# *EXCITATION OF NUCLEI BY SLOW CHARGED PARTICLES*

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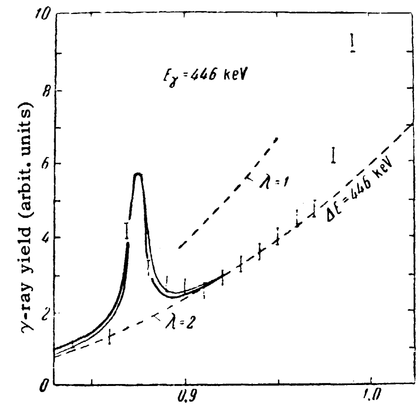
The cross section for excitation of nuclei by slow charged particles is obtained by taking into account not only Coulomb but also the nuclear forces which give rise to resonances. In a number of cases the role of interference in the total excitation cross section is shown to be small. The results agree with the experimental data.

K. ALDER et al.[1] cite the experimental results of Temmer and Heidenburg on the excitation of $Na^{23}$ by protons. These data (including the experimental errors) are shown in the figure by vertical lines. It follows from these results that for proton energies somewhat less than the Coulomb barrier height, nuclear forces become important and lead to the formation of peaks on a smooth curve. To the right and left of the indicated peaks the smooth curve coincides with the calculated Coulomb excitation cross section which is depicted on the figure by a broken line.




On the basis of this we computed the nuclear excitation cross section neglecting the nuclear forces in comparison with the Coulomb, except at resonances associated with the formation of an intermediate nucleus. Since we can apply perturbation theory to Coulomb interaction ($V^1$), we can use the dispersion formula[2] for inelastic scattering in a Coulomb field

$$d\sigma = \frac{M^2 v_f}{4\pi^2\hbar^4 v_i}\,\frac{1}{(2I_i+1)(2s+1)}$$

$$\times \sum_{M_i M_f \mu_i \mu_f}\Bigg|\int u^*_{\mu_f}\varphi^*_f F^{-*}_{\mathbf{k}_f} V^1 u_{\mu_i}\varphi_i F^+_{\mathbf{k}_i}\, d\tau_c + \frac{4\pi^{5/2}\hbar^3}{M^2 v_i^{1/2} v_f^{1/2}}$$

$$\times \sum_{l_f m_f} Y_{l_f m_f}(\mathbf{k}_f) \sum_{l_i r M} i^{l_i - l_f}(2l_i+1)^{1/2}\exp i\,[\eta_{l_f}+\eta_{l_i}]$$

$$\times \frac{H^{rM}_{il_i 0 s\mu_i}\, H^{rM*}_{fl_f m_f s\mu_f}}{W - W_{rJ} + i\,\dfrac{\gamma_{rJ}}{2}}\Bigg|^2 d\Omega. \qquad (1)$$

We represent (1) in the form

$$d\sigma = d\sigma_1 + d\sigma_{12} + d\sigma_2, \qquad (2)$$

where $d\sigma_1$ is the differential cross section for Coulomb excitation and coincides with the expression obtained by Alder et al.,[1] $d\sigma_{12}$ is the interference between the Coulomb and nuclear-resonant excitation[2], and $d\sigma_2$ is the nuclear-resonant excitation.[3]

For the calculation of $d\sigma_1$ and $d\sigma_2$ we have that[3]

$$H^{rM}_{flms\mu} = \sum_{j\mu_j} (-1)^{l-s+\mu_j}(2j+1)^{1/2}\begin{pmatrix} l & s & j\\ m & \mu & -\mu_j\end{pmatrix}$$

$$\times (-1)^{j-I+M}\,(2J+1)^{1/2}\begin{pmatrix} j & I & J\\ \mu_j & M_f & -M\end{pmatrix} U^{rJ}_{flj}, \qquad (3)$$

where the quantities $U^{rJ}_{flj}$ are real.[3,4]1) Also real (cf. [1]) are the radial matrix elements $M^{-\lambda}_{l_i l_f}$ and the reduced matrix elements for multipole transitions $\langle I_i \,\|\, \lambda \,\|\, I_f\rangle$, which enter into the Coulomb amplitude.

Substituting (3) in (1), summing over magnetic quantum numbers, and integrating over angles we obtain the following expressions for the interference term

1)The partial width $\gamma_f^{rJ}$ is associated with the quantities $U^{rJ}_{flj}$ by the relation

$$\gamma_f^{rJ} = 2\pi \sum_{lmM_f\mu} |H^{rM}_{flms\mu}|^2 = 2\pi \sum_{lj} (U^{rJ}_{flj})^2.$$

$$\sigma_{12} = \frac{32\pi^{5/2} v_f^{1/2} Z_1 e}{\hbar v_i^{3/2} (2I_i+1)(2s+1)} \sum_{\lambda=1}^{\infty} \sum_{l_i l_f j_i j_f J} \langle I_i \| \lambda \| I_f \rangle (-1)^{s-I_i-J}$$
$$\times [(2l_i+1)(2l_f+1)(2\lambda+1)^{-1}]^{1/2} \begin{pmatrix} l_i & l_f & \lambda \\ 0 & 0 & 0 \end{pmatrix}$$
$$\times M_{l_i l_f}^{-\lambda-1} [(2j_i+1)(2j_f+1)]^{1/2} (2J+1) \begin{Bmatrix} j_i & j_f & \lambda \\ I_f & I_i & J \end{Bmatrix}$$
$$\times \begin{Bmatrix} l_f & l_i & \lambda \\ j_i & j_f & s \end{Bmatrix} U_{il_i j_i}^{rJ} U_{fl_f j_f}^{rJ} \frac{E_i + W_{N_i} - W_{rJ}}{(E_i + W_{N_i} - W_{rJ})^2 + \gamma_{rJ}^2/4}; \quad (4)$$

and for $\sigma_2$, the nuclear-resonant excitation cross section (or, equivalently, the inelastic scattering cross section)

$$\sigma_2 = \frac{4\pi^3 \lambda\!\!\!^{-}{}_i^2}{(2I_i+1)(2s+1)}$$
$$\times \sum_{l_i j_i l_f j_f J} (2J+1) \left| \sum_r \frac{U_{il_i j_i}^{rJ} U_{fl_f j_f}^{rJ}}{E_i + W_{N_i} - W_{rJ} + i\gamma_{rJ}/2} \right|^2. \quad (5)$$

For incident particle energies near the level $|W_{rJ} - W_{N_i}|$ of the compound nucleus, $\sigma_2$ takes the form

$$\sigma_2 = \pi\lambda\!\!\!^{-}{}^2 \frac{2J+1}{(2I_i+1)(2s+1)} \frac{\gamma_i^{rJ}\gamma_f^{rJ}}{[E_i - (W_{rJ} - W_{N_i})]^2 + \gamma_{rJ}^2/4}. \quad (6)$$

From the deviation of $\sigma$ from $\sigma_1 + \sigma_2$ it is possible to obtain information about the matrix elements $U_{il_ij_i}^{rJ}$ and $U_{fl_fj_f}^{rJ}$, and to determine the sign of $\langle I_i \| \lambda \| I_f \rangle$. According to (4), the contribution of the interference term is zero for the S-wave, small for the P-wave, and noticeable only for the high levels of the intermediate nucleus when the large number of terms yield simultaneously an important contribution to $\sigma_1$ and $\sigma_2$. However, at resonance, i.e., for $E_i = W_{rJ} - W_N$, we have that $\sigma_{12} = 0$.

In conclusion we examine the case represented in the figure. From experiments[5,6] observing other decay channels of the excited state of the intermediate $Mg^{24}$ nucleus, which is excited when the incident proton energy $E_p = 0.877$ MeV, it is known that the spin and total width of this level are respectively $J = 1$, $\Gamma = 8 \pm 2$ kev. In addition, it is known[1] that $|\langle I_i \| \lambda \| I_f \rangle| = 0.228$e b. Substituting this additional data into (4) and (5), we obtain

$$\sigma_2 + \sigma_{12} = 11.94 \frac{\Gamma_i \Gamma_f}{\pi^2 \Gamma_r^2} \left[ 1 + 1.9 \cdot 10^{-4} \varepsilon \frac{\pi \Gamma_r}{(\Gamma_i \Gamma_f)^{1/2}} \frac{E_p - 877}{5.22} \right] \Big/ \left[ \left( \frac{E_p - 877}{5.22} \right)^2 + 1 \right] \text{b},$$

where $\epsilon = 0$ for $U_{i1\,1/2}^1 U_{i1\,3/2}^1$, $\epsilon = \pm 1$ for $U_{i1\,1/2}^1 U_{i1\,3/2}^1$, and $\langle I_i \| \lambda \| I_f \rangle = \mp 0.228$ e barns.

Constructing the corresponding curves by least squares, we find that the curve for $\epsilon = -1$, indicated by the heavy line, is in somewhat better agreement with the experimental data of Temmer and Heidenburg than the other two curves close to it. The curve corresponding to $\epsilon = 0$ is indicated by a thin continuous line. Thus, (4) and (5) are found to agree with the experimental data. For $\Gamma_i \Gamma_f / \Gamma_r^2$ we find the value $1.78 \times 10^{-5}$ which is in agreement with the result of Baumann et al [5]; in addition the small contribution $\sigma_{12}$ implies that the quantities $U_{i1\,1/2}^1$ and $U_{i1\,3/2}^1$ have opposite signs and that $\langle I_i \| \lambda \| I_f \rangle = +0.228$ e b.

We note that insofar as the influence of the higher order approximations in the Coulomb part of the cross section is very small [1] (for example, in the present case the contribution of the second approximation is two orders less than the contribution of $\sigma_{12}$ at the maximum) then the initial formula (1) describes the effect sufficiently well.

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