SOVIET PHYSICS JETP VOLUME 15, NUMBER 2 AUGUST, 1962

VERTICAL FOCUSING IN THE MICROTRON

V. N. MELEKHIN

Institute for Physical Problems, Academy of Sciences, U.S.S.R.

Submitted to JETP editor September 22, 1961

J. Exptl. Theoret. Phys. (U.S.S.R.) **42**, 622-624 (February, 1962)

We give the results of computations of the vertical motion in a microtron for various shapes of the transit holes in the accelerating cavity. It is shown that replacing the circular holes by slots may result in considerable improvement in the vertical focusing.

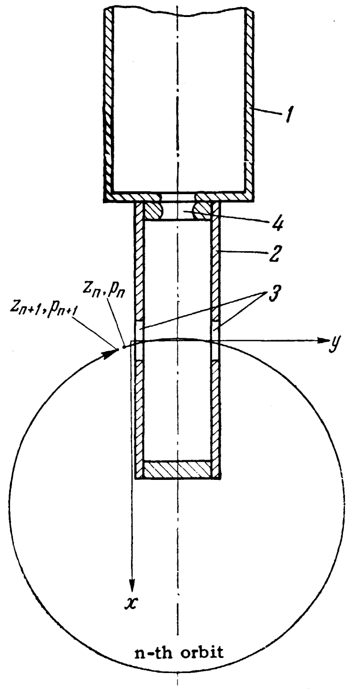
In constructing large microtrons, at energies of several times ten MeV, one encounters difficulties in maintaining vertical focusing. Computations show[1,2] that as the energy of the accelerated electrons increases, the vertical focusing properties of the accelerating cavity become poorer. For this reason, in the 29-MeV microtron,[2] an inhomogeneous magnetic field had to be used to guarantee vertical stability, although this was extremely undesirable because it interfered with the phase stability.

These difficulties are typical for resonators with circular flight holes. We shall show that changing the shape of the holes can significantly improve the focusing properties of the resonator and thus solve the problem of vertical focusing at high electron energies.

The figure shows a section of the microtron through the median plane and exhibits one of the orbits. In the vicinity of the flight holes, the vertical component E_z of the accelerating electric field acts on the electron, while the vertical force inside the cavity comes from the horizontal component H_x of the variable magnetic field. These forces determine the vertical motion.

It is not difficult to see that near the axis of a circular aperture one can approximate the field E_z by $E_z = -\frac{1}{2} z \, \partial E_y/\partial y$, whereas for a horizontal slot, $E_z = -z \, \partial E_y/\partial y$. Consequently a horizontal slot focuses the electrons twice as strongly as the circular hole. On the other hand a vertical slot has no effect on the focusing, since $E_z = 0$ near it.

Keeping these properties of slots in mind, we shall compute the vertical focusing. In making the computation, it is assumed that the region over which the inhomogeneous electric field extends in the neighborhood of the slots is small compared to the width of the resonator. This condition is satisfied for accelerating cavities of the new types.[3,4] because they are quite wide; inside the resonators themselves, we include the influence of the components E_y and H_x on the motion of the electrons.



Section of microtron through the median plane: 1 – waveguide, 2 – accelerating cavity, 3 – flight holes, 4 – coupling holes.

Let z_n and p_n be the vertical coordinate and momentum of the electron at its entrance into the resonator on the n-th orbit, and z_{n+1} and p_{n+1} the values of coordinate and momentum at this same point after the electron has completed one full turn (see the figure). We introduce the transformation matrix $\{a_{ik}\}$:

$$\begin{pmatrix} z_{n+1} \\ p_{n+1} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} z_n \\ p_n \end{pmatrix}. \qquad (1)$$

From the theory of such matrices[5] it follows that, for $-2 < S < 2$, $\operatorname{Det} a_{ik} = 1$, the motion described by formula (1) is a stable oscillation of the form $z_n = b \cos(\nu n + \delta)$, where the frequency ν is given by the relation

$$\cos \nu = S/2.$$

Here $S = a_{11} + a_{22}$ is the trace of the matrix a_{ik}. If $|S| \geq 2$, the motion is unstable, except for the case of the unit matrix.

The computation, dropping terms of order n^{-2} and higher, leads to the following expressions for the a_{ik}:

$$a_{11} = 2 + (2\pi c/eH)\, a_{21} - a_{22}, \qquad a_{12} = (2\pi c/eH)\, a_{22},$$

$$a_{21} = \eta' - \eta'' + \frac{1}{2n}\left[\eta' + \eta'' - \frac{2\omega L}{eH}\eta'\eta'' - \frac{eE^2\omega}{2cH}\int_{t'}^{t''} \cos^2\omega t \cdot dt\right],$$

$$a_{22} = 1 + \frac{1}{2n}\left(1 - \frac{2\omega L}{eH}\,\eta''\right). \qquad (2)$$

Here $E \cos \omega t$ is the accelerating electric field, H the intensity of the constant magnetic field, L the resonator thickness, e the absolute value of the electron charge. The values of all quantities are in absolute units. The symbol η denotes:

$$\eta = (eE/2c)(1 - 2\alpha) \cos \omega t,$$

where the parameter α is determined by the shape of the hole. We use a prime for quantities referring to the time of entrance and a double prime for those at the time of emergence from the resonator.

A horizontal slot gives the value $\alpha = 1$, while a vertical slot has $\alpha = 0$. In the case of a combination of two circular holes, $\alpha' = \alpha'' = \frac{1}{2}$, formula (2) simplifies and, for $H = mc\omega/e$, coincides with the expressions given previously.[1] In this case,

$$S = 2 - \frac{\pi E^2 \omega}{2n H^2}\int_{t'}^{t''} \cos^2 \omega t \cdot dt.$$

We see that on the later orbits the stability of the motion is reduced, since $S \to 2$. The oscillation amplitude increases like $n^{1/4}$.[1] Another difficulty is the following. As the computation shows, a tilt of the resonator axis at an angle θ from the horizontal plane results in a vertical drift of the electron beam from the center of the hole, proportional to θn. As a consequence, there is a rapid loss of particles in the acceleration process even for very small values of θ.[2]

Let us see the effect of a change in shape of the holes. Omitting terms of order $1/n$, which is permissible on the later orbits, i.e., for high energy electrons, we find from formulas (2),*

$$S = 2 + \pi\ [1 - (\alpha' + \alpha'')]\ \mathrm{tg}\,\varphi_s + \pi(\alpha'' - \alpha')\ \mathrm{ctg}\ (\omega L/2c). \qquad (3)$$

Here φ_s is the equilibrium phase, whose value can lie within the following limits: $0 < \varphi_s < \tan^{-1} 2/\pi$, where the optimum equilibrium phase is $\varphi_{opt} = \tan^{-1} 1/\pi$.[6]

In the case of circular holes, $S = 2$, giving our previous conclusion that the motion is unstable. If however $\alpha' = \alpha'' = 1$ (two horizontal slots), then $S = 1$, and there is a stable nonincreasing oscillation with a period of six revolutions. Here a tilt of the resonator axis gives a small constant shift of the electron beam relative to the symmetry plane of the resonator (by an amount of order $\lambda\theta/2$).

The computations show that a combination of horizontal slots guarantees not only vertical but also horizontal stability. Furthermore, such a hole shape allows for the horizontal drift of the orbits which arises because of magnetic field inhomogeneities and radiation reaction, and also permits one to change the mode of operation and, consequently, change the electron energy at the output of the resonator over wide limits.[4]

Using a cavity with horizontal slots in the microtron should certainly solve the problem of stable acceleration of electrons up to energies of several tens of MeV. At low energies this conclusion has been confirmed in preliminary experiments made on the microtron of the Academy of Sciences Institute for Physical Problems.[4]

In conclusion we mention the following point. If $\alpha' = 1$ and $\alpha'' = 0$, i.e., if there is a horizontal slot at the cavity entrance and a vertical slot at the exit, then we find from formula (3), $S \approx -3$, i.e., cross-focusing occurs. Then the vertical coordinate changes sign at each orbit and the oscillation amplitude increases rapidly. This system is unsuitable for the microtron; possibly, however, the use of crossed slots or a sequence of slots of some other orientation may enable one to get strong focusing in linear accelerators or other electron-optical systems.

I take this opportunity to express may profound gratitude to Academician P. L. Kapitza for his interest in this work, to S. P. Kapitza for directing it, and to L. A. Vaĭnshteĭn and V. P. Bykov for valuable discussions.

---

*tg = tan, ctg = cot.

[1] J. S. Bell, Proc. Phys. Soc. (London) **B66**, 802 (1953).

[2] Aitken, Heymann, Jennings, and Kalmus, Proc. Phys. Soc. (London) **77**, 769 (1961).

[3] Kapitza, Bykov, and Melekhin, JETP **39**, 997 (1960), Soviet Phys. JETP **12**, 693 (1961).

[4] Kapitza, Bykov, and Melekhin, JETP **41**, 368 (1961), Soviet Phys. JETP **14**, 266 (1962).

[5] P. A. Sturrock, Static and Dynamic Electron Optics, Cambridge University Press, 1955.

[6] A. A. Kolomenskii, ZhTF **30**, 1347 (1960), Soviet Phys.-Tech. Phys. **5**, 1278 (1961).

Translated by M. Hamermesh
94