not small for $kr \sim r/\delta \sim 1$ alone, since the spatial dispersion is weak in normal skin-effect* as a result of the inequality $kr \ll 1$ and it is therefore possible to expand $\omega(k)$; in anomalous skin effect, on the other hand, resonance is possible only in the magnetic field parallel to the metal surface, when $\bar{v}_z = 0$ and ω is finite as $k \to \infty$ (see [1]), so that $\omega(k)$ can be expanded in powers of $1/kr$.

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*In diamagnetic resonance in an inclined field, the Doppler effect produces a supplementary attenuation with $1/\tau_{eff} \sim \omega_1$ (quadratic dispersion or resonance for nonquadratic dispersion on a section off center) or $1/\tau_{eff} \sim \omega_1(\omega_1/\omega)$ (resonance on a central section).

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THE COLLAPSE OF A SMALL MASS IN THE GENERAL THEORY OF RELATIVITY

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A calculation of the equilibrium of a cold ideal Fermi gas in its own gravitational field made by Volkoff and Oppenheimer [1,2] led to the following result: for a small number of neutrons ($N < 0.35 \odot$) there is a single solution, for $0.35 \odot < N < 0.750 \odot$ there are two solutions, and for $N > 0.75 \odot$ there is no solution at all (the symbol $\odot$ here means the number of neutrons in the sun).

It was assumed that the unique solution for $N < 0.35 \odot$ is absolutely stable and that for such a value of N collapse is impossible. We shall show that this is not true.

By prescribing a sufficiently large density we can obtain for any given number N of particles a configuration with mass as close to zero as we please, and clearly less than the mass of the static solution. Such a configuration obviously cannot go over into the state of equilibrium (into the static solution), and consequently can only contract without limit.

Let us take an arbitrary spherically symmetrical distribution of motionless matter. We denote the particle density by n and the energy density by ϵ (ϵ includes the rest mass of the particles); n and ϵ are connected by the equation of state.

The metric is given by the expression (we everywhere set $c = 1$)

$$ds^2 = e^\nu dt^2 - e^\lambda dr^2 - r^2(\sin^2\theta\, d\varphi^2 + d\theta^2). \tag{1}$$

As is known from the equation for λ (cf. [3]) it follows that

$$e^{-\lambda(r)} = 1 - \frac{b}{r}\int_0^r \varepsilon(r) r^2 dr, \tag{2}$$

where $b = 8\pi k$. The mass of the star is given by the expression

$$M = 4\pi \int_0^\infty \varepsilon(r) r^2 dr, \tag{3}$$

and the number of particles by ($d\omega$ is an invariant volume element)

$$N = \int n\, d\omega = 4\pi \int_0^\infty n(r) e^{\lambda/2} r^2 dr. \tag{4}$$

Let us take the distribution of motionless matter given by the formulas

$$\varepsilon = a/r^2, \quad r < R; \quad \varepsilon = 0, \quad r > R. \tag{5}$$

Then

$$e^{-\lambda} = 1 - ab, \quad r < R, \quad e^{-\lambda} = 1 - abR/r, \quad r > R. \tag{6}$$

$$M = 4\pi aR, \quad N = \frac{4\pi}{\sqrt{1-ab}}\int_0^R nr^2\, dr. \tag{7}$$

For an ultrarelativistic gas

$$\varepsilon = \hbar(3/\pi^2)^{1/3} n^{4/3}, \quad n = (\pi^2/3)^{1/4}(\varepsilon/\hbar)^{3/4}. \tag{8}$$

Substituting Eqs. (5) and (8) in Eq. (7), we get

$$N = \text{const}\; a^{3/4}R^{3/2}/\sqrt{1-ab},$$
$$R = \text{const}\; N^{2/3}a^{-1/2}(1-ab)^{1/3}, \quad M = \text{const}\; N^{2/3}a^{1/2}(1-ab)^{1/3}. \tag{9}$$

It follows from this that $M \to 0$ for $a \to 1/b$, whatever the value of N.* This proves the assertion made above.

For a rough estimate of the energy barrier which separates the equilibrium solution with $M \le Nm$ (m is the mass of the neutron) from the collapsing state, let us find the maximum M from Eq. (9). We get

$$M_{max} \approx N^{2/3}\sqrt{\hbar/k}, \quad M_{max}/Nm \sim N^{-1/3}\sqrt{\hbar/k}/m \approx N/N_{cr}, \tag{10}$$

where mN_{cr} is of the order of the maximum mass for which a solution exists, i.e., of the order of the mass of the sun. Consequently for systems consisting of a small number of neutrons collapse may indeed be possible, but the height of the barrier is many times larger than the initial rest energy of the system. Since the barrier $\sim N^{2/3}$, its absolute value decreases (although the required density increases) when part of the body in question is compressed. All of the conclusions remain qualitatively unchanged when one takes interaction between the neutrons into account, and in particular even for the equation of state $\epsilon \sim n^2$, which is the most rigid relation consistent with the theory of relativity.[4]

In the use of the expressions (1)—(4) it is not assumed that $n(r)$ and $\epsilon(r)$ with zero velocity, $v = 0$, correspond to the static solution; the field equations give nonvanishing values of $\dot{\lambda}$, $\dot{\nu}$, $\dot{v}$, where the dot means differentiation with respect to time. Outside the body $(r > R)$ we have $\dot{\lambda} = 0$, so that the mass M measured from the external gravitational field remains unchanged during the process of evolution which ensues for a prescribed initial distribution which does not satisfy the conditions for equilibrium.

The distribution (8) used for the proof has singularities: $\epsilon \to \infty$ for $r = 0$; ϵ has a discontinuous change from a/R^2 to 0 at $r = R$. It is easy to verify, however, that the result is not changed when one smooths out these singularities, for example by replacing Eq. (5) by

$$\varepsilon = a/\alpha^2 R^2 \text{ for } r < \alpha R; \quad \alpha \ll 1,$$
$$\varepsilon = \frac{a}{r^2}\frac{R(1+\beta)-r}{2\beta R}, \quad R(1-\beta) < r < R(1+\beta); \quad \beta \ll 1,$$
$$\varepsilon = a/r^2, \quad \alpha R < r < R(1-\beta). \tag{11}$$

In the initial distribution (5) used in our argument, and also in the smoothed distribution (11) we have everywhere $e^{-\lambda} > 0$, $e^{\nu} > 0$, i.e., the metric is not singular and there are no difficulties of the sort associated with the Schwarzschild singularity $(e^{\lambda} \to \infty,\ e^{\nu} = 0)$.

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*For small a one must not use the ultrarelativistic equation (8). For $a \to 0$, the mass $M \to Nm$.

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FERROELECTRIC ANTIFERROMAGNETICS

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THE discovery of complex ferroelectrics with perovskite structure and with a considerable number of Fe^{3+} ions at octahedral sites prompted the suggestion that some of these ferroelectrics have antiferromagnetic properties.[1]

This suggestion was studied in the case of $Pb(Fe_{2/3}W_{1/3})O_3$ and $Pb(Fe_{1/2}Nb_{1/2})O_3$; the bracketed ions were at the octahedral positions. X-ray diffraction at room temperature showed no ordering of the ions at the octahedral sites, i.e., these compounds were disordered solid solutions based on orthoferrites.

We investigated the electrical and magnetic properties of monocrystals of these compounds. The monocrystals were grown from a solution in molten lead oxide by spontaneous crystallization on cooling. Chemical analysis showed that the compositions of the two compounds corresponded to the specified chemical formulas.

Electrical properties were measured on thin monocrystals and magnetic properties on powders of fine monocrystals, because large crystals were not obtained. The results are shown in the figure. Magnetization of both compounds was a linear function of the magnetic field intensity (H_{max} = 8000 Oe). No residual magnetic moments were found throughout the temperature interval used in the tests.

The ferroelectric phase-transition temperatures, Θ_c, were determined approximately from the maxima of ϵ: they were 178°K for $PbFe_{2/3}W_{1/3}O_3$ and 387°K for $PbFe_{1/2}Nb_{1/2}O_3$. In these two compounds, as in the majority of solid solutions, phase transitions from the paraelectric into the ferroelectric state occurred over a range of temperatures. The paramagnetic-antiferromagnetic phase transitions also occurred over a range of temperatures. The curves representing $\chi(T)$ and $\chi^{-1}(T)$ had kinks at 363°K for $PbFe_{2/3}W_{1/3}O_3$ and at 143°K for $PbFe_{1/2}Nb_{1/2}O_3$; these kinks were assumed to represent the antiferromagnetic transitions. Similar dependences have been reported for antiferromagnetic crystals of CrSe[2] and for the antiferromagnetic solid solutions $Mn_{1-x}Mg_xO$,[3] in which