FLUCTUATIONS OF THE NUMBER OF PARTICLES IN AN ELECTRON-PHOTON SHOWER, WITH EFFECTS OF IONIZATION LOSSES INCLUDED

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Expressions in analytic form are obtained for the mean square number of particles in an electron-photon shower, with ionization losses taken into account. Curves are given of the ratio of the mean square particle number to the square of the mean, as a function of the depth, for various energies of the primary electron or photon and for various energies of the secondary particles.

A method developed in a previous paper [1] allows us to calculate the mean square deviation of the number of particles in an electron-photon shower in approximation A. [2,3] This problem is treated here with ionization losses taken into account. We assume as usual that the ionization loss does not depend on the energy and is equal to β per radiation unit of length.

We give the derivation of the equations for the mean moments of the particle number, with ionization losses taken into account. We define the function $\Psi(E_0, E, t, N)$ as the probability that in traversing the depth t a particle of energy E_0 produces N particles with energies larger than E. Then we have for the mean moment of any order

$$\overline{N^r}(E_0, E, t) = \sum_{N=0}^{\infty} N^r \Psi(E_0, E, t, N). \tag{1}$$

The probability that at the depth t+dt an electron with energy E_0 has formed N particles is composed (see Fig. 1 in [1]) of the probability for traversing the thickness dt without interacting and then producing the shower, and the probability of emitting a photon in the layer dt and then having two independent showers produced by the electron of energy $E_0 - E'$ and the photon of energy E'. We have

$$\Psi(E_0, E, t+dt, N_e) = \Psi(E_0 - \beta dt, E, t, N_e)\left[1 - dt\int_0^{E_0} W_e(E_0, E')\,dE'\right] + dt\sum_{N_{e'}=0}^{\infty}\sum_{N_\gamma=0}^{\infty}\delta_{N_{e'}+N_\gamma, N_e} \times \int_0^{E_0} W_e(E_0, E')dE'\Psi(E_0 - E', E, t, N_{e'})\,\Psi(E', E, t, N_\gamma). \tag{2}$$

In the first term on the right account has been taken of the fact that in traversing the layer dt the energy of the electron, which thereafter is the source of the shower, is decreased by the amount βdt by ionization losses. The symbol $\delta_{N_{e'}+N_\gamma, N_e}$ assures that the numbers $N_{e'}$ and N_γ, each of which may have any value, shall have the fixed sum N_e (the indices indicate the primary particle).

Analogous arguments for the case of a primary photon give the equation

$$\Psi(E_0, E, t+dt, N_\gamma) = \Psi(E_0, E, t, N_\gamma)[1 - dt\,\sigma_0] + dt\sum_{N_{e''}=0}^{\infty}\sum_{N_{e'}=0}^{\infty}\delta_{N_{e''}+N_{e'}, N_\gamma}\int_0^{E_0} W_p(E_0, E')dE'\Psi \times (E_0 - E', E, t, N_{e''})\Psi(E', E, t, N_{e'}). \tag{3}$$

By means of Eqs. (2) and (3) one can easily obtain the equations for arbitrary moments. The equations for the mean squares differ from the corresponding equations in [1] by the presence of a term $-\beta\partial\overline{N_e^2}(E_0, E, t)/\partial t$. Furthermore the free term in the right member of the equations now involves the mean number of electrons calculated with ionization losses taken into account.

We use the solutions of Belen'kiĭ [3]:

$$\overline{N}_e(E_0, E, t) = \frac{H_1(s)\,D(s)\,G(s, \varepsilon)}{s\sqrt{2\pi\lambda_1''(s)\,t}}\exp[y_0 s + \lambda_1(s)\,t], \tag{4}$$

$$\overline{N}_\gamma(E_0, E, t) = \frac{M(s)\,D(s)\,G(s, \varepsilon)}{s\sqrt{2\pi\lambda_1''(s)\,t}}\exp[y_0 s + \lambda_1(s)\,t], \tag{5}$$

$$t = -\frac{y_0}{\lambda_1'(s)}, \qquad y_0 = \ln\frac{E_0}{\beta}, \qquad \varepsilon = \frac{E}{\beta}f[\lambda_1(s)]. \tag{6}$$

These forms for $\overline{N}_e$ and $\overline{N}_\gamma$ are valid under the conditions $\beta/E_0 \ll 1$ and $E \lesssim \beta$, and $D(s)G(s, \epsilon)$

is a function which varies slowly with s and ϵ. Using Eqs. (4) and (5) for the free terms in the right members of the equations, we get [1]

$$R_e = 2\overline{N}_e(E_0, E, t)\,\overline{N}_\gamma(E_0, E, t)\,\gamma_1(s) \sim (E_0/\beta)^{2s}\exp\{2\lambda_1(s)\,t\}, \quad (7)$$

$$R_\gamma = 2\overline{N}_e^2(E_0, E, t)\,\gamma_2(s) \sim (E_0/\beta)^{2s}\exp\{2\lambda_1(s)\,t\}. \quad (8)$$

It is natural to assume that the desired solutions are of the same form as the free terms. Set

$$\overline{N_e^2}(E_0, E, t) = f_1(t)\,D(s)\,G(s, \varepsilon)\,(E_0/\beta)^{2s}\exp\{2\lambda_1(s)\,t\}, \quad (9)$$

$$\overline{N_\gamma^2}(E_0, E, t) = f_2(t)\,D(s)\,G(s, \varepsilon)\,(E_0/\beta)^{2s}\exp\{2\lambda_1(s)\,t\}. \quad (10)$$

Then

$$\frac{\partial f_1(t)}{\partial t} = f_1(t)\left[2\lambda_1(s) + A(s) + 2s\frac{\beta}{E_0}\right] + f_2(t)\,C(s) + \frac{H_1(s)\,M(s)\,\gamma_1(s)}{\pi s^{3/2}\lambda_1''(s)\,t}, \quad (11)$$

$$\frac{\partial f_2(t)}{\partial t} = f_2(t)\,[\sigma_0 + 2\lambda_1(s)] + f_1(t)\,B(s) + \frac{H_1^2(s)\,\gamma_2(s)}{\pi s^2\lambda_1''(s)\,t}. \quad (12)$$

It can be seen that in Eq. (11) we can neglect the term $2s\beta/E_0$ in the coefficient of $f_1(t)$. Then we get as the equations for $\overline{N_e^2}(E_0, E, t)$ and $\overline{N^2}(E_0, E, t)$ (with $s < 1.47$):

$$\overline{N_e^2}(E_0, E, t) = \frac{2\{\overline{N}_e(E_0, E, t)\,\overline{N}_\gamma(E_0, E, t)\,\gamma_1(s)\,[2\lambda_1(s) + \sigma_0] + C(2s)\,\gamma_2(s)\,\overline{N}_e^2(E_0, E, t)\}}{[2\lambda_1(s) - \lambda_1(2s)]\,[2\lambda_1(s) - \lambda_2(2s)]}, \quad (13)$$

$$\overline{N_\gamma^2}(E_0, E, t) = \frac{2\{\overline{N}_e(E_0, E, t)\,\overline{N}_\gamma(E_0, E, t)\,\gamma_1(s)\,B(s) + [2\lambda_1(s) + A(2s)]\,\gamma_2(s)\,\overline{N}_e^2(E_0, E, t)\}}{[2\lambda_1(s) - \lambda_1(2s)]\,[2\lambda_1(s) - \lambda_2(2s)]}. \quad (14)$$

These expressions differ from those obtained in [1] with ionization losses neglected, in that the new equations contain the $\overline{N}_e$ and $\overline{N}_\gamma$ given by the formulas (4) and (5) of the present paper, and not by Eqs. (11) and (12) of [1]. In the ratios, however,

$$\delta_e \equiv \frac{\overline{N_e^2(E_0, E, t)}}{\overline{N}_e(E_0, E, t)^2}, \quad \delta_\gamma \equiv \frac{\overline{N_\gamma^2(E_0, E, t)}}{\overline{N}_\gamma(E_0, E, t)^2}$$

the function $D(s)G(s, \epsilon)$, in which the expressions differ, drops out, and therefore the quantities δ_e and δ_γ as functions of s are just the functions given in Eqs. (37), (38) of [1]. These functions are shown in Fig. 1.

As in [1], our equations (13) and (14) lose their meaning at $s = 1.47$, when $2\lambda_1(s_0) = \lambda_1(2s_0)$. In this region of s the character of the solution changes. The procedure for finding it is analogous to that used in [1]. We have

$$\overline{N_e^2}(E_0, E, t) = \frac{[\sigma_0 + \lambda_1(2s_0)]\,H_1(s_0)\,M(s_0)\,\gamma_1(s_0) + C(2s_0)\,H_1^2(s_0)\,\gamma_2(s_0)\,s_0^{-1/2}}{2\sqrt{\pi\lambda_1''(s_0)\,t_0}\,[\lambda_1(2s_0) - \lambda_2(2s_0)]\,[\lambda_1'(2s_0) - \lambda_1'(s_0)]\,s^{3/2}}$$
$$\times D(s_0)G(s_0, \varepsilon)\exp[2ys_0 + 2\lambda_1(s_0)\,t]\left\{1 + \mathrm{ert}\left(-\frac{(t - t_0)\,\lambda_1'(s_0)}{\sqrt{\lambda_1''(s_0)\,t_0}}\right)\right\}$$
$$-2\,\frac{[\sigma_0 + \lambda_2(2s)]\,\overline{N}_e(E_0, E, t)\,N_\gamma(E_0, E, t)\,\gamma_1(s) + C(2s)\,\overline{N}_e^2(E_0, E, t)\,\gamma_2(s)}{[2\lambda_1(s) - \lambda_2(2s)]\,[\lambda_1(2s) - \lambda_2(2s)]}, \quad (15)$$

$$\overline{N_\gamma^2}(E_0, E, t) = \frac{B(2s_0)\,H_1(s_0)\,M(s_0)\,\gamma_1(s_0) + [\lambda_1(2s_0) + A(2s_0)]\,H_1^2(s_0)\,\gamma_2(s_0)\,s^{-1/2}}{2\sqrt{\pi\lambda_1''(s_0)\,t_0}\,[\lambda_1(2s_0) - \lambda_2(2s_0)]\,[\lambda_1'(2s_0) - \lambda_1'(s_0)]\,s_0^{3/2}}$$
$$\times D(s_0)\,G(s_0, \varepsilon)\exp[2ys_0 + 2\lambda_1(s_0)]\left\{1 + \mathrm{erf}\left[-\frac{(t - t_0)\,\lambda_1'(s_0)}{\sqrt{\lambda_1''(s_0)\,t_0}}\right]\right\}$$
$$-\frac{2\{B(2s)\,\overline{N}_e(E_0, E, t)\,\overline{N}_\gamma(E_0, E, t)\,\gamma_1(s) + [\lambda_2(2s) + A(2s)]\,\overline{N}_e^2(E_0, E, t)\,\gamma_2(s)\}}{[\lambda_1(2s) - \lambda_2(2s)]\,[2\lambda_1(s) - \lambda_2(2s)]}. \quad (16)$$

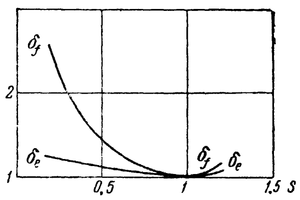

FIG. 1

Here, as in Eqs. (13) and (14), t is determined by Eq. (6) and $t_0 = -y_0/\lambda_1'(s_0)$. The corresponding values of δ_e and δ_γ vary slowly with E in the region $s > 1.5$. Naturally the fluctuations are larger for particles with larger energies. Figure 2 shows the dependence on t of $\ln\delta_e$ (solid curves) and $\ln\delta_\gamma$ (dashed) for various values of E_0 with $\beta = 72$ MeV and $E = \beta$. Figure 3, a and b, drawn

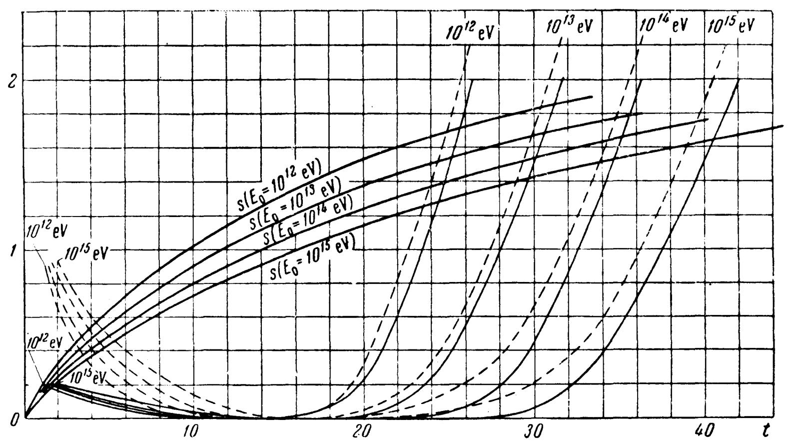

FIG. 2

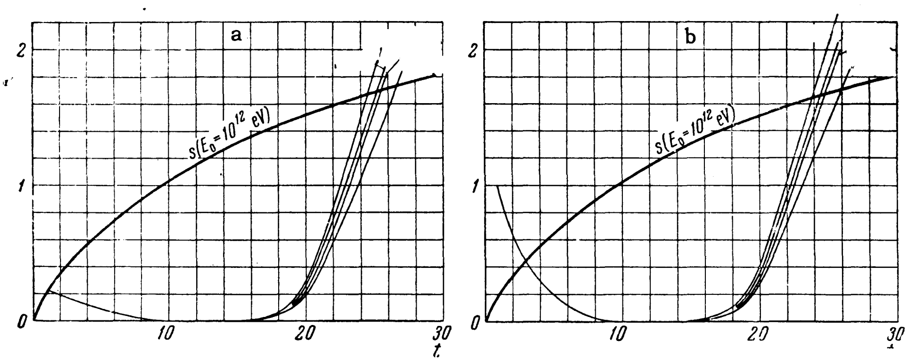

FIG. 3

for $E_0 = 10^{12}$ eV, illustrates the "splitting" of $\ln \delta_e$ and $\ln \delta_\gamma$ for various values of E. The steepest curves are for $E/\beta = 3$; the others are for $E/\beta = 1$, 0.5, and 0.05. In the last three diagrams we also give for convenience the dependence of s on t on the same scale (heavy curves).

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