

MUON BEAMS IN EXTENSIVE AIR SHOWERS

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Results of an investigation of irregularities in the spatial distribution of μ -meson beams in extensive atmospheric showers are presented. The distribution of meson tracks in the observation plane is analyzed by the Monte Carlo method. The existence of beams (narrow groups) of μ mesons is confirmed. The spectrum of the extensive atmospheric showers accompanying these beams is obtained. The possibility of production of μ -meson beams in nuclear interaction acts is discussed.

INTRODUCTION

In an earlier investigation [1] we obtained experimental data on the possible existence of groups of genetically related muons underground. We did not show conclusively, however, that the event isolated with the aid of the Geiger-counter detector was indeed a group of muons.

The interpretation offered in [1] for the observed events raises, at the same time, the more general question of whether the genetic connections between the muons contained in extensive air showers (EAS) can be ascertained by studying the irregularities of the lateral distribution of the muons. We have therefore continued the study of the irregularity in the lateral distribution of the muons underground.

EXPERIMENTAL SETUP

The EAS on the earth's surface were investigated with a hodoscope consisting of a large number of Geiger counters. The distribution of the counter positions and the number of counters of different sizes in each position are shown in [2].

The muon fluxes were registered in a previously described [1] underground chamber (depth 40 m.w.e.) located under the central chamber of the above-ground portion of the array (see Fig. 1 in [2]).

The muon detectors in the underground chamber were Geiger-counter trays connected to form a hodoscope and shielded with lead and iron. Unlike the arrangement in [1], the muon detector consisted of three trays of Geiger counters, the counters of the central tray being separated by lead layers 2.5 cm thick. Figure 1a shows a section through the muon detector, while Fig. 1b shows the plan of the detectors in the underground chamber. The upper

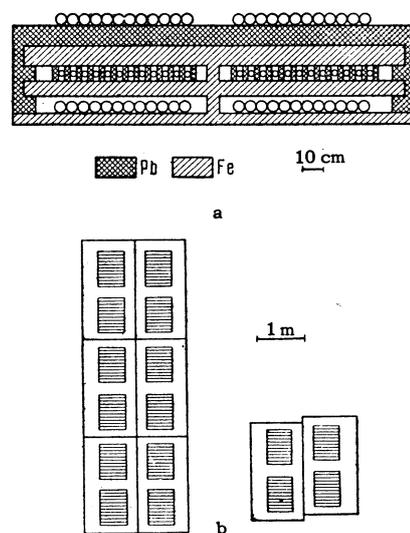


FIG. 1. Diagram of the setup: a—muon detector, b—arrangement of muon detectors in the underground chamber.

and lower layers consisted of counters measuring $6 \times 55 \text{ cm}^2$ and the counters of the central layer measured $3 \times 55 \text{ cm}^2$. Each layer contained 192 counters, making the sensitive area of the detectors 6.3 m^2 in the upper and lower layers and 3.15 m^2 in the central layer.

We used the same hodoscopic system as in [1], the hodoscope cells having a 10–20 μsec resolution time.

The array was triggered by two systems. One trigger system (system 1) served to select the EAS on the earth's surface and comprised a coincidence circuit of six counter groups, each 0.099 m^2 in area. The counters of this system were located in the central chamber above ground.

The second system (system 2) served to select special events registered in the muon detectors underground and accompanied by extensive air

Table I

Type of event								Number of events, %
I.	-	-	0	0	0	0	-	90.9
	-	0	-	0	0	-	0	
	0	0	0	0	-	-	-	
	(16.2%)	(10%)	(21%)	(23%)	(3%)	(15.7%)	(2%)	
II.	-	-	0	0				4.5
	00	00	00	00				
III.	-	-	0	0				0.5
	000	000	000	000				
IV.	00	00	00	00				3.2
	-	0	0	0				
V.	000	000	000	000				0.4
	-	0	0	-				
VI.	-	0	-	0				0.45
	00	00	00	00	00			
VII.	-	-	-	-	0	0	0	0.05
	000	000	000	000	000	000	000	
	-	0	00	000	-	0	00	

Note. The symbol zero denotes operation of the counters in the detector of Fig. 1, the minus sign denotes that the counter did not operate.

shower on the earth's surface. Trigger system 2 operated whenever the following two events occurred simultaneously: 1) operation of any three counters in succession from among the 144 counters of the central detector row shown in Fig. 1a, and 2) operation of at least one counter from among the 36 in the central chamber above the earth's surface (the total area of these counters was 1.2 m²).

RESULTS

We undertook to study the distribution of the muon tracks in the plane of observation, using the described array. As a quantitative characteristic of this distribution we chose the distribution of the distances D between muons. The coordinates of the muon tracks in the plane of observation were determined with the aid of the detectors of Fig. 1 (accurate to the dimensions of the Geiger counters).

The muon-detector readings may, in principle, be distorted by the electron-photon (ep) component generated by the muons in the ground or in the detector screen. Table I lists the types of events observed when individual muons passed through the detector of Fig. 1 'without interaction' (row 1), and events connected with formation of muons by δ-electrons or secondary electron-photon showers. The total number of events represented in Table I is 4 × 10⁴ (after 2340 hours of operation with trigger system 1). It follows from Table I that the counters of the central detector layer were operated by the electron-photon component with the

least frequency. Therefore the coordinates of the muon tracks were determined from the readings of the second-layer counters and the distances D were measured from the centers of the counters in this layer. It was always required that the corresponding counters in the upper and lower layers also operate.

Distributions over the distances D were obtained for showers having different numbers (M) of operating counters in the middle layer. For each shower we determined with the aid of the array on the earth's surface the number of particles N in the shower and the distance R on the earth's surface from the shower axis to the vertical line passing through the muon detectors. The distributions over the distances D were obtained for two groups of showers: a) for showers with R < 30 m and b) for showers with R > 50 m. The number of particles in the showers with R < 30 m ranged from 5 × 10⁴ to 10⁶, while those in the showers with R > 50 m ranged from 10⁶ to 2 × 10⁷. The shower distribution for these groups with respect to the number M of operated counters was as follows:*

M:	2	3	4	5
Event R < 30:	127	60	55	17
Event R > 50:	51	27	25	8

*The number of events with different numbers of operated counters was chosen for different operating times of the array. The distribution over M given here differs therefore from the real M-spectrum of the showers.

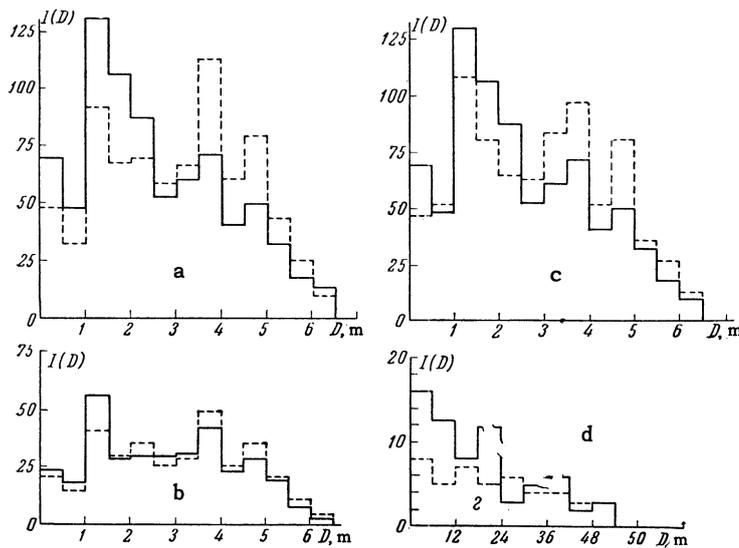


FIG. 2. Distribution of the distances D between the muon-registering counters: a—for $R < 30\text{m}$; b—for $R > 50\text{m}$; c—same as a, but theoretical distribution calculated with allowance for the angles of incidence of shower axes on the muon detector; d—part of the distribution a for $D < 0.5\text{m}$. Continuous curves—experiment, dashed—theory.

Figure 2 shows the distributions over D for both cases and for all showers with M between 2 and 5.*

To check the D -distribution that would be obtained from the statistical scatter of the muon trajectories, we calculated the analogous D -distributions for artificial showers, in which the muon trajectories are independent, using the Monte Carlo method. The numbers of the 'operating' counters for the artificial showers were obtained with the aid of a table of random numbers. The total number of 'operating' counters and the number of corresponding artificial showers were chosen to be equal to the observed values. The resultant theoretical distributions are shown dotted in Fig. 2. It is apparent from Fig. 2 that the distribution for $R > 50\text{ m}$ agrees with the theoretical distribution [the agreement probability is $P(\chi^2) \sim 17\%$], whereas the distribution for $R < 30$ contradicts the theoretical value (the agreement probability is $P(\chi^2) \ll 10^{-2}\%$).

It is possible that the distribution for $R > 30\text{ m}$ is connected with the influence of the showers with axes far from the muon detectors. The muon distribution in such showers, should have a gradient over the detector area, which can lead to the observed D -distribution. To verify whether the distributions for $R < 30\text{ m}$ agree with this assumption, we calculated the D -distribution for artificial showers with axes passing through or very close to the muon detector. The number of 'operating' counters was determined with the aid of a random-number table modified to make the average distri-

bution of the 'operating' counters correspond to the lateral distribution of the muons for a specified position of the shower axis. The lateral distribution of the muons near the shower axis ($r = 1-30\text{ m}$) was assumed to be $f(r) \sim r^{-0.7}$ [3]. The contribution of the showers with axes passing at a distance r from the center of the muon detector was determined from the formula

$$I(r) dr = \text{const} \cdot r [f(r)]^{\kappa'} dr, \quad (1)$$

where κ' —muon spectrum exponent of the shower, the value of which was shown by one of the authors [3] to be two. Figure 2c shows the D -distribution obtained from such a calculation. We see that the theoretical distribution, as in the preceding case, does not agree with the experimental one [the agreement probability is $P(\chi^2) < 10^{-2}\%$]. More events with $D < 2.5\text{ m}$ are observed experimentally than expected theoretically.

The experimental distribution for $R < 30\text{ m}$ can be readily understood by assuming that pairs or groups of muons, so connected with each other that the distances D for these muons cannot exceed a certain value d , meet at the center of the EAS (when $D < d$ these mesons are independently distributed). Then the muon pairs or groups for which d is larger than the characteristic dimensions of the array (in our case, when $d > 2.5\text{ m}$) yield a distribution that corresponds to independent muon trajectories. Meson pairs or groups for which $d < 2.5\text{ m}$ violate this condition and change the D -distribution.

The latter is possible only if the number M of the muons registered in the individual shower is not much greater than the number m_μ of the muons connected by the relation $D < d$. In the case when $M \gg m_\mu$ the distribution with respect to D

*The maxima of the distributions shown in Fig. 2, corresponding to distances D of 1–1.5, 3.5–4 and 4.5–5 meters, are connected with the geometry of the muon detectors (see Fig. 1b).

Table II

Type of event				Number of events
A	00	00	00	60, 62, 57
	-	0	00	
	00	00	00	
B	00	00		20, 12
	-	0		
C	000	000		16, 8
	-	0		
	00	00		
D	000	000	000	9, 22, 10
	-	0	00	
	000	000	000	
E	00	000		20, 17
	00	00		
	000	00		
F	00	00	000	2, 2, 1, 12
	000	000	000	
	00	000	00	
			000	

is determined completely by the distribution of the independent muons, and the possibility of separating pairs or groups of related muons is lost. We can therefore expect the experiments to disclose in this case a sharp dependence of the D-distribution on the number of registered muons.

Let us compare the D-distributions for different values of M . We choose as the characteristic of the D-distribution the ratio

$$\nu = I(D < 2.5 \text{ m}) / I(D > 2.5 \text{ m}).$$

For M ranging from 2 to 6, the experimental and theoretical values of ν (calculated for distributions obtained with allowance for the influence of the shower axis) are

M :	2	3	4	5	6
ν_e :	1.8 ± 0.3	1.9 ± 0.3	1.1 ± 0.1	1.3 ± 0.2	0.9 ± 0.1
ν_t :	0.7 ± 0.1	0.7 ± 0.1	0.8 ± 0.1	0.8 ± 0.1	0.9 ± 0.1

We see that the theoretical value ν_t changes little with M , whereas the experimental value ν_e decreases rapidly with increasing M . When $M \sim 6$, ν_e is close to its theoretical value.

Thus, the experimental D-distribution and the experimentally observed dependence of this distribution on the value of M show that pairs or groups of genetically related muons exist in the center of the EAS ($R < 30 \text{ m}$). The connection between them manifests itself in the experiment in the form of the condition $D < d$ imposed on the distribution of their trajectories in space.

The distances d can be quite small. Figure 2d shows the experimental and theoretical distributions of events with $D < 0.5 \text{ m}$. We see that the number of events with $D \leq 0.2 \text{ m}$ also exceeds the number of events expected theoretically for independent muon trajectories.

The "narrow" ($d \sim 0.2 \text{ m}$) pairs or groups of muons, unlike the "extensive" ones, can be observed directly in individual showers, by selecting events which theoretically have little likelihood (if the muons are independent). We consider below the totality of the data obtained on "narrow" pairs and groups of muons, with the aid of the detectors of Fig. 1.

Table II lists the types of events that can be classified as a narrow pair or group of muons, and their number, observed over the entire detector area of Fig. 1 in 2340 hours of operation with the array triggered by system 1. The events listed in Table II were selected from showers with a sufficiently small number M of registered muons, since it was clear beforehand that in showers with large M the events of Table II may be the result of statistical fluctuations in the distribution of the muon trajectories.

Table III shows the distribution of events of the type shown in Table II among showers having different M . The table lists also the number of these events expected to result from statistical fluctuations, calculated from the formula

$$I_{\text{expect}} = G(M) n^{-m} C_M^m (1 - n^{-1})^{M-m} \Phi, \quad (2)$$

where n —total number of counters in a given detector layer, M —total number of counters operated in this layer, $G(M)$ —experimental spectrum of the events with respect to M (see Table III), m —number of counters operated in the chosen layer in events of the type listed in Table II, and Φ —factor that accounts for the possible combinations of m counters for a given array; in our case $\Phi = [12 - (m - 1)] \times 16$. It follows from Table III that the number of events of type D and F of Table II cannot be attributed to statistical fluctuations.*

Such events can, in principle, be related to interactions of individual muons. The large number of observed events represented in Table II enables us to check this possibility quantitatively. Events of type D and F can be due to the following:

- the muon produces in the ground an electron-photon shower capable of penetrating through the detector screen,
- successive production of two electron-photon showers by the muon, one in the ground and the other in the detector screen,
- the muon produces in the ground an electron-nuclear (e.n.) shower penetrating through the de-

*It must be noted that Table III lists all the showers independently of the observed distance R , and therefore the "background" of random groupings is higher here than in the distribution a of Fig. 2.

Table III

M	m = 2 in central layer			m = 3 in lower layer			m = 3 in central layer		
	G (M)	I _{expect}	I _{exper}	G (M)	I _{expect}	I _{exper}	G (M)	I _{expect}	I _{exper}
2	1560	7	9						
3	980	13	17				980	0,02	1
4	525	14	15	1120	0,18	3	525	0,04	1
5	265	11	13	935	0,18	2	265	0,06	3
6	165	11	9	466	0,18	1	165	0,06	1
7	86	7	5	334	0,20	5	86	0,06	1
8	65	8	6	213	0,18	2	65	0,06	1
9	38	5	9	147	0,18	1	38	0,04	—
10—12	57	7	6	266	0,8	4	57	0,2	1
13—19	41	8	5	165	2	9	41	0,2	4
20—40				74	4	14	17	0,8	4

tector filter. When checking these three possibilities, it is particularly useful to have data on the operation of the counters of the central layer of the detector, which are separated by lead, since the operation of counters separated by lead during the passage of an electron-photon shower is much less probable than operation of counters not separated by lead.

A comparison of the events of type VII of Table I and events of type F of Table II (in which three counters in the middle layer operate in succession) shows that explanations a) and b) contradict the experimental facts.

Indeed, 17 events of type F in Table II and 29 events of type VII in Table I were observed altogether. The latter events can be regarded as electron-photon showers produced by the muon in the detector screen. According to explanation a) events of type F should be electron-photon showers generated in the ground, which have at the level of the middle counters the same number of particles as events VII. An estimate shows that the minimum energy of the electron-photon shower responsible for event VII should be $E_e \sim 10$ BeV. The maximum development of an electron-photon shower with $E_e \sim 10$ BeV occurs at a depth of 4 t units (radiation thicknesses). The detector screen between the upper and the middle layer of the detector is 26 t-units thick (lead + iron). Consequently the electron-photon showers responsible for the F events should be generated at a level approximately 22 t-units higher than the level where the electron-photon showers responsible for the type VII events are generated. The energy of the former showers should furthermore be about 100 times higher than that of the latter (see the cascade curve of Ivanenko and Samosudov^[4]). The energy spectrum of the electron-photon showers in equilibrium with the muon spectrum $F(> E_\mu) \sim E_\mu^{-0.5}$ (which was experimentally observed when triggering system 1, was used) should not be harder

than $F(> E_e) \sim E_e^{-0.5}$. Therefore, if explanation a) is correct, then the number of type F events should be at least 10 times smaller than the number of type VII events, which contradicts the experimental facts.*

Explanation b) presupposes that the type F event of Table II is obtained upon successive production of events of type V or VII of Table I by a single muon. We calculated the expected number of events of type F due to successive production of events of type V and VII, using the theoretical formulas for the production of δ -electrons, electron-positron pairs, and photon bremsstrahlung by a muon of specified energy E .^[5] These formulas were used to calculate the dependence of the probability of production of type V and VII events on the muon energy. The resultant probability was absolutely normalized to the observed number of these events for a definite type of energy spectrum of the registered muons. Two muon energy spectra were chosen:

$$F(E) dE = kE^{-1.5} dE \quad 10 \text{ BeV} \leq E \leq 10^5 \text{ BeV}, \quad (3)$$

$$F(E) dE = \begin{cases} k_1 E^{-1.5} dE & 10 \text{ BeV} \leq E \leq 10^3 \text{ BeV} \\ k_2 E^{-2.5} dE & 10^3 \text{ BeV} \leq E \end{cases} \quad (4)$$

[variant (3) corresponds to an extremely hard spectrum, which does not contradict the experimental data obtained with our apparatus in the range $E = 5-10$ BeV; see^[3]]. The muon energy spectrum was absolutely normalized to the total muon flux registered over the detector area during the operation of the array (see Table I).

Figure 3 shows the contribution of muons of different energies to the number of type V or VII events. By determining the probabilities of these

*Explanation a) also contradicts the fact that the number of observed events of type D in detectors with filters 16 and 32 t-units thick is approximately the same. Theoretically, explanation a) calls for the frequency of type D events to change at least by 10 times.

Table IV

Type of event	Number of showers with given N											
	$10^4 N$			$10^5 N$			$10^6 N$			$10^7 N$		
	1-3	3-6	6-10	1-3	3-6	6-10	1-3	3-6	6-10	1-3	3-6	6-10
D (Table II)	0	0	1	4	4	2	12	4	10	4	0	1
F (Table II)	0	0	0	3	2	0	6	1	2	3	0	0
VII (Table I)	0	0	2	9	8	4	4	3	0	0	0	0

events we were able to calculate the number of type F events expected to result from successive production of type V and VII events by a single muon; this is also shown in Fig. 3. The expected number of type F events is seen to be one-tenth the observed value.

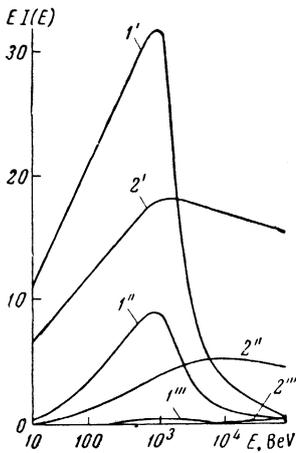


FIG. 3. Theoretically expected number of electron-photon showers produced by muons of energy E . 1—curves calculated for the muon spectrum given by (3), 2—the same for muon spectrum (4). Single prime denotes events of type V, Table I; double prime—events of type VII, Table I; triple prime—events of type F, Table II.

Explanation c) was also checked by comparing the number of type F events observed in the EAS with the number of the same events observed without the EAS. We selected the type F events by using the part of trigger system 2 connected with the middle counters (without requiring operation of the counters on the earth's surface). Their frequency was found to be $0.18 \pm 0.03 \text{ h}^{-1} \text{ m}^{-2}$. The intensity and the energy spectrum of the single muons in our underground chamber were investigated experimentally^[6]. Using the dependence of the effective cross section σ for the production of an electron-nuclear shower by a muon of energy E , namely $\sigma(E, E') \sim \ln(E/E') dE'/E'$, where E' is the energy transferred to the electron-nuclear shower^[7], we can obtain the theoretical frequency of the expected electron-nuclear shower in the single muon spectrum and normalize it to the experimentally observed frequency. Using the obtained absolute probability of occurrence of a type F event per muon of given energy, we can calculate the theoretically expected number of the same events in the EAS, for which the energy spectrum of the muons is taken in one of the variants given

above. The theoretically expected number of type F events turned out to be 72 ± 22 times less than that observed.

Finally, all the explanations in which the F and D events are attributed to muon interaction contradict the experimentally observed fact that these events are accompanied by extensive showers.

Table IV lists the distribution of the number of showers with N particles, accompanying events of type F and D, for the case when the array was triggered by system 1 (and for the same 2340 hours of operation). The same table shows for comparison the particle-number distribution for events of type VII of Table I. Assuming that the number of the considered events varies with the number of shower particles as N^β , and taking into account the efficiency of registering a shower with N particles by triggering system 1, we can obtain from Table IV the values of β for events types F, D, and VII:*

$$\beta_F = 1.5 \pm 0.3, \quad \beta_D = 1.5 \pm 0.2^\dagger, \quad \beta_{VII} = 0.8 \pm 0.2.$$

The values obtained for β apply in practice to value of N between 10^5 and 10^7 . Triggering system 2 made it possible to investigate the accompaniment of events of type VII and F in the region $N < 10^5$, where system 1 has a low efficiency of shower selection. After 1500 hours of operation of trigger system 2, 42 type VII and 2 type F events were obtained over an area of 2 m^2 in showers with $N = 10^4 - 10^5$ particles. Comparing these numbers with the number of the same events in showers with $N = 10^6 - 10^7$, obtained with the aid of system 1 (which selects showers with $N \geq 10^6$ accompanying the events F and VII with nearly 100% efficiency) we obtain $\beta_F = 1.5 \pm 0.2$ and $\beta_{VII} = 0.9 \pm 0.1$.

*The value of β was obtained by comparing the theoretically expected distributions and the experimental distributions of Table IV with the aid of the χ^2 criterion. The values of $\beta \pm \Delta\beta$ have a probability of 10%.

†It follows from Table III that some of the type D events can result from statistical fluctuations. A calculation of the values of β_D under the extreme assumption that all the D events connected with the statistical fluctuations are encountered in showers with $N > 10^6$ leads to a value $\beta = 1.3 \pm 0.2$.

Table V

\bar{p}_\perp		H, m							
		100	250	500	1000	3000	5000	10000	17000
Correspond to Coulomb scattering	$10^{-10} E_\pi, \text{eV}$	4,4	1,8	3,3	9,3	33	63	120	170
	n_π	30	15	20	45	75	100	100	90
$3 \cdot 10^8 \text{eV}/c$	$10^{-10} E_{n.a. \text{ min}}, \text{eV}$	40	20	66	400	$2,5 \cdot 10^3$	$6,3 \cdot 10^3$	$1,2 \cdot 10^4$	$1,5 \cdot 10^4$
	$10^{-10} E_\pi, \text{eV}$	12	30	60	120	360	600	$1,2 \cdot 10^3$	$2 \cdot 10^3$
$10^8 \text{eV}/c$	n_π	260	260	260	300	720	10^3	10^3	10^3
	$10^{-10} E_\mu, \text{eV}$	4	10	20	40	20	200	400	680
	$10^{-10} (E_{n.a. \text{ min}}), \text{eV}$	12	30	60	120	360	600	$1,2 \cdot 10^3$	$2 \cdot 10^3$

Thus, in the range of N from 10^4 to 10^7 the events of type F and VII are accompanied by different particle-number spectra in the shower. The dependence of the number of events of type VII on N has in this case the form $N^{0,8 \pm 0,2}$, in good agreement with the dependence of the number of muons on the number of particles in the shower (see^[3]). The N -dependence of the number of F and D events has the form $N^{1,5 \pm 0,2}$, indicating that such events are not the result of muon interaction.

It remains to assume that the type F and D events are due to the passage of a group of muons through the detector of Fig. 1. As shown above, the observed number of such events cannot be attributed to random accumulations of muons. Consequently, the muons observed in the groups should be genetically related.

For each shower in which an event of type F or D was observed, we know the total number of muons registered on the detector area and the total number of particles in the shower, N . This enabled us to estimate the distance r from the shower axis at which an event of type F or D is observed, using the known average lateral distribution function of the muon flux in the EAS (see^[3]). It was found that 33 out of 41 type D events were observed at distances $r < 10$ m while 8 were observed at distances $r > 10$ m. All 17 type F events were observed at $r < 10$ m. This r -distribution of the events again confirms that type F and D events are not connected with the statistical fluctuations in the distribution of the muon trajectories. Indeed, in the latter case the r -distribution should have the form (1) and the number of events considered at $r < 10$ m should be only one-third of the total number of events, in contradiction with experiment.

The rapid growth in the number of type F and D events with increasing number of particles in the shower causes several type F or D events to be expected for each shower with $N > 10^6$. In most showers with $N > 10^6$, however, experiment has shown not more than one type F or D event over the muon detector area. This means that the distances between the groups of muons in showers

with $N > 10^6$ are greater than the linear dimensions of the underground array (~ 3 m). In the cases when two or three type D or F events were observed in a single shower, the distances between them were actually quite large, ranging from 1 to 5.5 meters.

The foregoing results can be summarized as follows:

1. In the central part of the EAS ($r < 30$ m from the shower axis) the muon trajectories cannot be regarded as independent. Pairs and groups of interrelated mesons are present in this region.
2. A characteristic phenomenon in the central region of the EAS is the appearance of narrow muon groups or beams with dimension $d \sim 0.2$ m. The number of muon beams per shower increases rapidly with increasing number of particles in the shower. In showers with $N > 10^6$ particles several beams are encountered in a single shower and the distances between beams reach several meters.

DISCUSSION

In an earlier paper^[3] we considered different explanations for the existence of narrow muon beams. That analysis has demonstrated that the occurrence of such beams is connected with the following possibilities: 1) there is a small probability of nuclear interactions in which the multiplicity and distribution of the transverse momenta of pions experiencing $\pi \rightarrow \mu$ decay differ appreciably from the usual ones, namely the multiplicity is several times greater than usual and the mean value of the transverse momentum is one order of magnitude smaller than usual; 2) there is likewise a small probability of nuclear interactions in which muon production is either direct or the result of decay of some short-lived particles with lifetimes that are a few hundredths of the pion or K-meson lifetime; 3) interactions similar to 2) are realized, except that the cross section for the interactions responsible for the muon beam is close to the geometric cross section, but the interaction energy is greater than $10^{14} - 10^{15}$ eV.

Let us consider the first of these possibilities. Table V lists data on the minimum multiplicity

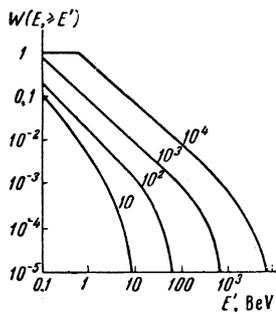


FIG. 4. Expected number of bursts with $n = E'/\beta$ particles, produced by a muon with energy E under thick layers of iron. The numbers on the curves represent the muon energy in BeV.

$n_\pi = 3W^{-1}$, necessary to produce narrow muon beams for different values of the average transverse momentum \bar{p}_\perp and level H of pion production (it is assumed here that the muon beam consists of at least three particles, and W denotes the probability of the $\pi \rightarrow \mu$ decay at a specified muon energy). We note that momenta p_\perp on the order of $\sim 3 \times 10^7$ eV/c are acquired by the muons by Coulomb scattering practically independently of H . For comparison we give the required multiplicity for the case when $\bar{p}_\perp = 3 \times 10^8$ eV. As can be seen from the table, such a multiplicity is improbable from the point of view of the conservation laws, even if we consider the production of secondary particles by a particle with an energy equal to the total shower energy ($E_0 \sim 10^{15} - 10^{16}$ eV).

The values of n_π enable us to determine the minimum energy of the nuclear-active particle responsible for the muon beam, using the formula $E_{n.a. \min} = n_\pi E_\pi$. The exact value of $E_{n.a.}$ is difficult to determine. On the one hand, it is clear that it should be greater than the value listed in Table V, since we disregard in the determination of $E_{n.a. \min}$ the creation of π_0 mesons or of secondary particles of any other energy. On the other hand, since the pion decay in the beam is random, the minimum number of pions producing a beam with a specified number of muons (say, three) can be also less than $3W^{-1}$. Since precise calculations presuppose furthermore a quantitative knowledge of the character of the fluctuations, we confine ourselves to the assumption that $E_{n.a.} = E_{n.a. \min}$. Under this assumption, starting from the experimental data on the energy spectrum of the nuclear-active particles in the EAS and the number of muon beams per shower, we can estimate the fraction α of the nuclear-interaction acts responsible for the muon beam. This fraction is found to be $\alpha < 10\%$.

Let us consider also the second possibility. In this case it is sensible to assume that $\bar{p}_\perp \approx 10^8$ eV. The values $E_{n.a. \min} = 3E_\mu$ listed in Table V for this case should be certainly increased by several times, owing to the creation of π_0 and π^\pm mesons.

It is therefore reasonable to assume the same α as in the preceding case, i.e., on the order of several per cent. We note that these estimates of α do not contradict the numerous experimental data obtained with emulsions and cloud chambers regarding nuclear interactions in the energy range $10^{11} - 10^{13}$ eV.

Cases 1) and 2) have several common features. First, in either case it is natural to expect the existence of not only narrow groups of muons ($d \sim 0.2$ m) but also groups of large dimensions. Second, muon groups of a given dimension can result from interaction between nuclear-active particles at different levels H , and the mean energy of the particles in the group can thus fluctuate from case to case over a wide range (although we have at the same time $E_\mu \sim d^{-1}$). This distinguishes cases 1) and 2) from case 3), in which E_μ should always be large and correspond to the level at which the shower was initiated.

Apparently, cases 1) and 2) should differ the most in the predicted dependence of the number of meson groups per shower on the group dimension d . It is qualitatively clear that the connection between the dimension of a muon group and the energy $E_{n.a.}$ is not the same in cases 1) and 2). If the level where the meson group is produced is fixed, we have in case 1) $E_{n.a. \min} = m_\mu W^{-1} E_\pi$ (m_μ is the number of mesons in the group), with W approximately proportional to E_π^{-1} and $E_\pi \sim d^{-1}$, and thus $E_{n.a. \min} \sim d^{-2}$. In case 2) we obtain $E_{n.a. \min} = m_\mu E_\mu$ and $E_\mu \sim d^{-1}$, and consequently $E_{n.a. \min} \sim d^{-1}$.

The energy spectrum of the nuclear-active particles in an extensive air shower near sea level has the form $F(> E_{n.a.}) \sim E_{n.a.}^{-1}$. Knowing the energy spectrum of the nuclear-active particles and the dependence of $E_{n.a.}$ on d , we obtain the dependence of the number of muon groups per shower on the group dimension in the form $F(\leq d) \sim d^2$ in case 1) and $F(\leq d) \sim d$ in case 2).

The obtained variation of $F(d)$ can change somewhat with the level H . An exact calculation of $F(d)$ calls for a refinement of the initial assumptions concerning the character of the fluctuations of the average transverse momentum of the secondary particles in the nuclear interaction acts. We confine ourselves only to the estimate given above for $F(d)$.

In order to distinguish between possibilities 2) and 3) it is necessary to measure at least the average meson energy in the beams. For this purpose we can measure in ionization chambers the spectrum of the bursts produced by the muon

beams. Figure 4 shows the spectrum of the burst expected from muons of different energies. As seen from Fig. 4, one can separate sufficiently accurately mesons with $E_\mu > 10^{12}$ eV from mesons with $E_\mu < 10^{11}$ eV. Such an experiment calls for an array with rather large area (~ 10 m²), in view of the low intensity of the muon beams.

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