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ON THE DETECTION OF LOW FREQUENCY GRAVITATIONAL WAVES

M. E. GERTSENSHTEĬN and V. I. PUSTOVOĬT

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It is shown that the sensitivity of the electromechanical experiments for detecting gravitational waves by means of piezocrystals is ten orders of magnitude worse than that estimated by Weber.[1] In the low frequency range it should be possible to detect gravitational waves by the shift of the bands in an optical interferometer. The sensitivity of this method is investigated.

THE problem of the discovery of gravitational waves has recently begun to be discussed in the literature,[1,2] where discussion is given of the electromechanical experiments. However, the nonrelativistic bodies at the disposal of the experimenter interact very weakly with the gravitational wave. Let us consider the equations of motion of a particle in the nonrelativistic approximation in the presence of an external electromagnetic field F^{ik}:[3]

$$mc\left[\frac{du^\alpha}{ds}+\Gamma^\alpha_{ik}u^i u^k\right]=\frac{e}{c}F^\alpha{}_k u^k=\frac{e}{c}F^{\alpha k}u^k+\frac{e}{c}F^{\alpha s}h_{sk}u^k. \quad (1)$$

A plane gravitational wave ($g_{00} = -1$, $g_{0\alpha} = 0$) does not change the proper time of the nonrelativistic ($u^\alpha = 0$) body, $\Gamma^\alpha_{00} = 0$; an uncharged nonrelativistic particle does not sense the wave. If the field F^{ik} is produced by specified nonrelativistic charges and currents which do not change under the action of the passing gravitational wave, then the field F^{ik} likewise does not change as seen directly from the equations of motion[3]

$$F^{ik}{}_{;k}=\partial F^{ik}/\partial x^k=(4\pi/c)\,j^i, \quad (2)$$

since $\sqrt{-g} = 1$ in the gravitational wave. The presence of a gravitational wave leads to the appearance of an additional force on the right hand side of (1), $eF^{\alpha s}h_{sk}u^k$, which vanishes in the nonrelativistic approximation ($k = 0$). Detection of gravitational waves by a nonrelativistic body (as well as by a piezoelectric one) is but weakly effective.

Starting out from the general relations for linear processes[1)] we shall show below that the high sensitivity of detection reported in the work of Weber[1] is too high by 10 orders of magnitude. Linear processes, which dominate in weak fields, determine the limiting sensitivity of the experiments for the discovery of weak gravitational waves. If the equations of an arbitrary linear system are reversed in time, then a relation results connecting the energy losses by radiation and the effective diameter σ in reception:

$$\sigma=\sigma_0 G\eta, \qquad \eta=Q_0/Q_R, \quad (3)$$

where σ_0 is the effective cross section of an ideal lossless antenna, G is the gain resulting from the directivity, η is the efficiency of the antenna in the transmission mode, Q_0 is the quality factor of the real antenna, and Q_r is the quality factor associated with radiation.[1] The cross section is $\sigma_0 \cong \lambda^2$ with accuracy to within a factor of order unity. A relation of the type (3), known in the theory of antennas,[4] can be obtained from the formulas of the paper of Weber,[1] and also follows from the principle of detailed balancing (see [5], Sec. 117). Experiment shows that the results of Weber do not satisfy the relation (3), and the divergence amounts to $\sim 10^{10}$. Thus, for example, according to Weber,[1] for a wave with $\lambda = 100$ cm, the power fed to the crystal in the radiation mode is equal to 10^8 W, the power radiated is 10^{-13} erg/sec, whence $\eta = 10^{-28}$, the radiation is quadrupole and G = 15. In the reception mode for an ideal antenna, $\sigma_0 = 3 \times 10^3$ cm^2; for a real antenna Eq. (3) gives $\sigma = 4 \times 10^{-24}$ cm^2. At threshold, the value of energy flux according to Weber is P $= 10^{-3}$ erg/sec-cm^2, the electromagnetic energy received is equal to 4×10^{-34} W $= -334$ dB/W, which is lower by 110 dB than the threshold power for an ideal receiver with a noise temperature of 3°K and bandwidth of 1 cps. The time required for detection of a signal which is lower than threshold by 110 dB exceeds 10^4 years.

1)It is well known that a rotating rod or a binary star radiates in quadrupole fashion, the frequency of the radiation being twice as great as the frequency of motion existing in the system. Such processes in which double frequencies appear are not linear and are not considered here.

Calculation of the radiated power was carried out by Weber according to the "quadrupole" formula,[3] the validity of which there is no reason to doubt. Therefore, it follows from Eq. (3) that the absorbed power was incorrectly computed by him. The reason for the mistake lies in the fact that in a piezocrystal, the piezoelectric stresses are compensated by mechanisms which were not taken into account by Weber. Piezoelectric stresses do not satisfy the virial theorem.[3] Since the reception of gravitational waves is a relativistic effect, one should expect that the use of an ultrarelativistic body—light—can lead to a more effective indication of the field of the gravitational wave. The optics of rays in a gravitational field is determined by the eikonal equation[3]

$$g^{ik}\frac{\partial\psi}{\partial x^i}\frac{\partial\psi}{\partial x^k} = \left(\frac{\partial\psi}{\partial x^i}\right)^2 - h^{\alpha\beta}\left(\frac{\partial\psi}{\partial x^\alpha}\right)\left(\frac{\partial\psi}{\partial x^\beta}\right) = 0, \tag{4}$$

where ψ is the eikonal. This is equivalent to a medium with index of refraction

$$n = 1 + \tfrac{1}{2}h_{\alpha\beta}n^\alpha n^\beta, \tag{5}$$

where n^α is a unit vector along the propagation of the ray. For propagation of the ray along and perpendicular to the gravitational field, we have

$$n_\parallel = 1, \quad n_\perp = 1 + \tfrac{1}{2}h_{22}\cos 2\varphi + \tfrac{1}{2}h_{23}\sin 2\varphi,$$
$$\cos\varphi = n_2. \tag{6}$$

In apparatus such as the Michelson interferometer, the relative difference of the optical lengths of the light rays traveling along and perpendicular to the gravitational wave will be

$$\Delta l/l_0 = \tfrac{1}{2}h_{\alpha\beta}n^\alpha n^\beta, \tag{7}$$

where l_0 is the unperturbed length of the interferometer arm. We note that Eq. (7) for the Michelson interferometer can be obtained directly. In a gravitational field, the optical length of the interferometer arm is changed and the relative difference ([3], Sec. 84) is equal to

$$\frac{\Delta l}{l_0} = \frac{1}{l_0}\int_0^{l_0}\sqrt{g_{22}}\,dx_2 - \frac{1}{l_0}\int_0^{l_0}\sqrt{g_{11}}\,dx_1 \cong \frac{1}{2}h_{22}. \tag{7a}$$

In the derivation of Eq. (7), it was assumed that the period of the gravitational wave is much greater than the time of flight of the ray in the interferometer.

Thus the gravitational wave produces a periodic displacement of the interference bands. We express (7) in terms of the radiation energy flux P, first rotating the Ox_2 and Ox_3 axes so as to eliminate the component h_{23}. In this system only $h_{22} = -h_{33} = h$ is different from zero, and therefore the flux of gravitational energy is equal to

$$P = \omega^2c^3h^2/16\pi k = \omega^2ch^2/2\varkappa; \quad \varkappa = 8\pi k/c^2, \tag{8}$$

where $\varkappa$ is the Einstein gravitational constant. Making use of (7), we have

$$\frac{\Delta l}{l_0} = \frac{1}{2\omega}\sqrt{\frac{2\varkappa P}{c}} = \frac{8.1\cdot 10^{-20}}{f\,(\text{cps})}\sqrt{P\ \text{erg/cm}^2\cdot\text{sec}}. \tag{9}$$

The minimum measured Δl with ordinary light sources[6,7] amounts to 10^{-3} Å, which is equal to 10^{-11} cm, for a time constant of the apparatus $\tau \sim 1$ sec. It can be expected that application of strong sources and amplifiers of monochromatic directed light radiation (lasers[8]) will make it possible to decrease this factor by at least three orders of magnitude.

Assuming the interferometer arm to be $l^0 \approx 10^3$ cm, we have for the minimum observable change $\Delta l/l_0 \approx 10^{-14}$—$10^{-17}$; $\tau \sim 1$ sec.

Thus the interferometer makes it possible, at least in principle, to detect the very weak gravitational waves. For $f_0 = 10^{-3}$ cps and $P = 1$ erg/cm²-sec we get $\Delta l/l \approx 8\times 10^{-14}$, which is approximately 10^7—10^{10} times better than the possibilities of electromechanical experiments.[1]

Further gain in the sensitivity can be obtained by increase in the time of observation and by known methods for the separation of a weak signal from the noise background. Evidently, times of observation $\tau \sim 10^4$—10^5 sec are realistic; in this case $P_{min} \sim 10^{-4}$ erg/cm²-sec. Bernstein,[6,7] detected a monochromatic sinusoidal signal against the noise. In the case of a useful signal with a complex spectrum, these estimates must be changed somewhat but we shall not dwell on this subject since these changes are the same for both the interferometer and the electromechanical experiments.[1] Technical experiments with an interferometer on the discovery of gravitational waves of low frequencies existing in nature are very complicated. It is necessary to have stable apparatus, and the air must be pumped out along all the optical paths. Inasmuch as neither the frequency nor the polarization nor the direction of propagation of the wave are known, it is necessary to have several interferometers and to study the correlation among them.[1]

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