent case of an isotropic medium with phase velocity $\beta^*_{ph}$.

We now consider in somewhat greater detail the condition under which (4.8) can be satisfied; evidently, (4.8) establishes a definite relation between the direction of the wave normal and the dielectric constants $\epsilon_\perp$ and $\epsilon_\parallel$. This relation is

$$\cos^2\theta = \varepsilon_\perp^2 (1 - \varepsilon_\parallel)/(\varepsilon_\parallel - \varepsilon_\perp)(\varepsilon_\perp\varepsilon_\parallel - \varepsilon_\parallel - \varepsilon_\perp). \quad (4.9)$$

The necessary condition for particle-wave resonance (4.8) can only be satisfied when the following inequalities obtain:

$$0 < \varepsilon_\perp^2 (1 - \varepsilon_\parallel)/(\varepsilon_\parallel - \varepsilon_\perp)(\varepsilon_\perp\varepsilon_\parallel - \varepsilon_\parallel - \varepsilon_\perp) < 1. \quad (4.10)$$

This requirement imposes a definite limitation on suitable values of $\epsilon_\perp$ and $\epsilon_\parallel$. Using (4.9), we plot the region on the ($\epsilon_\perp$, $\epsilon_\parallel$) plane that satisfies (4.10) and corresponds to possible resonance motion of a particle in a plane wave propagating in an anisotropic medium. There are two such regions. These regions are the two quarter-planes bounded by the lines

$$\varepsilon_\perp = 1, \; \varepsilon_\parallel = 1 \quad (4.11)$$

and the regions have one common point ($\epsilon_\perp = \epsilon_\parallel = 1$) (cf. Fig. 2). This point is the only allowed one (in the sense indicated above) for an isotropic medium, which, as indicated above, must be a vacuum. It is evident that the possibilities for wave-particle resonance are much broader in anisotropic media.

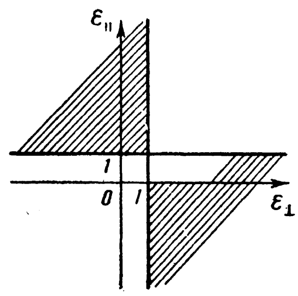



FIG. 2

As an example we consider briefly the case in which the medium is an electron plasma in a fixed uniform magnetic field $\mathbf{B}_0$. Such a plasma is essentially a uniaxial anisotropic (gyrotropic) crystal and propagates plane waves that are elliptically polarized. The magnetic field $\mathbf{B}_0$, which gives the plasma the properties of a gyrotropic crystal, must simultaneously serve to produce the resonance between the wave and the particle. Considerations that follow directly from the above show that the electric vector (more precisely, the plane in which the electric vector rotates, describing an ellipse) must be perpendicular to the direction of $\mathbf{B}_0$; on the other hand the wave must propagate in this direction with the velocity of light. Analysis shows that in general it is impossible to satisfy these two requirements simultaneously.

It is evident from the above considerations that a resonance interaction between waves and particles can be realized in different ways. From the point of view of enhancing efficiency, it is of interest to consider more complicated field configurations than those considered here, for example, nonuniform (including alternating) magnetic fields, electric fields alternating in time and so on.

[1] L. Landau and E. Lifshitz, Teoriya polya (Theory of Fields), 2nd edition, Fizmatgiz, 1960.

[2] Ya. Frenkel', Sobranie izbr. trudov, 1, Elektrodinamika (Selected Works, Volume 1, Electrodynamics), AN SSSR, 1956.

[3] A. A. Kolomenskiĭ and A. N. Lebedev, DAN SSSR **145**, 1259 (1962), Soviet Phys. Doklady **7**, 745 (1963).

[4] F. Ollendorf, Bull. Res. Council Israel **3**, 348 (1954).

[5] Ya. B. Faĭnberg and V. I. Kurilko, ZhTF **29**, 939 (1959), Soviet Phys. Tech. Phys. **4**, 855 (1960).

[6] M. V. Vol'kenshteĭn, Molekulyarnaya optika (Molecular Optics), Gostekhizdat, p. 141, 1951.

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