POLARIZATION OF INELASTICALLY SCATTERED NUCLEONS

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It is shown that for a certain type of inelastic scattering the neutron polarization can be computed within the framework of the optical model. Polarization in elastic scattering by deformed nuclei is considered.

IN the conventional optical model, [1] the cross section for the interaction of nucleons with nuclei is divided into two parts: the cross section for the optical elastic scattering and the reaction cross section, which is identified with the cross section for the formation of the intermediate nucleus. All inelastic scattering processes are included in the second cross section, i.e., are regarded as going through the compound nucleus. This model allows only for the prediction of the polarization due to the optical elastic scattering. The nucleons scattered through the compound nucleus mechanism are assumed to be unpolarized on the average (as a result of the averaging over the random orientations of the spins of the nucleons emitted by the decaying compound nucleus). In the orthodox optical model, therefore, the inelastically scattered nucleons are unpolarized.

On the other hand, it is known that inelastic processes may occur without formation of an intermediate nucleus. These are the so-called direct processes. Chase, Wilets, and Edmonds [2] have proposed a theory for deformed nuclei which permits the description of a certain type of direct reactions within the framework of the optical model. The type of direct process considered by these authors is the direct excitation of nuclear rotational levels in the scattering of neutrons. If this theory [2] is generalized to include the spin-orbit interaction it permits the computation of the polarization of the inelastically scattered neutrons. Moreover, such a model leads to a considerably improved agreement between theory and experiment for the polarization in elastic scattering in the region of atomic numbers $150 < A < 190$, where the model of a spherical nucleus is inadequate.

The Hamiltonian for our model is written as

$$H = -\frac{\hbar^2}{2m}\nabla^2 + T_{rot} + V(r, \theta') + H_{SO}. \qquad (1)$$

Here T_{rot} is the operator for the rotational motion of the target nucleus with eigenfunctions $D^I_{MK}(\theta_i)$, $(r\theta'\varphi')$ are the coordinates of the neutron relative to the principal axes of the nucleus, and $V(r, \theta')$ is the potential representing the interaction between the neutron and the deformed nucleus.

The Hamiltonian (1) differs from the corresponding Hamiltonian in [2] by the presence of the spin-orbit interaction H_{SO}, which is chosen in the form

$$H_{SO} = \frac{\varkappa}{r}\frac{\partial V(r, \theta')}{\partial r}(\mathbf{ls}). \qquad (2)$$

For simplicity we shall restrict ourselves to the discussion of even-even nuclei. Then the initial spin of the target nucleus is $I = 0$ and $K = 0$. The wave function for the incoming partial wave with momentum j ($\mathbf{j} = \mathbf{l} + \mathbf{s}$) and projection M on the z axis (M coincides with the projection of the spin of the incoming neutron, since $m_l = 0$) in the channel $I = 0$ is

$$\Psi_{jM} = \sum_{l'j'} \frac{1}{r}\psi^{lj0}_{l'j'}(r)\, Y^{jM}_{l'j'}(\theta, \varphi; \theta_i; s_z). \qquad (3)$$

The function $Y^{jM}_{l'j'}$, which gives the dependence of the solution on the angular variables and the spins of the neutron and the target nucleus, is of the form

$$Y^{jM}_{l'j'} = \sum_{m_{I'}} (-)^M C^{m_{I'}\, M-m_{I'}\, M}_{I'\; j'\; j} \times \left(\frac{2I+1}{4\pi}\right)^{1/2} D^I_{M0}(\theta_i)\, \Phi_{l'sj',\, M-m_{I'}}(\theta, \varphi; s_z), \qquad (4)$$

where Φ_{lsj,m_j} is the spin-angle function. [3] We expand the potential $V(r, \theta')$ in a series,

$$V(r, \theta') = \sum_{\lambda=0,\, 2\ldots} v_\lambda(r) P_\lambda(\cos\theta') \qquad (5)$$

and restrict ourselves to the first two terms. [2] Substituting (3) and (5) in the equation $H\Psi_{jM} = E\Psi_{jM}$, we obtain the following system of differential equations for the radial functions $\psi^{lj0}_{l'j'}(r)$:

$$\left\{-\frac{\hbar^2}{2m}\left[\Delta_r - \frac{l'(l'+1)}{r^2}\right] + \frac{\hbar^2}{2J}I'(I'+1) + v_0(r) - E + \frac{\varkappa}{r}\frac{\partial v_0}{\partial r}b_{j'}\right\}\psi^{lj0}_{I'j'}(r) + \sum_{I''j''}\left\{v_2(r) + \frac{\varkappa}{r}\frac{\partial v_2}{\partial r}b_{j''}\right\}a^j(I'j';\ I''j'')\psi^{lj0}_{I''j''}(r) = 0, \quad (6)$$

where the matrix element has the form ($\hat{a} \equiv \sqrt{2a+1}$)

$$a^j(I'j';\ I''j'') = (-)^{j'+j''+j+1/2}\cdot 5^{-1/2}\hat{I}'\hat{j}'\hat{l}'\hat{I}''\hat{j}''\hat{l}''C^{000}_{I'I''2}C^{000}_{l'l''2}W \times (j'l'j''l'';\tfrac{1}{2}2)W(j'I'j''I'';\ j2) \quad (7)$$

and $b_j = j - 1/2$ or $-(j + 3/2)$ depending on whether $j = l + 1/2$ or $j = l - 1/2$.

The asymptotic form of the radial wave functions at large distances is

$$\frac{1}{r}\psi^{lj0}_{0j}(r) = h^{(2)}_l(k_0 r) + \eta^{lj0}_{0j}h^{(1)}_l(k_0 r),$$

$$\frac{1}{r}\psi^{lj0}_{I'j'}(r) = \sqrt{\frac{k_0}{k_{I'}}}\,\eta^{lj0}_{I'j'}h^{(1)}_{l'}(k_{I'}r), \qquad I' \neq 0. \quad (8)$$

It is easily shown that the total scattering amplitude is the sum

$$f(\theta) = \sum_{I'} f_{I'}(\theta), \quad (9)$$

where $f_{I'}(\theta)$ is the amplitude for scattering into the angle θ with excitation of the rotational level I'. For $I' = 0$ we have the amplitude for optical elastic scattering f_0. The matrix element $\langle f_0^*\sigma f_0\rangle$ determines the elastic polarization, as usual. The essential point is, however, that the quantities $\langle f_{I'}^*\sigma f_{I'}\rangle$ do not vanish for $I' \neq 0$ as a consequence of the coherence of the amplitudes $f_{I'}$.

We define the relative polarization of the nucleons scattered inelastically with excitation of the level I' of the target nucleus in the following way:

$$P_{I'}(\theta) = \frac{1}{s}\frac{\langle f_{I'}^*\sigma f_{I'}\rangle}{d\sigma_{I'}/d\Omega}. \quad (10)$$

The differential scattering cross section with excitation of the level I' is

$$d\sigma_{I'}/d\Omega = \langle f_{I'}^* f_{I'}\rangle + d\sigma_{I'}^{\text{comp}}/d\Omega, \quad (11)$$

where the first term on the right corresponds to the optical scattering and the second term to scattering with compound nucleus excitation. Evidently, if $I' = 0$, formula (10) gives the usual optical elastic polarization P_0. Using the usual rules for the addition of angular momenta, it is easy to express (10) and (11) in terms of the coefficients η.

We have carried out computations of the polarization of neutrons with an energy of 380 MeV, using an electronic computer. The potential $V(r, \theta')$ was chosen of the form

$$V(r,\theta') = -V_0\left[\left[1 + \exp\frac{r - R(\theta')}{a}\right]^{-1} + i\zeta\exp\left\{-\left(\frac{r - R(\theta')}{b}\right)^2\right\}\right], \quad (12)$$

where

$$R(\theta') = R_0[1 + \beta Y_{20}(\theta')], \quad R_0 = 1.245A^{1/3}\,\text{F};$$

$$a = 0.65\,\text{F}, \quad b = 1\,\text{F}, \quad \zeta = 0.1.$$

Figure 1 shows the function $P_{I'}(\theta)$ for tungsten. It is seen that, in absolute value, $P_{I'}$ with $I' \neq 0$ is not small compared to P_0. This is explained by the fact that, although the denominator in (10) decreases sharply as I' is increased, the numerator falls off just as rapidly.

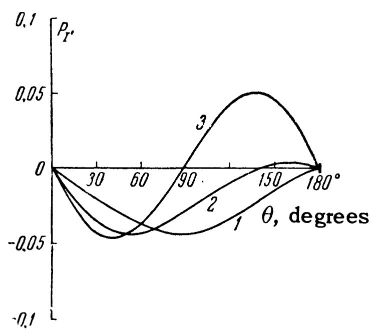

FIG. 1. Function $P_{I'}(\theta)$ for tungsten. 1 – elastic polarization, 2 – polarization in inelastic scattering with excitation of the level I = 2, 3 – the same with excitation of the level I = 4.

Unfortunately, the absence of experimental data on the polarization of inelastically scattered neutrons does not allow us yet to compare theory and experiment. But such a comparison can be made for P_0. The results of the computation of P_0 for tungsten and erbium are shown in Figs. 2 and 3. The dotted line indicates the polarization computed with the model of a spherical nucleus. It is seen that the agreement with the experimental data is considerably improved if the deformation is taken

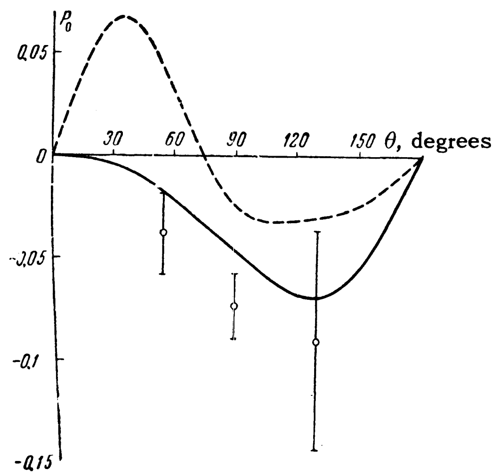

FIG. 2. Polarization in elastic scattering on tungsten.

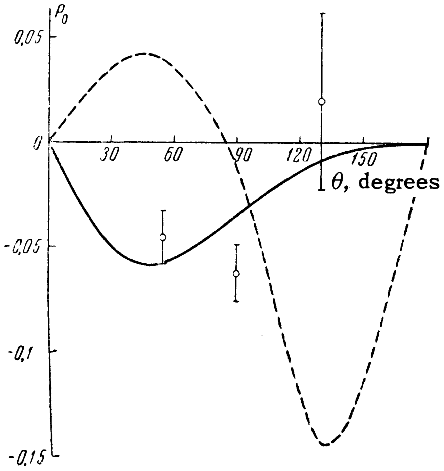

FIG. 3. Polarization in elastic scattering on erbium.

into account.[4] The parameters used are $V_0 = 46.3$ MeV, $\beta = 0.21$ and $3\hbar^2/J = 110$ keV for tungsten and $V_0 = 46.5$ MeV, $\beta = 0.29$ and $3\hbar^2/J = 80$ keV for erbium. On the other hand, there is considerable disagreement with experiment in the case of U^{238}, using the conventional values for the radius and deformation.

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