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NUCLEON-NUCLEON SCATTERING IN THE TWO-MESON APPROXIMATION WITH ACCOUNT OF PION-PION INTERACTION

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The two-meson absorptive part of the nucleon-nucleon scattering amplitude is computed for momentum transfers $4\mu^2 \leq t < 4m\mu$ up to terms of the order $\sim t/4m\mu$ and p^2/m^2 (μ and m are the pion and nucleon masses, respectively, and p is the nucleon momentum). This is done employing the $\pi + \pi \rightarrow N + \overline{N}$ amplitude derived in [7] and taking into account the effects of pion-pion interaction in S, P and D states. The pion-pion interaction contributes significantly only in the presence of kinematic resonances, when the pion-pion amplitude has a zero near the resonance. In the simplest case of a kinematic P-resonance at an energy $\sqrt{t_r} \approx 750$ MeV (the effect of the ρ meson) it is impossible to obtain agreement with experiment on the basis of the two-meson approximation simultaneously for the electromagnetic form factors and for the elastic nucleon-nucleon amplitude.

1. INTRODUCTION

THE method of computing scattering amplitudes by means of an expansion in terms of increasing masses in the intermediate states has been widely used in recent times (cf., e. g.,[1]). In the language of dispersion theory this method is equivalent to taking into account successively singularities which are more and more distant in the complex plane of the invariants. If the lowest term of the expansion is a pole term (one-particle exchange), it can be computed in an elementary manner, but even the following term (two-particle exchange) can be obtained only by overcoming considerable technical difficulties.

An actual computation of the two-meson contribution to the nucleon-nucleon scattering amplitude has been performed up to the present [2,3] only for partial amplitudes and phase shifts with large orbital momenta $l \gg 1$ (peripheral interactions), and only the lowest order term of the expansion of the two-meson absorptive part in powers of

$$x = (t - 4\mu^2)/4\mu^2 \qquad (1)$$

has been obtained. This guaranteed the calculation of the two-meson phase shifts to an accuracy of the order of

$$1/L = \sqrt{1 + p^2/\mu^2}/(l + 1). \qquad (2)$$

Here t is the momentum transfer, μ the pion mass, and p the momentum of the nucleon in the center-of-mass system.

Evidently, only not too large values of l are of practical value, when the expansion parameter (2) is close to unity (weak peripheralism[1)]) and the asymptotic formulae one derives can be considered only as a rough estimate of the order of magnitude of the two-meson phase shifts. It is interesting to note that in this case there are special reasons for the low accuracy of the asymptotic formulae: the central and spin-orbit two meson forces turn out to be anomalously small due to some strong compensation, which reduces the main terms approximately by a factor of $\epsilon^2 = (\mu/m)^2$ (m is the nucleon mass). This compensation could disappear in the absorptive part for $x \sim 1$ and then the resulting formulae would be completely unacceptable in the case of weak peripheralism.

For a more accurate computation of the two-meson contribution to the NN-amplitude one is forced to give up the expansion in terms of the quantities (1) and (2) and one must derive the absorptive part at least for $4\mu^2 \leq t \lesssim 9\mu^2$. A general method for such calculations has been developed by several authors [4,5] on the basis of Mandelstam's formalism [6,1]. The main problem arising in this case is the calculation of the pion-nucleon amplitude in the nonphysical region, which determines the absorptive part of the NN-amplitude.

In the present paper the two-meson contribution is calculated on the basis of the pion-nucleon am-

[1)]In this case the effective interaction of the nucleons takes place at distances $r \sim 1/\mu$ and the absorptive part contributes effectively to the phase shifts for $x \sim 1$.

plitude obtained by Galanin and Grashin [7], taking into account the amplitudes for pion-pion scattering with isospin I = 0 for even l and isospin I = 1 for odd l in the form[2)]

$$\lambda_l(x) = e^{i\delta_l(x)}\sin\delta_l(x) = Q^{(l)}(x)\sqrt{x}/[X^{(l)}(x) - iQ^{(l)}(x)\sqrt{x}];$$

$$\sqrt{x}\cot\delta_l(x) = X^{(l)}(x)/Q^{(l)}(x);\quad l = 0\,(S),\ 1\,(P),\ 2\,(D), \tag{3}$$

where $X^{(l)}(x)$ and $Q^{(l)}(x)$ are arbitrary polynomials in the square of the meson c.m. three-momentum. An expansion in terms of the small parameter $\epsilon = \mu/m = 0.15$ is consistently carried out, which considerably simplifies the computations and allows one to obtain the absorptive part of the NN amplitude in a sufficiently large domain $4\mu^2 < t < 4m\mu$ with an accuracy to $\sim \epsilon x \approx t/4m\mu$. Strictly speaking the resulting formulae are not applicable in a small neighborhood $0 \leq x \lesssim \epsilon^2/4$ near $t = 4\mu^2$, since besides expanding in terms of ϵx one also expands in terms of the quantity $\epsilon(1 + 2x)/2\sqrt{x}$ [3)]. From asymptotic formulae derived earlier [2,3] it is, however, evident that this neighborhood gives a considerable contribution only for very large orbital momenta $l \gtrsim 4/\epsilon^2 = 180$. The calculations have been carried out in the nonrelativistic approximations, up to corrections of the order p^2/m^2.

2. CALCULATION OF THE ABSORPTIVE PART

In the two-meson approximation the absorptive part of the NN-amplitude A(E, t) is expressed via the unitarity condition (cf. figure) in terms of the pion-nucleon amplitude

$$T_{\alpha\beta}(\nu, t) = \delta_{\alpha\beta}\{A^{(+)}(\nu, t) + \hat{k}B^{(+)}(\nu, t)\} + \frac{[\tau_\alpha\tau_\beta]}{2}\{A^{(-)}(\nu, t) + \hat{k}B^{(-)}(\nu, t)\}, \tag{4}$$

in the following manner:

$$A(E, t) = \frac{m}{8E(4\pi)^3}\sqrt{\frac{t - 4\mu^2}{t}}\int\langle p_1'|T_{\alpha\beta}(\nu_1, t)|p_1\rangle \times \langle p_2'|T_{\alpha\beta}(\nu_2, t)|p_2\rangle^+ d\Omega_1. \tag{5}$$

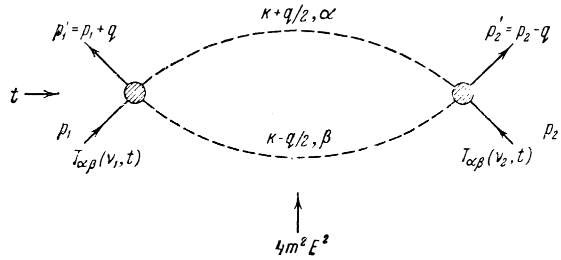
When integrating over $d\Omega_1 = dz_1 d\varphi_1$ (along the vector k) the pion-nucleon amplitudes are regarded as functions of the following invariants

$$\nu_1 = (s_1 - m^2 - \mu^2 + t/2)/2m = \mu\sqrt{-x}Wz_1,$$
$$\nu_2 = (s_2 - m^2 - \mu^2 + t/2)/2m = \mu\sqrt{-x}Wz_2,$$

where

$$z_2 = zz_1 + \sqrt{1 - z^2}\sqrt{1 - z_1^2}\cos\varphi_1,\quad z = 1 + 2P^2(x)/W^2,$$
$$P^2(x) = p^2/m^2 + \varepsilon^2(1 + x),\qquad W = \sqrt{1 - \varepsilon^2(1 + x)}.$$

The non-pole contributions $A^{(\pm)}(\nu, t)$, $[B^{(\pm)}(\nu, t) - B_p^{(\pm)}(\nu, t)]$ to the amplitude (4) can be expanded in powers of ν^2, since in fact the expansion is in terms of quantities $\lesssim \epsilon\nu^2$ (cf. [7])



and goes over into an expansion in terms of ϵx after integrating over $d\Omega_1$ in (5). Hence, up to terms of order ϵx the absorptive part (5) depends only on the invariant pion-nucleon amplitudes for $\nu = 0$:

$$\alpha(x) = mA^{(+)}(0, t)/g_r^2,$$

$$\beta_1(x) = \mu^2[B^{(-)}(0, t) - B_p^{(-)}(0, t)]/g_r^2,$$

$$\beta_2(x) = \mu^2[A^{(-)}(\nu, t)/\nu g_r^2]_{\nu=0},$$

$$\gamma(x) = \mu^3[(B^{(+)}(\nu, t) - B_p^{(+)}(\nu, t))/\nu g_r^2]_{\nu=0},$$

$$g_r^2 = 4\pi g^2 = 4\pi\cdot 14.5. \tag{6}$$

It is convenient to replace $\beta_2(t)$ by the function

$$\beta(x) = -\beta_1(x) - \beta_2(x) = -\mu^2[F^{(-)}(\nu, t)/\nu g_r^2]_{\nu=0}, \tag{7}$$

which is expressed through the helicity amplitude $F^{(-)}(\nu, t)$ [7] and occurs in the absorptive part of the electric form-factor of the nucleon [8].

The calculations lead to the following expression for the operator in the nucleon spin and isospin spaces, corresponding to the absorptive part (5) [with an accuracy $\sim \epsilon x$, $\epsilon(1 + 2x)/2\sqrt{x}$]:

$$A(E, t) = \frac{3g^4}{8mE}\sqrt{\frac{t - 4\mu^2}{t}}\{A^S + im^{-2}\,\mathrm{S}[\mathrm{p'p}]\,A^{LS} + [(\sigma^{(1)}\mathrm{l})(\sigma^{(2)}\mathrm{l}) - \sigma^{(1)}\sigma^{(2)} - 2]\,A^T + [1 + (\sigma^{(1)}\mathrm{n})(\sigma^{(2)}\mathrm{n})]\,A^{LL}\};$$

[2)]The calculations in [7] have been carried through only in the particular case $Q^{(l)}(x) = x^l$, but the formulae obtained are in effect valid for arbitrary $Q^l(x)$.[8,9]

[3)]In distinction from the expansion in terms of ϵx, this other expansion is not a matter of principle; however, retaining corrections of the order $\epsilon(1 + 2x)/2\sqrt{x}$ would go beyond the limit of precision and would lead to unnecessary complication of the formulae.

$$A^S = |\alpha(x) - 1|^2 + (\mathrm{Re}\,\alpha(x) - 1)\frac{\pi\varepsilon(1+2x)}{2\sqrt{x}} + 2\varepsilon^2(1+x)$$
$$+ \frac{2}{3}\lambda_\tau\left\{\varepsilon^2\frac{(1+2x)^2}{4x} - \frac{x}{3\varepsilon^2}|\beta(x)|^2 + (1+2x)\,\mathrm{Re}\,\beta(x)\right\}$$
$$+ \frac{\pi\varepsilon(3-2\lambda_\tau)}{24E\sqrt{x}v},$$

$$A^{LS} = -\frac{1}{2}|\alpha(x) - 1|^2 - \left(\mathrm{Re}\,\alpha(x) + \frac{3-2\lambda_\tau}{6EP^2(x)}\right)\frac{\pi\varepsilon(1+2x)}{4\sqrt{x}}$$
$$+ \frac{2}{3}\lambda_\tau\left\{\frac{x}{6\varepsilon^2}(|\beta(x)|^2 + 4\,\mathrm{Re}\,\beta^*(x)\beta_1(x))\right.$$
$$\left. + \frac{\pi\sqrt{x}}{2\varepsilon}\mathrm{Re}\,\beta(x) - (1+2x)\,\mathrm{Re}\,\beta_1(x)\right\}$$
$$+ \frac{\pi(1+2x)\sqrt{x}}{4}\mathrm{Re}\,\gamma(x) + \frac{\pi\varepsilon(1+2x)^2(3-2\lambda_\tau)}{24EP^2(x)\sqrt{x}v},$$

$$A^T = \varepsilon^2(1+x)\left\{1 - \frac{\pi\varepsilon(1+2x)(3-2\lambda_\tau)}{24EP^2(x)\sqrt{x}}\right.$$
$$+ \frac{2}{3}\lambda_\tau\left(\frac{x}{3\varepsilon^2}|\beta_1(x)|^2 + \frac{\pi\sqrt{x}}{2\varepsilon}\mathrm{Re}\,\beta_1(x)\right)$$
$$\left. - \frac{x^2}{15\varepsilon^2}|\gamma(x)|^2 - \frac{2x}{3\varepsilon}\mathrm{Re}\,\gamma(x) + \frac{\pi\varepsilon(3-2\lambda_\tau)}{24EP^2(x)\sqrt{x}}v\right\},$$

$$A^{LL} = \frac{\pi\varepsilon^3(1+x)(3-2\lambda_\tau)}{24EP^2(x)\sqrt{x}}\left\{-2(1+2x) + v + \frac{(1+2x)^2}{v}\right\}, \quad (8)^*$$

where $\mathbf{S} = (\sigma^{(1)} + \sigma^{(2)})/2$, $\sigma^{(1)}$ and $\sigma^{(2)}$ are Pauli matrices in the spin-space of the nucleons, $\mathbf{1}$ and $\mathbf{n}$ are unit vectors along $\mathbf{q} = \mathbf{p}' - \mathbf{p}$ and $\mathbf{p}' \times \mathbf{p}$, respectively, λ_T is the eigenvalue of the operator $\tau^{(1)}\tau^{(2)}$ (1 or -3 for isospins T = 1 or 0, respectively) and the function $v = \sqrt{1 - 4xp^2/\mu^2}$ has a singularity for $t = 4\mu^2 + \mu^4/p^2$ and contributes to the imaginary part of the amplitudes.

The pion-nucleon amplitude obtained in [7] leads to the following expressions for the functions (6) and (7):

$$\alpha(x) = 1.15 + 0.2x + i\lambda_S(x)\left\{0.15 + 0.2\,x\right.$$
$$\left. + \frac{\varepsilon(1+2x)}{2\sqrt{-x}}\ln\sqrt{-x} + \frac{L^{(S)}_{n+1}(x)}{Q^{(S)}(x)\sqrt{-x}}\right\}, \quad (9)$$

$$\beta_1(x) = 0.083 + i\lambda_P(x)\left\{0.083\right.$$
$$\left. + \frac{3}{2}\varepsilon\left[\frac{\ln\sqrt{-x}}{\sqrt{-x}} - \frac{\varepsilon(1+2x)}{x} + \frac{M^{(P)}_{m+1}(x)}{Q^P(x)\sqrt{-x}}\right]\right\},$$

$$\beta(x) = -\frac{3}{2}i\varepsilon^2\lambda_P(x)\left\{\frac{1+2x}{x} + \frac{L^{(P)}_{m+1}(x)}{Q^{(P)}(x)\sqrt{-x}}\right\}, \quad (10)$$

$$\gamma(x) = 5i\varepsilon\lambda_D(x)\left\{\frac{1}{x} + \frac{L^{(D)}_{k+1}(x)}{Q^{(D)}(x)\sqrt{-x}}\right\}. \quad (11)$$

The polynomials $L^{(S)}_{n+1}(x)$, $M^{(P)}_{m+1}(x)$, $L^{(P)}_{m+1}(x)$, $L^{(D)}_{k+1}(x)$ (with degrees n + 1, m + 1, and k + 1) in Eqs. (9)—(11) are obtained from the vanishing of the curly brackets at the point $x = -1$ and at the n, m, and k poles of the pion-pion S-, P-, and D-amplitudes (3), respectively, and also the vanishing of the first derivatives of the curly brackets at the point $x = -1$ (cf. also [7]). The branch $\mathrm{Re}\sqrt{-x} \geq 0$ has been chosen for the root $\sqrt{-x}$, and in analogy to Eq. (3) for the pion-pion amplitude, Eqs. (9)—(11) contain the value $\sqrt{-x} = -i\sqrt{x}$ on the upper side of the branch cut $x \geq 0$.

Let us discuss briefly the general properties of the absorptive part (8), without considering, for the time being, the contributions due to the pion-pion interaction. Due to the presence of the combination $\alpha(x) - 1 \sim \epsilon(1+x)$ the scalar part $A^S \sim \epsilon^2(1+x)^2$. Hence the already mentioned [2,3] strong compensation of the contributions from pole terms and non-pole terms in the pion-nucleon amplitude (rescattering corrections) is violated for $t > 4\mu^2$, owing to the factor $\sim(1+x)^2$. For $x \sim 1$ the absorptive part A^S acquires an additional factor of ~ 4 as compared to the region $x \approx 0$, hence for the case of weak peripheralism the asymptotic formulae underestimate the contributions of the central forces by a factor of several units.

The spin-orbital part is of the form $A^{LS} \sim \epsilon^2(1+x)$, whereas the contribution $|\alpha(x) - 1|^2$ constitutes an insignificant correction $\lesssim \epsilon x$ to the principal terms. The asymptotic formulae underestimate the contribution of the spin-orbit forces by a factor of two or three.

For isospin T = 0, due to a compensation within the first two terms (arising from the fourth-order diagram), the tensor part A^T becomes sensitive to whether terms of the order of x which have been neglected in the asymptotic formulae, are taken into account or not. This leads to an increase by almost one order of magnitude of the corresponding contributions to the NN-phase shifts.

The absorptive part A^{LL} which corresponds to a potential of the form $(\sigma^{(1)}\mathbf{L})(\sigma^{(2)}\mathbf{L})$, contains only the contribution from the fourth-order diagram and is computed to a better relative accuracy compared to A^S, i.e., corrections of the order $\lesssim [\epsilon(1+2x)/2\sqrt{x}]^3$, p^4/m^4 have been omitted.

The anomalously large terms in A^T and A^{LS}, which are proportional to $1/p^2(x)$ are also due to the fourth order diagram and have been computed to the same degree of higher relative accuracy. In the construction of the equivalent two-meson potential [10] all these terms cancel out with the second iteration of the one-meson potential. This leads, in particular, to the disappearance from the

*[p′p] = p′ × p.

potential of forces of the type $(\sigma^{(1)} L)(\sigma^{(2)} L)$, which are contained only in corrections of the orders ϵ^2, p^2/m^2 to the other types of forces.

The taking into account of the pion-pion interaction by means of models in which the polynomials $Q^{(l)}(x)$ have no zeros in the physical region (e.g., scattering length models, or Breit-Wigner type resonance models) brings about only an insignificant change of the results. The absorptive part remains a relatively smooth function of the momentum transfer t. Thus, for instance, a scattering length model for the S-wave pion-pion amplitude with a positive scattering length a leads to a more rapid decrease of $A^S(x)$, i.e., displaces the effective integration region for the contribution of the central forces towards lower momentum transfers, but only slightly modifies the integral contribution to the NN-scattering phase shifts. A Breit-Wigner type resonance model for the pion-pion interaction in the P-state leads to the appearance of an inessential oscillation of the absorptive part, the integrated contribution of which disappears in the limit of an infinitely narrow resonance.

Essentially different results are obtained when one uses models for the pion-pion interaction, in which the polynomials $Q^{(l)}(x)$ and $X^{(l)}(x)$ have near-lying zeros in the physical region (the case of a resonance with a nearby zero of the pion-pion amplitude). The simplest model of this kind is the model of a kinematic resonance:

$$X^{(l)}(x) = (x_r - x)(x_0 + x),$$
$$Q^{(l)}(x) = a(x_0 - x),\ x_0 > x_r,\ m = 1 \qquad (12)$$

which has been treated earlier [8,9]. The case of a narrow resonance in the P-wave amplitude of the pion-pion scattering leads to large additive delta-function type contributions to A(E, t). In the limit of an infinitely narrow resonance $a \to 0$, $x_0 \to x_r = 6$, $a/(x_0 - x_r) \to 0.2$ (the effect of the ρ-meson), which allowed one to obtain agreement with the experimental data for the electromagnetic form factors of the nucleon [8], the contributions to A(E,t) are of the form:

$$A_\rho^S = -4\pi\lambda_\tau\delta(x-6), \qquad A_\rho^{LS} = -20\pi\lambda_\tau\delta(x-6),$$
$$A_\rho^T = 5.5\pi\lambda_\tau\delta(x-6). \qquad (13)$$

The contribution of the pion-pion interaction to the forces of the form $(\sigma^{(1)} L)(\sigma^{(2)} L)$ contains terms which are smaller by $\sim \epsilon^2$, p^2/m^2 than the other types of forces and therefore has not been taken into account here. The contribution of the pion-pion interaction to the D-state can become essential in A^{LS} and A^T only if the D-amplitude has a kinematical resonance for $t_r < 4m\mu$.

3. THE SCATTERING PHASE SHIFTS

The following quantities are conveniently used as partial NN-amplitudes:

$$\eta_l = (S_l - 1)/2i \qquad \text{for the singlet,}$$
$$\eta_l^J = (S_{l,l} - 1)/2i \qquad \xi_J = S^J_{J-1,J+1}/2i, \text{ for the triplet,} \qquad (14)$$

where S_l, $S^J_{l,l'}$ is the S-matrix in the J-representation. For $|\eta_l|$, $|\eta_l^J|$, $|\xi_J| \ll 1$ these amplitudes coincide up to second order corrections with the nuclear bar phase shifts $\bar\delta_l$; $\bar\delta_{l,J}$; $\bar\epsilon_J$ introduced in [11]. The transition from the operator (8) to the phase shifts (14) is carried out by means of an elementary technique [3,12] and leads to the following expressions for the singlet phase shift η_l and the mixing parameter ξ_J, respectively

$$\eta_l = \frac{3g^4\varepsilon\mu}{4\pi pE}\int_{4\mu^2}^{\infty}\sqrt{\frac{t-4\mu^2}{t}}\,A^S Q_l\frac{dt}{4\mu^2}, \qquad (15)$$

$$\xi_J = \frac{3g^4\varepsilon\mu\sqrt{J(J+1)}}{2\pi pE(2J+1)}\int_{4\mu^2}^{\infty}\sqrt{\frac{t-4\mu^2}{t}}\Big\{A^T Q_J + \frac{p^2A^{LL}/m^2 - P^2(x)A^T}{\varepsilon\sqrt{1+xP(x)J(J+1)}}Q_J^{(1)}\Big\}\frac{dt}{4\mu^2}, \qquad (16)$$

where $Q_l \equiv Q_l(z)$, $Q_l^{(1)} \equiv Q_l^{(1)}(z)$ are Legendre functions of the second kind of the argument $z = 1 + t/2p^2$. The triplet phase shifts can be written in the form

$$\eta_l^{l-1} = \eta_l + 2\eta_l^{LL} - 3\eta_l^T - (l+1)\varphi_l^{LS} - \sqrt{\frac{l-1}{l}}\,\xi_{l-1} + \eta_{l-1}^T,$$

$$\eta_l^l = \eta_l + \frac{\eta_l^T - 2\eta_l^{LL}}{2l(l+1)} - 2\eta_l^T - \varphi_l^{LS} + \frac{2l-1}{2(l+1)}\Big(\sqrt{\frac{l-1}{l}}\,\xi_{l-1} - \eta_{l-1}^T\Big) + \frac{2l+3}{2l}\Big(\sqrt{\frac{l+2}{l+1}}\,\xi_{l+1} - \eta_{l+1}^T\Big), \qquad (17)$$

$$\eta_l^{l+1} = \eta_l + 2\eta_l^{LL} - 3\eta_l^T + l\varphi_l^{LS} - \sqrt{\frac{l+2}{l+1}}\,\xi_{l+1} + \eta_{l+1}^T,$$

where

$$\varphi_l^{LS} = \frac{3g^4\varepsilon^2}{2\pi pEl(l+1)}\int_{4\mu^2}^{\infty}\sqrt{t-4\mu^2}\,P(x)\,Q_l^{(1)}A^{LS}\frac{dt}{4\mu^2} \qquad (18)$$

is the contribution of the spin orbit interaction and the quantities η_l^{LL} and η_l^T are obtained from the integral (15) by means of the substitutions $A^S \to A^{LL}$ or A^T, respectively. Strictly speaking, the expressions (15), (16), and (17) have to be multiplied by a projecting function with values 0 or 1, so that only the two-nucleon states which are effectively realized remain.

The connection between the scattering operators, and the phase shifts defined by Eqs. (15), (16), and (17) is of a general nature. In an expansion in terms of the number of mesons we must consecutively consider the domains of integration $4\mu^2 \leq t \leq 9\mu^2$, $9\mu^2 \leq t \leq 16\mu^2$ etc. In the two-meson approximation it makes sense to calculate only the contribution of the first interval. However, taking into account that the expressions for the two-meson absorptive part obtained in the present paper can be used [4] for $t \lesssim 16\mu^2$, we shall integrate, in the computation of the two-meson phase shifts, up to $t_{max} = 16\mu^2$. This integration is also justified by the fact, that the three-meson absorptive part gives a fundamental contribution in the case of weak peripheralism, probably only beyond $t > 16\mu^2$.

For large orbital momenta $l \gg 1$ the integrals in (15), (16), and (18) can be calculated by means of the method of steepest descent, which leads to asymptotic formulae, which have been obtained earlier [2,3]. In place of the constants $\alpha = 1.2$, $\beta = 0.004$, $\beta_1 = 0.025$ and $\beta_2 = -0.029$ the formulae will contain the quantities [5]

$$\alpha(0) = 1{,}15 - L_{n+1}^{(S)}(0)/X^{(S)}(0),$$

$$\beta(0) = 3\varepsilon^2 L_{m+1}^{(P)}(0)/2X^{(P)}(0),$$

$$\beta_1(0) = 0{,}083 - 3\varepsilon M_{m+1}^{(P)}(0)/2X^{(P)}(0),$$

$$\beta_2(0) = -\beta(0) - \beta_1(0), \tag{19}$$

which depend on the S- and P-scattering lengths for pion-pion scattering. Note that the substitution $\lambda_P = 0$, $X^{(P)}(0) = \infty$ in Eq. (19) does not lead, within the errors, to the numerical results obtained earlier [2] for the quantities β, β_1 and β_2. This discrepancy is due to the presence of subtractions in the dispersion integrals for the pion-nucleon amplitudes $A^{(-)}$ and $B^{(-)}$, which were used in the more recent calculations [7]. The modification of the result due to the introduction of subtractions is an indication either of an essential contribution of the high energies (beyond the region of the 33-resonance), or of uncertainties in the pion-nucleon scattering lengths which have been used for the computation of the subtraction terms and also of the errors indicated for these lengths. This modification leads approximately to a doubling of the contribution of the tensor forces for isospin T = 1 and turns out to be negligible for the other types of forces.

4. NUMERICAL RESULTS AND COMPARISON WITH EXPERIMENTS

For numerical computations it is necessary to substitute into the obtained expressions the S-, P-, and D-amplitudes for pion-pion scattering. At present there are no reliable data available on the pion-pion interaction and therefore we use the simplest models with a minimal number of parameters. This gives a possibility of at least qualitatively investigating the dependence of the NN-amplitude on the pion-pion interaction.

Besides, it is meaningful to separate the contributions of the smooth components of the absorptive part A(E, t) which correspond to the non-resonant pion-pion interaction (or to the absence of pion-pion interaction) from the delta-like components which correspond to possible kinematic pion-pion resonances. The effect of the non-resonant pion-pion P- and D-amplitudes can be neglected, since the functions (10) and (11), which depend on these amplitudes do not contribute essentially to the absorptive part (8). For the pion-pion S-wave phase shift we shall make use of a scattering length (effective range) approximation $X^{(S)}(x)/Q^{(S)}(x) = 1/a$. In the case of a negative $\underline{a}$ the amplitude (3) has one pole in the point $x_1 = 1/a^2$ (n = 1) and for positive $\underline{a}$ there is no pole (n = 0).

Table I lists the NN phase shifts (14) in degrees, obtained through numerical integration of the smooth part of Re A(E, t) for a = 1, $\lambda_P = \lambda_D = 0$, between $4\mu^2 \leq t \leq 16\mu^2$. An extension of the domain of integration up to $t_{max} = 4m\mu$ would yield a maximum variation in the case of the D- phase shifts at an energy E = 300 MeV. In place of the values given in Table I we would obtain the values $^1D_2 = 14°$, $^3D_1 = 2.5°$, $^3D_2 = 18°$, $^3D_3 = 24°$, $\xi_2 = 3.2°$. Going over to lower energies and higher phase shifts the convergence of the integrals improves. The convergence of the contributions from the scalar absorptive part A^S also depends essentially on the scattering length. Thus, for a = 0 we would obtain $^1D_2 = 9°$ for $t_{max} = 16\mu^2$ and 18° for

[4] For $t > 16\mu^2$ the two-meson absorptive part can be modified essentially by taking into account the four-meson contribution to the NN-amplitude and the corresponding modification of the functions (6) and (7). We stress the fact that this would lead to additional corrections, not connected with the expansion in terms of ϵx.

[5] Equation (9) yields for $\alpha(x)$ a logarithmically divergent expression (as $x \to 0$) due to the expansion in terms of $\epsilon(1 + 2x)/2\sqrt{x}$, therefore $\alpha(0)$ has been derived from the more exact expression given in [7].

Table I

E, MeV	40	100	200	300	E, MeV	40	100	200	300
	$T=1$					$T=0$			
$^1D_2$	0.6	2.7	6.7	10	$^1F_3$	0.05	0.4	1.3	2.4
$^1G_4$	0.006	0.1	0.5	1.0	$^1H_5$	0.0006	0.015	0.1	0.3
ξ_2	0.04	0.3	1.0	1.8	$^3D_1$	−0.5	−3.6	−5.3	0.1
$^3F_2$	0.03	0.15	0.4	0.6	$^3D_2$	1.2	4.5	8.3	12
$^3F_3$	0.05	0.4	1.4	2.5	$^3D_3$	0.9	4.4	11	20
$^3F_4$	0.05	0.4	1.5	2.6	ξ_3	−0.03	−0.2	−0.5	−0.7
ξ_4	0.0003	0.007	0.05	0.12	$^3G_3$	0.004	−0.04	−0.15	−0.08
$^3H_4$	0.002	0.01	0.06	0.12	$^3G_4$	0.01	0.13	0.59	1.15
					$^3G_5$	0.008	0.14	0.70	1.5

$t_{max} = 4m\mu$. For a = 2, one obtains for $^1D_2$ the values 8° for $t_{max} = 16\mu^2$ and 10° for $t_{max} = 4m\mu$. For negative a there is no sense in calculating the D-phase shifts, owing to the slow convergence of the integrals. Note however, that the exact absorptive part should lead to a better convergence, as compared with the first order term of the expansion of A(E, t) in powers of x, considered in the present work.

In Table II are listed the supplementary contributions to the NN phase shifts, due to the delta-like absorptive parts (13), corresponding to a kinematic resonance in the P- amplitude for pion-pion scattering (the effect of a ρ-meson with mass $(t_r)^{1/2} \approx 750$ MeV). The results obtained for this case have an essentially qualitative character, since the position of the resonance under consideration is located at the frontier of the convergence domain of the expansion in terms of $t/4m\mu$ utilized here.

Adding the one-meson [12] and two-meson phase shifts (from Tables I and II) we are led to results which are seriously at variance with the experimental data [13]. This disagreement is produced mainly by the presence of the strong central forces $A_\rho^S \sim -\lambda_T\delta(x-6)$, which lead to negative contributions to the phase shifts for isopin T = 1 (repulsion) and to large positive contributions for T = 0 (strong attraction). Besides, the isospin dependence of the type $A_\rho^{LS} \sim \lambda_T$ due to strong spin orbit forces from the ρ-meson is apparently also in contradiction with experiment. We call attention to the fact that the experiments require supplementary contributions to the even triplet phase shifts (T = 0), to the sum "one-meson + nonresonant two-meson phase shifts" (Table I) which have opposite signs as compared to the ones in Table II. The tensor absorptive part A^T yields a supplementary contribution in both isospin states T = 0 and 1, which has the correct sign but is too large in magnitude by a factor of two—four. We think that the mentioned strong disagreement with experiment indicates that the three-meson absorptive part plays an essential role and possibly indicates the necessity of introducing a more complicated model for the pion-pion interaction, for instance, a model with resonances in the S- and (or) D-amplitudes.

5. CONCLUSIONS

1. The asymptotic estimates obtained earlier [2,3] can underestimate by a factor of 5—10 the values of the two meson phase shifts in the case of weak peripheralism, when the peripheral parameter (2) is close to unity, and the absorptive part gives an effective contribution in a sufficiently wide region of momentum transfers $t \approx (9 \text{ to } 16)\mu^2$.

Table II

E, MeV	40	100	200	300	E, MeV	40	100	200	300
	$T=1$					$T=0$			
$^1D_2$	−0.2	−1.5	−5.6	−10	$^1F_3$	0.02	0.3	2.0	5.2
$^1G_4$	0	−0.006	−0.06	−0.2	$^1H_5$	0	0.001	0.03	0.1
ξ_2	0.4	2.7	10	21	$^3D_1$	7	45	160	350
$^3F_2$	−0.05	−1.0	−5.5	−15	$^3D_2$	2.5	14	70	120
$^3F_3$	−0.03	−0.2	−2.6	−6.6	$^3D_3$	2.5	14	50	95
$^3F_4$	−0.02	−0.3	−2.0	−3.3	ξ_3	−0.03	−0.4	−3.2	−8.5
ξ_4	0	0.01	0.1	0.3	$^3G_3$	0.002	0.1	1.5	5.1
$^3H_4$	0	−0.007	−0.08	−0.4	$^3G_4$	0.003	0.1	0.8	2.4
					$^3G_5$	0.001	0.07	0.4	1.3

2. The already indicated [2,3,10] strong compensation of the central two-meson peripheral forces is violated in the region $1/m < r \lesssim 1/\mu$, due to the quick increase with decreasing distance of the factor $\sim(\mu r)^{-4}$, which is equivalent to a violation of the compensation in the absorptive part for $4\mu^2 < t < 4m\mu$ due to the factor $(t/4\mu^2)^2$. The two-meson central forces, computed without considering the resonant pion-pion interaction, agree with experiment and, on the other hand, differ strongly from the central potential obtained on the basis of perturbation theory [14] (the fourth-order diagram [6)]).

3. The calculation of the two-meson spin-orbit and tensor forces without considering the resonant pion-pion interaction leads to an amplification by a factor of several units of the estimates obtained on the basis of the asymptotic formulae [3,10] but does not yield agreement with the phenomenological forces. One can carry out a comparison with experiment, taking into account that the spin orbit forces give the essential contribution to the difference $\eta_l^{l+1} - \eta_l^{l-1} \approx (2l+1)\varphi_l^{LS}$ and the tensor forces—to the mixing parameter ξ_J. In addition to Table I we indicate that the LS forces yield a contribution $3\varphi_1^{LS} = 5, 15, 25$ and $35°$ to the difference $^3P_2 - {}^3P_0$ for E = 40, 100, 200, and 300 MeV, respectively.

In order to obtain agreement with experiment, the two-meson LS forces have to be increased by a factor of two to four, i.e., one needs supplementary positive contributions. For T = 0, on the contrary, the two meson LS forces have to be compensated, i.e., one needs supplementary negative contributions about twice as small as the supplementary terms for T = 1. The tensor two-meson forces have also to be increased several times in both isospin states.

4. The nucleon-nucleon amplitude depends weakly on the non-resonant pion-pion amplitudes. Only the S-amplitude gives a certain effect (isospin T = 0) in the central NN forces, however even in this case the experimental data on NN-scattering do not yield essential information about the parameters of the pion-pion amplitude.

5. Large contributions to the NN amplitude, in particular to the spin-orbit and tensor forces, are given only by the pion-pion amplitudes in the presence of kinematic resonances, when the pion-pion amplitude has a zero in the neighborhood of the resonance. In the simplest case of a kinematic P-resonance at an energy $\sqrt{t_r} \approx 750$ MeV (the effect of the ρ-meson) one cannot obtain agreement with experiment in the two meson approximation both for the electromagnetic form factors of the nucleons [8] and the elastic nucleon-nucleon scattering amplitude.

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[6)]In similar calculations of the spin-orbit forces corresponding to the fourth-order diagram[15] Gupta has committed an error, connected with an incorrect nonrelativistic limit under the Feynman integrals, which is equivalent to identifying the matrix elements of the Dirac matrices γ_0 and 1.

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