AN INVESTIGATION OF THE SHAPE OF ONE CROSS SECTION OF THE FERMI SURFACE OF TIN

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The diameters of the cross section of the Fermi surface of the fourth-hole zone in tin is measured with great accuracy. Two effects, based on a comparison of the dimensions of the orbits of the electrons in the metal in a magnetic field with the specimen dimensions are used: (1) the cutting-off of cyclotron resonance and (2) the cutting-off of nonresonance electron orbits by the surface of the specimen. The shape of the Fermi surface section found agrees in detail with the model of the Fermi surface based on the assumption of nearly free conduction electrons.

THE first experiments on the measurement of electron momenta by observing the "cutting-off" of high order cyclotron resonances were described earlier.[1] This occurs when the magnetic field H_y applied to the specimen is reduced sufficiently for the diameter of the electron orbit in spatial coordinates to become equal to the thickness D_z of the specimen. The diameter of the orbit in momentum space, i.e., the diameter of the corresponding cross section of the Fermi surface, is

$$2|p_x| = H_y D_z e/c. \quad (1)$$

To achieve high accuracy in the measurement of p_x, the cutting-off of the high order resonances 26—27 were observed in the previous experiments[1] on a single crystal of tin ~ 1 mm thick, so that the uncertainty in the required value of H_y was ± 1%.

The possibility of observing resonances of such high order for the section of the Fermi surface being studied in tin is, however, limited to a range of approximately ± 26° of rotation of the field (and of the plane of the electron orbit).[1,2] In [1] this was used to construct part of the section of the Fermi surface. For an extension of the study of this section, thinner specimens were required. In the present paper results are given of experiments carried out by the same frequency modulation method[3] as before,[1] on tin single crystals, prepared in the same way but thinner, at a wavelength of ~ 3.2 cm.

CYCLOTRON RESONANCE CUTOFF

In Fig. 1 is shown an example of the record of an experiment to measure the "cutoff field," which is the field strength for which the diameter of the electron orbit becomes equal to the thickness of a 0.39 mm specimen. In this experiment the specimen was a rectangular single crystal strip of tin with dimensions 13 × 6 × 0.39 mm. The plane of the strip surface had an (010) orientation. The records were made for various angles of rotation, φ, of the magnetic field relative to the [001] axis in the (010) plane of the specimen surface. The high-frequency current flow in the plate was parallel to the [101] axis. It can be seen that cyclotron resonances for the effective mass studied were observed for all directions of the field up to the ninth order; resonances of higher order are missing.

It is easy to follow how the "cutoff field" increases with the angle φ; in the present case this can be done up to $\varphi = 33°$. The limiting value of the angle φ up to which cutting-off of resonances can be followed depends, of course, to a large extent on the perfection of the single crystal used. On another specimen of the same orientation, in the shape of a 0.41 mm thick disk, for example, it was possible to determine the value of the "cutoff field" up to $\varphi = 37°$, i.e., over the whole range of field directions within which cyclotron resonance is observed for electrons of the effective masses designated by the number 1 in Fig. 4 of [2].

The results of these experiments allow the diameters of the section of the studied Fermi surface, lying in the directions $\varphi + \pi/2$, to be calculated from (1), and the construction of this section can be extended to 37°. The experimental points are shown in Fig. 2; the centers of the orbits of electrons of mass 1 lie at the point 1.0 on the [100]

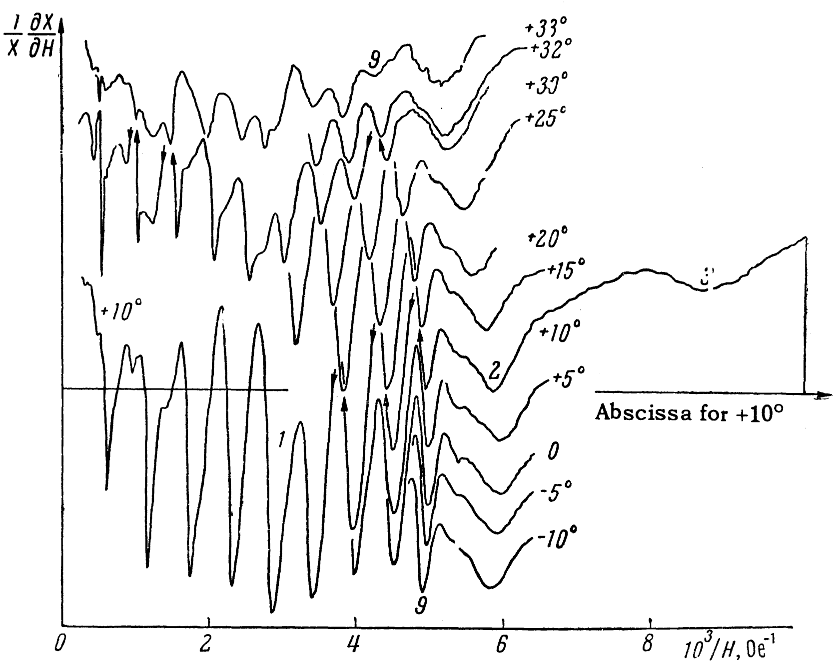

FIG. 1. Record of an experiment to measure the "cutoff field" for cyclotron resonances in a 0.39 mm thick tin single crystal. The angle of rotation of the magnetic field relative to the [001] axis, φ, is shown at the right of the curves; 9 is the order of the last visible resonance for the effective mass studied; 1, 2 and 3 are resonances of mass ~0.1 m_e. The direction of the high frequency currents J ∥ [101].

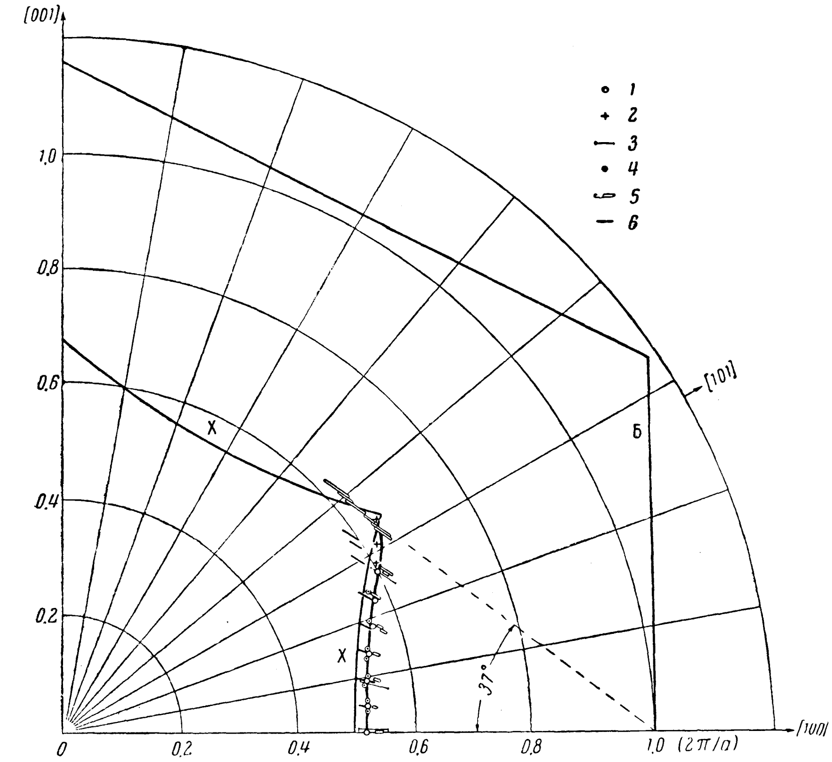

FIG. 2. Section of the fourth-hole zone Fermi surface of tin. Results of the following measurements: 1, 2 – method of cutting-off non-resonance orbits; 3–6 – method of cutting off cyclotron resonances. Specimens: 1–3 – 0.18 mm strip, J ∥ [101]; 4 – 0.98 mm thick disk, J ∥ [100]; [2] 5 – 0.41 mm disk, J ∥ [100]; 6 – 0.39 mm strip, J ∥ [101], X – section constructed by Harrison's method,[6] B – boundary of the fundamental Brillouin zone.

axis. We should point out that the new data are less accurate, since the possible error in determining the "cutoff field" increases with decreasing specimen thickness and the reduction of the order of the cutoff resonance. In addition, as the field direction approaches the limiting value of the angle φ the effect of cutting-off the resonances becomes harder to observe, as a result of a sharp decrease in their depth. The accuracy of the results obtained is, however, sufficient to reveal certain features of the Fermi surface, as can be seen from Fig. 2.

The correspondingly less accurate results of an experiment on cutting-off cyclotron resonances in a specimen of thickness 0.18 mm (strip 13 × 6 mm) are also shown in this figure.

ELECTRON ORBIT CUTOFF IN A NON-RESONANCE FIELD

Kaner [4] showed that it was possible to determine the diameter of the extremal electron orbits from the jump in the derivative of the surface impedance of the metal with respect to the field. If we neglect the non-monotonic changes due to cyclotron resonance, the surface impedance of a thick metal specimen in a magnetic field parallel to its surface should decrease with increasing field. This is explained by the contraction of the electron orbits as a result of which the conduction electrons return to the skin layer δ more often (Fig. 3a), and the fraction of each electron's free path within the skin depth increases. If the thickness of the specimen D_z is small, then for a convex Fermi surface (Fig. 3b) the diameters of the orbits of part of the electrons will be larger than D_z in a weak magnetic field (Fig. 3a) and the fall in surface impedance of the specimen with increasing field will result not only from the increase in the frequency of the return of each electron to the skin layer, but from the increase in the number of such electrons.

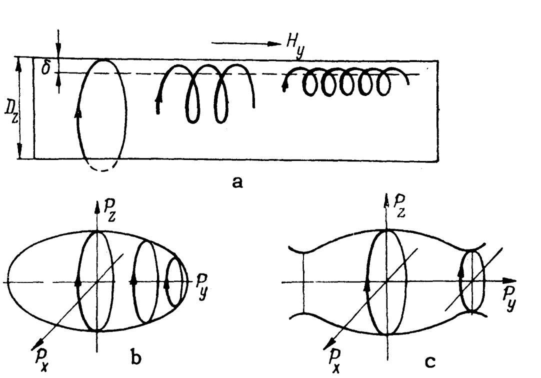

FIG. 3. Illustration of the phenomenon of cutting-off non-resonance electron orbits by the surface of a thin specimen.

The derivative of the surface impedance of a thin specimen with respect to the field will thus be greater (in absolute magnitude) than for a thick specimen, up to the value of field for which the diameter of the maximum orbit is equal to the specimen thickness D_z; at this value of the field the derivative drops discontinuously to its value for a thick specimen. For the case when the Fermi surface is not convex but has the shape shown in Fig. 3c, there is both a maximum and a minimum diameter of the electron orbits for a given field direction. The derivative of the surface impedance must then have another jump, but of the opposite sign, in the field for which the diameter of the minimum orbit becomes equal to D_z. All this applies to the resistive component as well as to the reactive component, the logarithmic derivative of which is recorded in the course of an experiment carried out by the method of frequency modulation. [3]

The record of an experiment on a tin single crystal 0.18 mm thick is shown in Fig. 4. The jump marked O can be clearly[1)] seen in all the curves which differ in the angle of rotation φ of the magnetic field from the direction of the [001] axis in the (010) plane. This jump determines with great accuracy the field for which the diameter of the maximum electron orbit belonging to the section of the Fermi surface being studied equals the specimen thickness. The jump of the opposite sign O′, visible only for φ equal to 0 and 5°, determines the cut-off field of the minimum orbit. The points of the Fermi surface deduced from this, using (1), with an accuracy of $\pm 1\%$, are shown in Fig. 2.

The existence of intense cyclotron resonances in the record shown in Fig. 4 makes it difficult to observe the jumps in the derivative of the surface impedance. However, similar jumps should be observed over the very wide frequency range for which the anomalous skin effect occurs. It is therefore more convenient to observe them at considerably lower frequencies in the absence of cyclotron resonance. Such an experiment is described by Gantmakher. [5]

DISCUSSION

The outline of the section of the Fermi surface by the (100) plane, constructed from the experimental points, is shown in Fig. 2. The line is drawn mainly according to the results of measuring the "cutoff field" of non-resonance orbits, as they embrace the whole section. Some systematic differences between the results on different specimens can be attributed to a difference in the crystallographic orientations of their surfaces and to inaccuracy in determining the crystallographic directions in them, and also to errors in determining the specimen thicknesses.

The section found refers to the fourth-hole zone Fermi surface. [1] This surface is shown in Fig. 5; the position of the section is shown on it by the thick line. The geometrical construction of the Fermi surface is made by Harrison's method. [6] Its appearance differs from that given by Gold and Priestley, [7] where the outline of the Fermi surface was arbitrarily smoothed out, as a result of

[1)]The amplitude of the jump decreases with increase in φ, as does the depth of the resonances for electrons belonging to the same part of the Fermi surface (these resonances of order from 1 to 4 can be seen in Fig. 4). This is connected with the decrease in the number of electrons effectively taking part in this phenomenon and also with the change in the direction of the magnetic field relative to the polarization of the high frequency field. Kaner[4] has given the basis for an accurate calculation of the surface impedance of a thin metal layer.

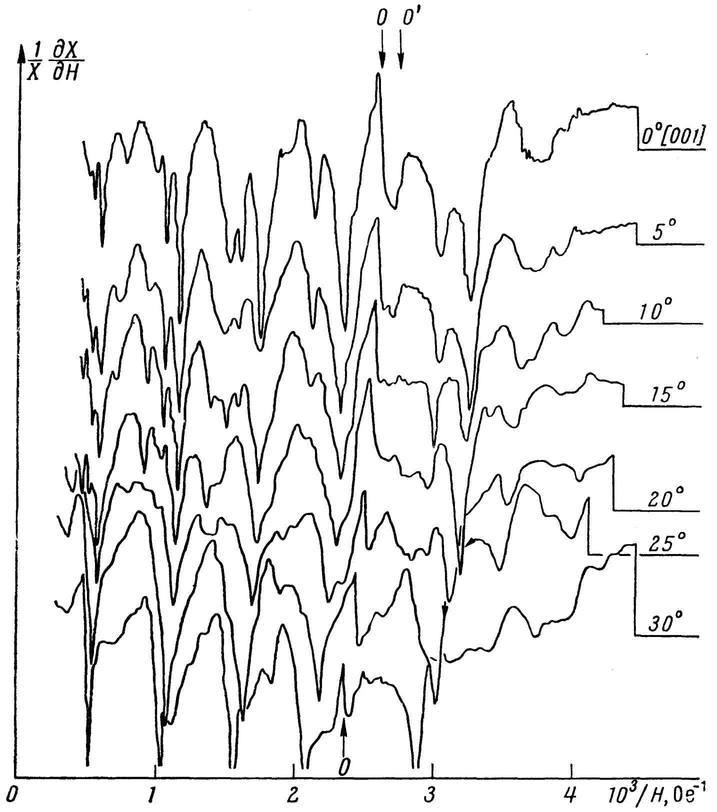

FIG. 4. Record of an experiment to measure the "cut-off field" for non-resonance electron orbits in a 0.18 mm thick tin single crystal. O and O′ are the jumps in the derivative of the surface impedance determining the "cut-off field." The angle of rotation of the magnetic field from the [001] axis, φ, is shown on the right of the curves. In the present work the cutting-off of cyclotron resonances of different masses, which could be observed in this figure, is not considered.

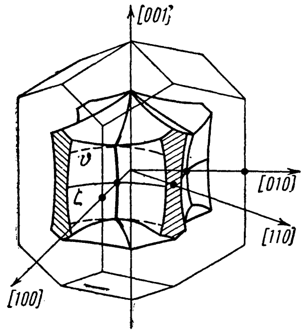

FIG. 5. The fourth-hole zone Fermi surface of tin referred to the principal Brillouin zone. The section studied is denoted by the thick line. The extremal orbits in a magnetic field parallel to [001] are: ζ – maximal, ϑ – minimal (two orbits; the approximate position is shown).

which the interesting details of its shape, found in the present experiments, disappeared. We refer to the "barrel-like" shape of the inner tubular parts of the surface (Fig. 5), which previously appeared to be nearly cylindrical [1] or even broadening to their open ends [7] (the possibility that the walls of these "tubes" were concave was noted in [7]).

Owing to the "barrel-like" shape of the Fermi surface, extremal orbits of two types of electrons can exist in a field parallel to the [001] axis: the maximal ζ (Fig. 5) in the section by the coordinate plane (001), and the minimal ϑ in the section by the plane parallel to it near the open end of the "barrel" (two orbits). Orbits of both types were observed by the method of cutting-off non-resonance orbits, as described above.

The existence of the orbits ζ and ϑ is confirmed by studies of cyclotron resonance. Resonance corresponding to an effective mass $0.56\,m_e$ (mass 1 in [2]) is observed in a field parallel to the (010) surface of the specimen, but it splits in two for a field inclination of 3′—10′ and the effective mass corresponding to one of them decreases and the other increases (this is described in Fig. 3 of [2]). Apparently the first resonance occurs for electrons of orbit ζ and the second for orbit ϑ. This view is also confirmed by the fact that on rotating a slightly inclined field in the (010) plane the second resonance is observed only within the limits of ~ 8°, while the first resonance is visible for a field rotation up to ~ 37°. The details of the behavior of the cyclotron resonance in an inclined magnetic field determined in this way, which are in agreement with their qualitative explanation in [2], can be used to distinguish the resonances for maximal and minimal effective masses.

Besides the results of the experiments, the section of the fourth-hole zone Fermi surface constructed by Harrison's method is also shown in Fig. 2. The arcs which bound the section have centers in the neighboring points of the reciprocal lattice of tin, and their radii, equal to 1.52 ($2\pi/a$), are the radii of a sphere in phase space filled by the four valence electrons of tin. The practically complete agreement between the experimentally determined shape of the section of the Fermi surface and the free electron model is apparent from Fig. 2. To this must be added the existence of cyclotron resonance for the elliptical saddle point in the [100] direction, observed earlier[2] (Fig. 4, mass 10 equal to $1.6 m_e$). The radius of curvature of the Fermi surface at the saddle calculated from this mass according to Kaner and Azbel'[8] is 1.7 ($2\pi/a$), i.e., agrees within the accuracy of calculation with the radius of a sphere of free electrons.

The "barrel form" of the section of the Fermi surface studied is not contradicted by Gold and Priestley's results[7] since, as they note, the period of the G-oscillations and consequently the area of the section calculated by them is little dependent on its curvature. Incidentally, the accuracy of their measurements is clearly insufficient for studying such fine details of the shape of the section of the Fermi surface. Olsen's results also agree completely with the results of the present work.

CONCLUSION

The geometrical method for constructing the Fermi surface of a metal, proposed by Harrison,[6] based on the assumption of nearly free conduction electrons, makes it possible to derive very easily a model which has the correct topological properties and the main features of the shape of the true Fermi surface of the metal considered. Harrison discusses in detail the significance of this fact which is confirmed by experiments to study the Fermi surface, carried out by different methods on several metals. Later work, in particular [10], also convincingly supports the closeness of such a model of the Fermi surface to the truth; on the other hand there are no experiments which definitely contradict this model in the respects mentioned above.

One of the sections of the Fermi surface of tin has been studied with great accuracy as a result of the present experiments. The detailed agreement between the shape found for the section of the fourth-hole zone of the surface and the model constructed on the basis of the nearly free electron assumption, is a new and convincing indication of the reasonableness of this model. In any case, this model can be taken as a starting model, close to the truth, in accurate experimental investigations of the Fermi surface, and requiring some corrections based on the experimental results. These corrections can be most noticeable near the lines of intersection of the spherical surfaces making up the model of the Fermi surface, either with one another or with the faces of the Brillouin zone, and especially near sharp corners arising in the geometrical construction of the model.

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