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ELASTIC SCATTERING OF 2.8 BeV/c π^- MESONS ON HYDROGEN

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Elastic scattering of 2.8-BeV/c π^- mesons on hydrogen nuclei was studied with a propane bubble chamber. The elastic interaction is shown to be predominantly of a diffraction nature. The elastic diffraction scattering cross section is $\sigma_d = (6.5 \pm 0.8)$ mb and the cross section for all elastic processes is $\sigma_e = (7.8 \pm 0.9)$ mb. The differential cross section for elastic diffraction scattering is in good agreement with the predictions of the optical model in which the nucleon is considered as a nonrefractive and absorbing sphere with a radius $R = (1.10 \pm 0.09) \times 10^{-13}$ cm and an absorption coefficient $K = (0.71 \pm 0.19) \times 10^{13}$ cm^{-1}.

INTRODUCTION

DURING the last few years, a number of papers have been published describing measurements of the total and differential cross sections for elastic scattering of pions on hydrogen at energies above 1 BeV.[1-11] In this energy region, the differential cross section for elastic scattering is appreciably peaked forward and its shape resembles somewhat the picture of elastic scattering of fast nucleons by complex nuclei. This circumstance provided the basis for the adoption of the optical model for the analysis of pion-nucleon scattering. According to this model, the nucleon is represented in the form of a sphere with sharp or diffuse edges and possesses a complex refractive index, while the peak of the differential elastic scattering cross section is interpreted as a diffraction maximum, from whose shape one can obtain an idea of the size and properties of the nucleon.

The analysis of πp elastic scattering in the energy region above 1 BeV is facilitated by the fact that the scattering amplitude here is almost purely imaginary, and consequently the real part of the refractive index can be considered as equal to unity.[12] Moreover, the wavelength of the incident pion in the c.m.s. of the colliding particles is much smaller than the dimensions of the nucleon, which makes it possible to apply the quasi-classical approximation with a good degree of accuracy.

The present experiment fills a gap in the energy scale of π^- mesons used for the study of πp elastic scattering.

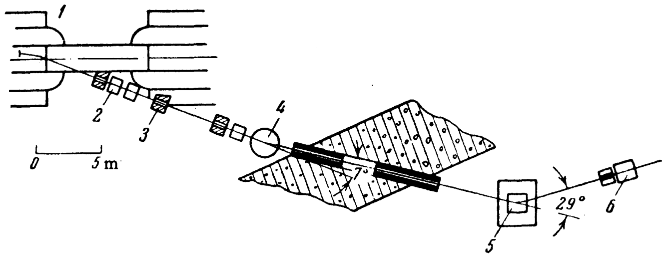

FIG. 1. Diagram of experimental arrangement: 1 – linear gap of proton synchrotron; 2 – focusing magnetic lens; 3 – shield; 4, 5 – bending magnets; 6 – bubble chamber.

1. EXPERIMENTAL METHOD

Elastic scattering of π^- mesons on hydrogen was studied with the aid of a $37 \times 10 \times 10$-cm pro pane bubble chamber, without a magnetic field, exposed to a beam of π^- mesons of momentum 2.8 $\pm$ 0.15 BeV/c[13] at the proton synchrotron of the Joint Institute for Nuclear Research (Fig. 1).

For the study, we selected two-prong stars produced by relativistic particles entering at an angle no greater than 2° relative to the horizontal axis of the chamber (in the projection on the plane of observation). The pictures were scanned by the method in which a transparent rule is placed on a greatly magnified picture of the particle tracks.[14] This method of scanning, similar to the scanning for stars along the track in nuclear emulsion, considerably reduces the loss of stars in scanning.

As a result, we selected 306 two-prong stars, of which 142 were of the type 1 + 1p (with one black and one relativistic track) and 164 of the type 0 + 2p (with two relativistic tracks) whose

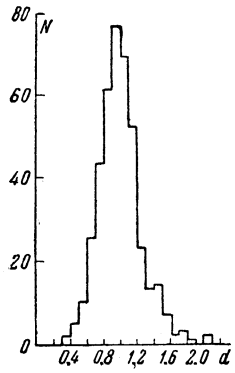

FIG. 2. Distribution of secondary particle tracks considered to be relativistic on basis of visual estimate as a function of the bubble density d.

secondary products were directed toward different sides of the plane passing through the primary particle track and the optical center of the camera objectives. Stars of the type 2 + 0p (with two black tracks) could not occur in the case of elastic scattering of 2.8-BeV/c pions on protons.

The spatial characteristics of the scattering events were reconstructed from measurements of the coordinates of the point of interaction and arbitrary points on each track in two pictures taken with the aid of a stereographic camera. As a check, we also measured the projections of the angles between the tracks. The coordinates in the photographed plane were measured to an accuracy of ~ 0.2 mm and the angles to an accuracy of 0.3—0.5°. The coordinates were measured with the aid of a millimeter grid on which a magnified picture of the chamber was projected. The results of the calculations of the angular projections based on the measured coordinates were compared with the data of the direct measurements. A suitable choice of the points on both views permitted a check on the correctness of the coordinate measurements.

With the aid of an electronic computer, we calculated the spatial characteristics of the interactions and also three quantities characterizing the deviation of the tracks from coplanarity: the projection of the angle between the secondary prongs on a plane perpendicular to the direction of the primary beam, the volume of the parallelepiped constructed on segments of the primary and secondary tracks of unit length, and the angle between the primary track and the plane determined by the secondary tracks.

The ionization of the interaction products was estimated by inspection. For part of the stars recorded in the chamber, the bubble density on tracks of primary and secondary relativistic particles was counted. Results of such measurements are shown in Fig. 2, in which it is seen that the visual estimate permits a sufficiently good separation of particles whose bubble density is more than double that of particles corresponding to protons of energy velow 225 MeV.[15]

The calculation of the spatial characteristics permitted us to separate and discard cases in which the prolongation of the primary particle tracks lay in the glass or bottom of the chamber as well as particles whose angle of entry was greater than 7° relative to the plane of observation, which, in all probability, are secondary particles coming from the chamber walls. The effective length of the chamber over which the efficiency for recording stars was constant proved to be 290 mm.

Cases of elastic scattering were separated from the remaining material on the basis of the coplanarity of the primary and secondary tracks, the correspondence of the scattering angles of the pion and recoil protons with the kinematical curve, and on the basis of the ionization and range of the recoil proton. Analysis showed that of the three calculated quantities characterizing the deviation from coplanarity, the angle δ between the primary track and the plane of the secondary tracks gave the sharpest separation between cases of elastic and inelastic scattering. This parameter was subsequently used for the estimate of the coplanarity.

On the basis of double measurements of 31 events, we found that the standard deviation of the measurement error for the coplanarity parameter δ was $\Delta\delta = 1.25°$; for the space angle of the scattered pion it was $\Delta\vartheta_\pi = 0.94°$; for the recoil proton it was $\Delta\vartheta_p = 1.80°$; and for the track length of the recoil proton it was $\Delta L_p = 1.8$ mm.

The distribution of the two-prong stars as a function of the coplanarity parameter is shown in

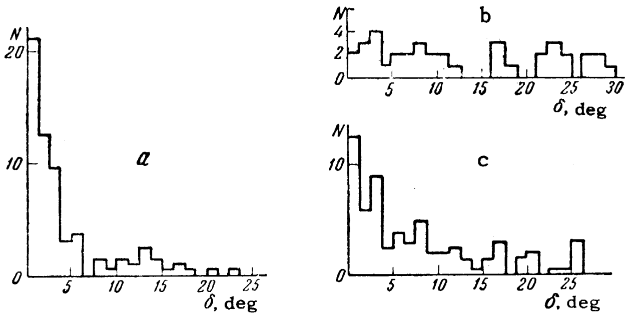

FIG. 3. Distribution of two-prong stars as function of coplanarity parameter δ: a – stars of type 1 + 1p, b – stars of type 0 + 2p, c – only inelastic stars of type 1 + 1p and 0 + 2p.

Fig. 3. Cases with the $\delta \leq 3\Delta\delta = 3.75°$ were accepted as coplanar. We discarded as inelastic 110 two-prong stars not satisfying this requirement. It can be shown that the distribution of cases of quasi-elastic scattering as a function of the angle between the primary track and the plane of the secondary tracks is quite narrow, and therefore the exclusion of noncoplanar cases eliminates only part of the quasi-elastic background.

Furthermore, we constructed the distribution of the remaining cases as a function of the deviation θ from the kinematical curve (Fig. 4), where the smaller angle of emission of the interaction products is given along the abscissa axis and the larger angle, along the ordinate axis, since, strictly speaking, it is not known which of the secondary tracks belong to the recoil proton and which to the pion. Then, for stars of the type 1 + 1p, we measured the deviation from the segment of the kinematical curve corresponding to the slow recoil protons (relative bubble density greater than 1.5 times that of the primary pion tracks), while for stars of the type 0 + 2p, we measured the deviation from the remaining part of the curve. The deviation of elastic events from the kinematical curve is almost independent of the energy spread in the pion beam and was determined chiefly from the measurement errors of the pion and recoil-proton scattering angles, which leads to a standard deviation of $\Delta\theta = 2°$. If we assume that the distribution of the points about the kinematical curve is Gaussian, then we can limit our selection to the values $\theta \leq 3\Delta\theta = 6°$. The discarded cases were considered as background produced by quasi-elastic and inelastic $\pi^- p$ collisions. The extrapolation of the background to the region of deviations less than 6° makes it possible to estimate the contribution from these processes among the cases of πp elastic scattering. The estimates of the background in this region by linear extrapolation showed that it consisted of five interactions out of 64 cases of the type 1 + 1p and eight interactions out of 21 cases of type 0 + 2p. We note that the contamination from cases of a quasi-elastic character does not change the shape of the differential cross section and was considered only in the estimate of the total elastic scattering cross section.

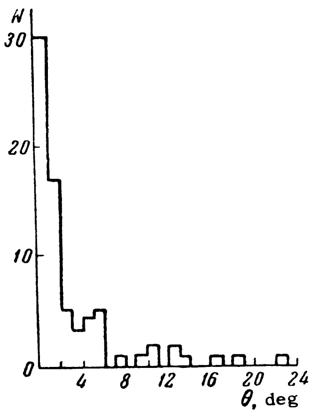

FIG. 4. Distribution of two-prong coplanar stars of type 1 + 1p as a function of the deviation θ from the kinematical curve.

The next selection criterion was the comparison of the range of the slow recoil protons with the expected range corresponding to the measured angles of emission of the secondary particles. For this, we used the calculated range-energy curves for protons in propane computed by Samoĭlov (see also [16]). The propane density during the chamber operation was measured by two methods and was equal to 0.40 ± 0.01 g/cm^3. The calculated and measured ranges were compared for cases in which the slow proton track stopped in the chamber. The range of the slow recoil proton emitted at a given angle weakly depends on the energy spread in the primary beam. The uncertainty in the range was due to measurement errors of the angles and track lengths of the secondary particles. The deviation of the recoil proton range from the calculated range was considered significant if it was more than three standard deviations. We observed six such cases. In two cases, this discrepancy could be explained by errors resulting from the short lengths of the secondary tracks and the unfavorable orientation of the scattering plane. The remaining four cases were considered as background and were eliminated from the resulting distribution. Since the background intensity predicted on the basis of extrapolation consisted of five interactions, the exclusion of four of them reduced the background among stars of type 1 + 1p to one case.

It is important to note that if the selection were restricted to the limits $\delta \leq 5\Delta\delta$ and $\theta \leq 5\Delta\theta$ (i.e., if five standard deviations were taken instead of three), then the distribution would contain only one additional interaction because of the low accuracy of measurement of the parameters resulting from the very small (1.1 mm) length of the secondary-proton track. This justifies the choice of the limits for the allowable deviations of the values of δ and θ.

The azimuthal angle distribution of the scattering events does not give any definite indications of a drop in the scanning efficiency for unfavorable orientations of the scattering plane, and for this reason, we estimate that less events are missed in the small angle region than in other methods.

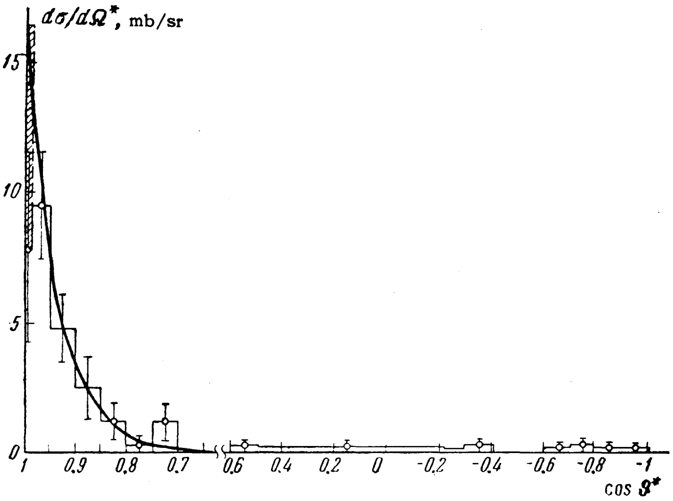

FIG. 5. C.m.s. differential cross section for elastic scattering of 2.8-BeV/c π^- mesons on hydrogen. The shaded region represents events missed at small angles. The smooth curve was constructed on the basis of the optical model for a nucleon considered as a sphere of radius $R = 1.10 \times 10^{-13}$ cm with an absorption coefficient $K = 0.71 \times 10^{13}$ cm^{-1}.

Neglecting the very small errors contributed by the uncertainties in the extrapolation procedure, we can conclude that the number of acts of elastic scattering of π^- mesons on free hydrogen, after allowance for the background, is 60 ± 8 for stars of type 1 + 1p and 13 ± 5 for stars of type 0 + 2p.

2. CROSS SECTION FOR π^-p ELASTIC SCATTERING

On the basis of the obtained data, we calculated the differential and total cross sections for the elastic scattering of 2.8-BeV/c π^- mesons on free hydrogen. Here, we took into account the fact that the μ-meson contamination in the primary beam was $(27 \pm 4)\%$.[14]

Figure 5 shows the differential cross section for π^-p meson elastic scattering in the c.m.s. of the colliding pion and proton. The rapid rise of the differential cross section in the small-angle region is evidence that the elastic scattering process connected with a small momentum transfer from the incident pion is of a diffraction nature. The small number of events of type 0 + 2p represents incoherent elastic scattering, whose cross section is 1.3 ± 0.5 mb, which is not in disagreement with the predictions of the isobaric variant of the statistical theory of multiple production (3.1% of the cross section for all inelastic processes).[17] Despite the low accuracy of the measurements of the elastic scattering differential cross section in the region beyond the diffraction maximum, the results of the present experiment are in agreement with the data obtained by Lai, Jones, and Perl[11] for 2.53-BeV/c π^- mesons.

The dip in the histogram of Fig. 5 in the angular region $\cos\vartheta^* > 0.99$ indicates the missing of events with a scattering angle less than 3° in the laboratory system (track length of recoil proton < 3 mm). Comparison of this part of the histogram with the calculated value of the forward scattering differential cross section

$$\frac{d\sigma}{d\Omega^*}(\vartheta^* = 0^0) = \frac{k^{*2}\sigma_t^2}{16\pi^2} = 16.3 \text{ mb/sr}$$

(k^* is the c.m.s. wave vector of the incident pion) indicates that ~ 9% of the events were omitted. As a result, the total cross section for elastic diffraction scattering turns out to be

$$\sigma_d = 6.5 \pm 0.8 \text{ mb},$$

and the total cross section for all elastic processes is

$$\sigma_e = 7.8 \pm 0.9 \text{ mb}.$$

Using the value of the total interaction cross section[18] $\sigma_t = 30 \pm 1.5$ mb, we find that the total absorption cross section is $\sigma_a = 23.5 \pm 1.7$ mb, while the inelastic cross section is $\sigma_i = 22.3 \pm 1.7$ mb.

3. OPTICAL MODEL ANALYSIS OF THE EXPERIMENTAL DATA

For the analysis, we used a simpler model in which the nucleon is considered as a homogeneous sphere of radius R with a purely imaginary refractive index. On the basis of the well-known formulas[19]

$$\sigma_d = \frac{\pi}{4K^2}\{4K^2R^2 - 2(1 + 2KR)\exp(-2KR) + 16(1 + KR)\exp(-KR) - 14\},$$

$$\sigma_t = \frac{\pi}{2K^2}\{4K^2R^2 + 8(1 + KR)\exp(-KR) - 8\}$$

we determined the region of possible pairs of values of R and K corresponding to one standard deviation of the quantities σ_d and σ_t (Fig. 6). According to our calculations,*

$$R = (1.10 \pm 0.09)\cdot 10^{-13} \text{ cm},$$

$$K = (0.71 \mp 0.19)\cdot 10^{-13} \text{cm}^{-1},$$

which corresponds to an rms radius of the proton $\langle r \rangle = (0.85 \pm 0.07) \times 10^{-13}$ cm. It is interesting to note that, according to Hofstadter's data,[20] the rms radius of the proton determined from electron scattering experiments is $(0.77 \pm 0.10) \times 10^{-13}$ cm.

The smooth curve in Fig. 5 corresponds to a differential cross section for elastic scattering

*The inverted order of the error signs signifies that in order to satisfy the values $\sigma_d \pm \Delta\sigma_d$ and $\sigma_t \pm \Delta\sigma_t$ it is necessary to decrease R for an increase in K and vice versa.

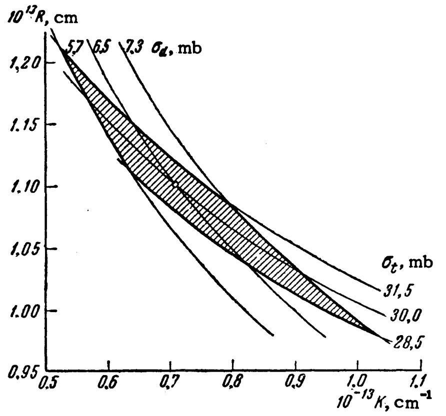

FIG. 6. Region of possible values of R and K corresponding to the elastic scattering differential cross section $\sigma_d = 6.5 \pm 0.8$ mb and total cross section $\sigma_t = 30 \pm 1.5$ mb.

of pions on a nucleus of radius R and absorption coefficient K calculated from the formula

$$\frac{d\sigma}{d\Omega^*} = \left| k^* \int_0^R [1 - \exp(K\sqrt{R^2 - \rho^2})] J_0(k^* \sin \vartheta_\pi^* \rho) \rho d\rho \right|^2 .$$

As seen from Fig. 5, the experimental and calculated values of the elastic scattering differential cross section are in good agreement with one another. We note that the curves constructed for other values of R and K taken within the bounds of the shaded region of Fig. 6 also satisfactorily approximate the experimental distribution. The area bounded by the smooth curve corresponds to a π^-p total elastic diffraction scattering cross section $\sigma_{d\,calc} = 6.4$ mb. The agreement within $\sim 2\%$ between the calculated and experimental values of σ_d confirms the correctness of the quasi-classical approximation ($\lambda^* = 0.187 \times 10^{-13}$ cm $\ll$ R).

If the scattering of pions on protons at high energies is determined only by the momentum transfer σ, then the differential scattering cross section in the coordinates $\frac{d\sigma}{dq^2}(q) \sim \lambda^{*2}\frac{d\sigma}{d\Omega}(q)$ should coincide for various energies. From the viewpoint of the optical model, this would mean that the optical parameters of the π^-p interaction do not change with energy.

In Fig. 7, the results of the present experiment are compared with the data of other authors [8,9] corresponding to π^--meson momenta of 5.17 and 6.8 BeV/c. The results of all experiments are in sufficiently good agreement with one another in the region of momentum transfers $q \sim \sin \vartheta_\pi^*/\lambda^*$ between 200 and 700 MeV/c.

Comparison with data obtained for 1.43-BeV/c π^- mesons [2] can be misleading because of the appreciable admixture of events from incoherent elastic scattering at this energy.

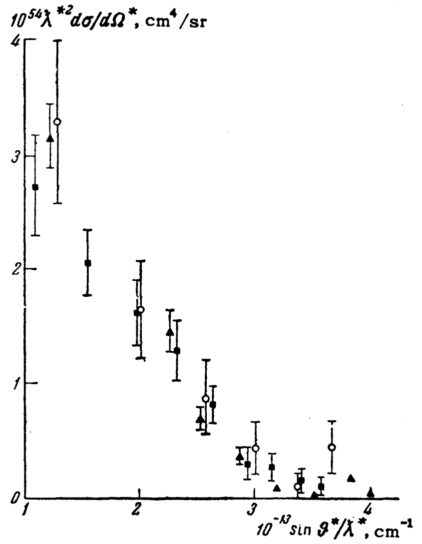

FIG. 7. π^-p elastic scattering differential cross section $d\sigma/dq^2 \approx \lambda^{*2}d\sigma/d\Omega^*$ as a function of the momentum transfer according to data of various authors: o – present work, p = 2.8 BeV/c; ▲ – [8] p = 5.17 BeV/c; ■ – [9], p = 6.8 BeV/c.

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