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DISPERSION RELATIONS FOR SCATTERING OF γ QUANTA ON NUCLEI

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The scattering of low energy photons on nuclei is considered. Dispersion relations for the forward scattering amplitude have been obtained taking into account only the nucleus + photon and nucleus + electron-pair intermediate states (the other states contribute corrections of higher order in the electric charge e). The contributions from nuclear excited states have been included. They were represented (by assumption) by poles on the second Riemann sheet of the elastic scattering amplitude. In first approximation in the final form of the dispersion relations only those terms of higher order in e were kept which represent the contributions from the resonances.

1. The application of dispersion relations (DR) to the analysis of the scattering of photons on nuclei is a problem which has received relatively little attention.

The first concrete results in applying DR to the scattering of light by nuclei have been obtained by Gell-Mann, Goldberger, and Thirring.[1] In that paper the forward scattering amplitude was investigated and the DR were proven within the framework of perturbation theory up to order e^2.

The DR were later used mainly for the Compton scattering on nucleons, while a rigorous proof of the DR for scattering angles $\theta \neq 0$ was also given only in the e^2 approximation.[2] The DR were applied to the elastic scattering of photons on nuclei in the dipole approximation by Fuller and Hayward[3] and by Penfold and Garvin[4], and to the singularities associated with channel thresholds by Lapidus and Chou Kuang-chao.[5] The latter authors suggested that the first maximum in the cross section $\sigma(\gamma, \gamma)$ is wholly due to dispersive threshold effects.[5] However, as shown by Kalinkin,[6] this maximum can also be due to nuclear resonance fluorescence, i.e., the scattering of photons on separate one-nucleon levels of the nucleus.

For a description of the nuclear resonance fluorescence by means of the radiation damping theory one has to take into account terms of higher order in e. One thus sees that DR limited to the consideration of the amplitudes in the e^2 approximation do not allow us to take into account the contribution from resonances that are due to unstable intermediate nuclear states. A general description of the process of nuclear photon scattering within the framework of DR that take resonances into account seems to be a rather involved problem. The aim of the present paper is to formulate the problem of, and indicate the possibility of, the treatment of the contributions of intermediate unstable nuclear states in the DR for the scattering angle $\theta = 0°$ and for energies below the threshold of the photonuclear reactions.

2. The retarded amplitude for photon scattering on a nucleus can be written in the form

$$T_{\alpha\omega}^{ret}(q', q) = 2\pi^2 i \int d^4x e^{i(q'+q)x/2} \times \left\langle \mathbf{p}', s' \left| \theta(x^0)\left[e^{\nu'}\cdot j\left(\frac{x}{2}\right), e^{\nu}\cdot j\left(-\frac{x}{2}\right)\right]\right| \mathbf{p}, s \right\rangle, \quad (1)$$

where $\alpha \equiv (\mathbf{p}, s, \nu)$, $\omega \equiv (\mathbf{p}', s', \nu')$; $p(p')$ is the 4-momentum of the nucleus before (after) the scattering, $q(q')$ is the photon 4-momentum before (after) the scattering, $s(s')$ indicates the discrete initial (final) nuclear states and $\nu(\nu')$ indicates the initial (final) photon polarization.

Analogously we have for the advanced amplitude

$$T_{\alpha\omega}^{adv}(q', q) = -2\pi^2 i \int d^4x e^{i(q'+q)x/2} \times \left\langle \mathbf{p}', s' \left| \theta(-x^0)\left[e^{\nu'}\cdot j\left(\frac{x}{2}\right), e^{\nu}\cdot j\left(-\frac{x}{2}\right)\right]\right| \mathbf{p}, s \right\rangle. \quad (2)$$

Because of a reason which will be given below we assume in the following that the photon has a certain small but finite rest mass μ. However, we do not consider the complications connected with the finite rest mass of the photon in the proof of the DR, for ultimately we go to the limit $\mu = 0$.[1)]

1)The infrared catastrophe does not appear here since we consider only the DR for forward scattering.

In the Breit coordinate system $(\mathbf{p}' + \mathbf{p} = 0)$ we have

$$T_{\alpha\omega}^{ret}(E, \lambda\mathbf{n}) = 2\pi^2 i \int d^4 x e^{i\,(Ex^0 - \lambda \mathbf{n}\mathbf{x})}$$

$$\times \left\langle \mathbf{p}', s' \left| \theta(x^0)\left[e^{\nu'}\cdot j\left(\tfrac{x}{2}\right), e^{\nu}\cdot j\left(-\tfrac{x}{2}\right)\right]\right| \mathbf{p}, s\right\rangle,$$

$$T_{\alpha\omega}^{adv}(E, \lambda\mathbf{n}) = -2\pi^2 i \int d^4 x e^{i\,(Ex^0 - \lambda \mathbf{n}\mathbf{x})}$$

$$\times \left\langle \mathbf{p}', s' \left| \theta(-x^0)\left[e^{\nu'}\cdot j\left(\tfrac{x}{2}\right), \quad e^{\nu}\cdot j\left(-\tfrac{x}{2}\right)\right]\right| \mathbf{p}, s\right\rangle. \tag{3}$$

Here

$$E = q^0 = q'^0, \quad \mathbf{q} = -\mathbf{p} + \lambda\mathbf{n}, \quad \mathbf{q}' = \mathbf{p} + \lambda\mathbf{n}, \quad \mathbf{n}^2 = 1,$$
$$\mathbf{p}\mathbf{n} = 0, \quad \lambda^2 = E^2 - \mathbf{p}^2 - \mu^2.$$

The analytic continuation of the retarded (advanced) forward $(\mathbf{p} = 0)$ scattering amplitude for a "physical" photon $(\mu = 0)$ into the upper (lower) half-plane of the complex variable E does not present any difficulties. To this end one can utilize the above given integral representations. Therefore the function defined in the usual way[7]

$$S\widetilde{T}_{\alpha\omega}(E, \lambda\mathbf{n}) = \begin{cases} ST_{\alpha\omega}^{ret}(E, \lambda\mathbf{n}), & \operatorname{Im} E > 0 \\ ST_{\alpha\omega}^{adv}(E, \lambda\mathbf{n}), & \operatorname{Im} E < 0 \end{cases} \tag{4}$$

will be regular on the whole E plane with the exception of those points on the real axis where the difference

$$ST_{\alpha\omega}(E, \lambda\mathbf{n}) = ST_{\alpha\omega}^{ret}(E, \lambda\mathbf{n}) - ST_{\alpha\omega}^{adv}(E, \lambda\mathbf{n})$$

does not vanish. In the following we shall not write out explicitly the dependence of the functions on $\lambda\mathbf{n}$ and on the indices α and ω when we do not need it.

Using the spectral properties and the translation invariance we find that the function $S\widetilde{T}(E)$ has two poles on the real axis at $E = \pm E_p$ and two branch cuts at $-\infty < E \le -E_1$, $E_1 \le E < +\infty$. At $\mathbf{p} = 0$ the expressions for E_p and E_1 have the form

$$E_p = \mu^2/2M, \qquad E_1 = \mu,$$

where M is the mass of the nucleus.

We also note the positions of the branch points $E = \pm E_2$ and $E = \pm E_3$ which correspond to the threshold of the photonuclear reactions and to the electron pair production threshold respectively:

$$E_2 = \Delta E_b + (\Delta E_b^2 - \mu^2)/2M \approx \Delta E_b,$$
$$E_3 = 2m + (4m^2 - \mu^2)/2M \approx 2m,$$

where ΔE_b is the difference between the binding energies of the nuclei in the initial and the final states and m is the electron mass.

We see that only in the case of a photon with a finite rest mass the basic requirements which have to be fulfilled by the spectral function ST(E) can be met if one considers intermediate photon states in the photon scattering, which suppose that the isolated poles do not occur within a continuous spectrum, and that two continuous spectra do not overlap. Then one can use the usual considerations[7] to obtain DR for the forward scattering amplitude.

The contribution connected with the intermediate state pairs, i.e., the Delbrück scattering amplitude can be calculated from perturbation theory[8] and therefore one can obtain the DR for the remaining part of the scattering amplitude:

$$S\widetilde{T}'(E) = S\widetilde{T}(E) - S\widetilde{T}_{\mathrm{Delbr}}(E). \tag{5}$$

Then the branch points $E = \pm E_3$ of the full scattering amplitude at the same time also get eliminated.

The poles of the scattering amplitude which according to assumption correspond to resonances in the cross section $\sigma(\gamma, \gamma)$ below threshold are amongst the singularities which in the DR have to be included by means of analytic continuation of the amplitude unto the nonphysical sheet and deformation of the path of integration in Cauchy's theorem. It has been shown by Oehme[9] and Zimmermann[10] that the scattering amplitude has a two-sheet branch point at zero kinetic energy of the colliding particles, i.e., the branch cut which corresponds to two-particle intermediate states connects two sheets of the Riemann surface of the scattering amplitude. In the indicated papers, the partial amplitudes for scattering of identical scalar particles were investigated. However it was noted that the given arguments could be generalized for the case of scattering of particles with unequal mass and nonzero spin.

If we limit ourselves to the approximation in which at most one photon appears in the intermediate state then the cut $E_1 \le E < E_2$ will connect two sheets of the elastic photon-nucleus scattering amplitude. The indicated approximation implies that we neglect all other channels by which the intermediate unstable state can decay except for the direct decay to the ground state by emission of a γ quantum. Following [9] we determine the analytical continuation of $S\widetilde{T}'(E)$ through the "elastic" cut $E_1 \le E < E_2$ to the second sheet by using the equations

$$S\widetilde{T}'^{\mathrm{I}}(E + i0) = S\widetilde{T}'^{\mathrm{II}}(E - i0),$$

$$S\widetilde{T}'^{\mathrm{I}}(E - i0) = S\widetilde{T}'^{\mathrm{II}}(E + i0), \qquad E_1 \leqslant E \leqslant E_2. \tag{6}$$

The superscripts I and II indicate on which sheet the respective quantities have to be taken. If one assumes that in the lower half-plane of the second sheet $ST'^{\mathrm{II}\,ret}(E)$ has poles of the form

$$ST'^{\,\mathrm{II}\,ret}(E)=\sum_n \frac{u^{(n)}(E)}{E-E_n}+Q(E),$$

where $E_n=\bar{E}_n-i\Gamma_n/2$ and the function $u^{(n)}(E)$ and $Q(E)$ are regular at the point $E=E_n$ then we have for $ST'^{\mathrm{II}\,adv}(E)$ in the upper half-plane

$$ST'^{\,\mathrm{II}\,adv}(E)=\sum_n \frac{u^{(n)+}(E^*)}{E-E_n^*}+Q^+(E^*).$$

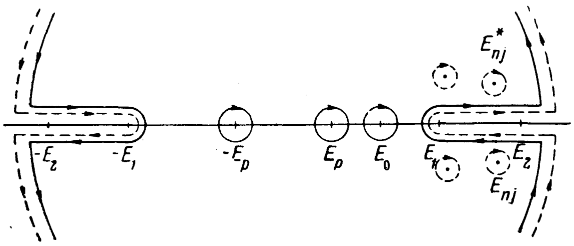
This follows from the fact that the relation

$$T^{ret+}_{\alpha\omega}(E^*)=T^{adv}_{\alpha\omega}(E)$$

is fulfilled in both sheets of the Riemann surface.

Applying Cauchy's integral formula to the function $S\tilde{T}'(E)/(E-E_0)^2$ with the contour indicated in the figure we obtain

$$\frac{S\tilde{T}'^{\mathrm{I}}(E)}{(E-E_0)^2}+\frac{S\tilde{T}'^{\mathrm{II}}(E)}{(E-E_0)^2}=\frac{1}{2\pi i}\left\{\oint_{C_\mathrm{I}} dE'\frac{S\tilde{T}'^{\mathrm{I}}(E')}{(E'-E)(E'-E_0)^2}+\oint_{C_{\mathrm{II}}} dE'\frac{S\tilde{T}'^{\mathrm{II}}(E')}{(E'-E)(E'-E_0)^2}\right\}. \tag{7}$$



The dashed line shows the contour C_{II} on the second sheet of the scattering amplitude.

Then we apply the same formula to the function $\varphi(E)\,S\tilde{T}'(E)/(E-E_0)^2$, where the function $\varphi(E)$ is chosen such that

$$\frac{S\tilde{T}'^{\mathrm{I}}(E)}{(E-E_0)^2}-\frac{S\tilde{T}'^{\mathrm{II}}(E)}{(E-E_0)^2}=\frac{1}{2\pi i}\left\{\oint_{C_\mathrm{I}} dE'\frac{\varphi^{\mathrm{I}}(E')}{\varphi^{\mathrm{I}}(E)}\frac{S\tilde{T}'^{\mathrm{I}}(E')}{(E'-E)(E'-E_0)^2}\right.$$

$$\left.-\oint_{C_{\mathrm{II}}} dE'\frac{\varphi^{\mathrm{I}}(E')}{\varphi^{\mathrm{I}}(E)}\frac{S\tilde{T}'^{\mathrm{II}}(E')}{(E'-E)(E-E_0)^2}\right\}. \tag{8}$$

Adding (7) and (8) we get rid of $ST'^{\mathrm{II}}(E)$:

$$\frac{S\tilde{T}'^{\mathrm{I}}(E)}{(E-E_0)^2}=\frac{1}{2\pi i}\left\{\oint_{C_\mathrm{I}} dE'P_+(E',E)\frac{S\tilde{T}'^{\mathrm{I}}(E')}{(E'-E)(E'-E_0)^2}\right.$$

$$\left.+\oint_{C_{\mathrm{II}}} dE'P_-(E',E)\frac{S\tilde{T}'^{\mathrm{II}}(E')}{(E'-E)(E'-E_0)^2}\right\}, \tag{9}$$

where

$$P_\pm(E',E)=\tfrac{1}{2}\,[1\pm\varphi^{\mathrm{I}}(E')/\varphi^{\mathrm{I}}(E)].$$

For $\varphi(E)$ we take the function $[(E^2-E_1^2)/(E^2-E_2^2)]^{-1/2}$ which is single-valued and regular in the complex E plane with cuts $-E_2\le E\le -E_1$ and $E_1\le E\le E_2$ along the real axis. Letting the radii of the large half circles of the integration contours go to infinity and of the small half-circles to zero we obtain

$$\frac{S\tilde{T}'^{\mathrm{I}}(E)}{(E-E_0)^2}=\frac{1}{\pi}\,\mathrm{P}\left\{\int_{-\infty}^{E_1}+\int_{E_2}^{\infty}\right\}dE'\frac{SA'^{\mathrm{I}}(E')}{(E'-E)(E-E_0)^2}$$

$$+\frac{1}{\pi}\,\mathrm{P}\left\{\int_{-\infty}^{E_1}+\int_{E_2}^{\infty}\right\}dE'P_-(E',E)\frac{SA'^{\mathrm{II}}(E')-SA'^{\mathrm{I}}(E')}{(E'-E)(E'-E_0)^2}$$

$$-\sum_n \mathrm{Res}\left[P_-(E',E)\frac{S\tilde{T}'^{\mathrm{II}}(E')}{(E'-E)(E'-E_0)^2}\right]_{E'=E_n}+C_1+C_2E. \tag{10}$$

Here $SA'(E)=[ST'^{ret}(E)-ST'^{adv}(E)]/2i$; C_1 and C_2 are subtraction constants.

Comparing (10) with the DR which take into account only the singularities on the first sheet, namely

$$\frac{S\tilde{T}'^{\mathrm{I}}(E)}{(E-E_0)^2}=\frac{1}{\pi}\,\mathrm{P}\int_{-\infty}^{\infty}dE'\frac{SA'^{\mathrm{I}}(E')}{(E'-E)(E'-E_0)^2}+C_1'+C_2'E, \tag{10'}$$

we see that the introduction of the "nonphysical" singularities is equivalent to the integral over the interval $E_1\le E\le E_2$ on the right hand side of (10′).

Since only elastic scattering can take place in the interval $E_1\le E\le E_2$, we must have $A'^{\mathrm{I}}(E)\sim e^4$ or higher and the second integral in (10) is of higher order than the first integral. Therefore we can neglect it in first approximation and keep only the "resonance" terms from the poles. Eliminating the integration over the unobservable negative energies by means of the known symmetries of the functions $SD_{\alpha\omega}(E,\lambda\mathbf{n})$ and $SA_{\alpha\omega}(E,\lambda\mathbf{n})$ [7], carrying out the evaluations for $E_0=0$ and, finally, going to the limit $E_1=\mu\to 0$ we obtain

$$D'_{\mathrm{even}}(E)-D'_{\mathrm{even}}(0)=\frac{2E^2}{\pi}\,\mathrm{P}\int_{E_2}^{\infty}dE'\frac{A'_{\mathrm{odd}}(E')}{E'(E'^2-E^2)}$$

$$-2\,\mathrm{Re}\sum_n\frac{1}{E_n}F(E,E_n)\,u^{(n)}_{\mathrm{even}}(E_n),$$

$$D'_{\mathrm{odd}}(E)-E\left(\frac{dD'_{\mathrm{odd}}(E)}{dE}\right)_{E=0}=\frac{2E^3}{\pi}\,\mathrm{P}\int_{E_2}^{\infty}dE'\frac{A'_{\mathrm{even}}(E')}{E'^2(E'^2-E^2)}$$

$$-2\,\mathrm{Re}\sum_n\frac{E}{E_n^2}F(E,E_n)\,u^{(n)}_{\mathrm{odd}}(E_n), \tag{11}$$

where

$$SD'(E) = \frac{1}{2}[ST'^{ret}(E) + ST'^{adv}(E)],$$

$$F(E, E_n) = \left[E^2 - \frac{E}{E_n}\sqrt{\frac{E^2 - E_2^2}{E^2 - E_n^2}}\right] \Big/ (E_n^2 - E^2),$$

$$D'_{\text{even/odd}} = \{1 \pm p_{\nu'\nu}) S_+ D'_{\alpha\omega}(E, \lambda\mathbf{n});\ (1 \mp p_{\nu'\nu}) S_- D'_{\alpha\omega}(E, \lambda\mathbf{n})\},$$

$$u^{(n)}_{\text{even/odd}} = \{(1 \pm p_{\nu'\nu}) S_+ u^{(n)}_{\alpha\omega}(E, \lambda\mathbf{n});\ (1 \mp p_{\nu'\nu}) S_- u^{(n)}_{\alpha\omega}(E, \lambda\mathbf{n})\},$$

$$A'_{\text{even/odd}} = \{(1 \mp p_{\nu'\nu}) S_- A'_{\alpha\omega}(E, \lambda\mathbf{n});\ (1 \pm p_{\nu'\nu}) S_- A'_{\alpha\omega}(E, \lambda\mathbf{n})\};$$

$p_{\nu'\nu}$ is the permutation operator for the polarization indices ν' and ν.

Adding to the left side of the equations in (11) the corresponding real parts of the Delbrück scattering amplitude we obtain the DR for the full photon-nucleus elastic scattering amplitude.

3. The physical interpretation of the poles lying on the nonphysical sheet and the determination of the values of their characterizing parameters (position of the pole $\bar{E}_n$, widths Γ_n, weight of the pole $u^{(n)}(E)$) depend strongly on the details of the assumptions one makes concerning the interaction mechanism which leads to the considered process. If one assumes the "one nucleon" mechanism of the radiative transitions in the scattering of low energy photons by the nucleus [6] then one can determine the parameters of the poles by assuming some definite model of the nucleus which then would permit the evaluation of the energy spectrum and the single particle wave functions of a given nucleus.

The obtained DR could be useful in the analysis of the threshold anomalies in the cross section $\sigma(\gamma, \gamma)$ since they allow the evaluation of the "resonance" and the "dispersive" contributions which reflect the effect of the inelastic processes on the amplitude for the elastic scattering, as well as the evaluation of interference terms.

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