SOVIET PHYSICS JETP VOLUME 15, NUMBER 2 AUGUST, 1962

DETERMINATION OF THE AMPLITUDES OF BETATRON AND SYNCHROTRON OSCILLATIONS OF ELECTRONS BY HIGH-SPEED MOTION PICTURE PHOTOGRAPHY

A. G. ERSHOV

Moscow State University

Submitted to JETP editor September 15, 1961

J. Exptl. Theoret. Phys. (U.S.S.R.) **42**, 606-609 (February, 1962)

Results of a theoretical calculation of the axial and radial distributions of the electron intensities over the cross section of a bunch are given. A method is presented which can be employed to determine the betatron and synchrotron oscillation amplitudes from the experimental intensity distributions. The method is based on the above-mentioned calculation. Some new data regarding the variation of the radial oscillation amplitudes in the C-60 synchrotron of the Academy of Sciences Physics Institute during the acceleration process are presented.

THE main difficulty in the investigation of betatron and synchrotron oscillations of electrons in a synchrotron by high-speed motion-picture photography[1] lies in determining the oscillation amplitudes from the distributions of the electron radiation brightness over the cross section of the bunch. To develop a method of determining the amplitudes, the brightness distribution was calculated theoretically for the two most important electron amplitude distributions, namely for a Rayleigh-Gauss distribution that is stationary, and for a Rayleigh-Gauss distribution that oscillates with a certain amplitude.

The first case was assumed to correspond to betatron and synchrotron oscillation amplitude distributions arising during injection and at the instant of synchrotron acceleration, or resulting from quantum excitation of these oscillations. We calculated the brightness distribution produced by electrons oscillating as

$$x = a \sin \omega t, \tag{1}$$

with amplitude distributions

$$p(a)da = (2N/\sigma^2)\, a \exp(-a^2/\sigma^2)\, da, \tag{2}$$

where x is the coordinate of the electron at the instant t, a is the amplitude, ω is the circular frequency of oscillation, N is the number of electrons, and $\sigma^2 = a^2$ is twice the value of the dispersion, equal to the mean square of the amplitude. It was assumed in the calculation that all the radiation emitted by each electron is concentrated within a narrow cone with the axis perpendicular to the oscillation direction and parallel to the observation direction.

The following expression was obtained for the relative brightness

$$B(u) = \exp(-u^2), \tag{3}$$

where $u = x/\sigma$. A plot of B(u) is shown in Fig. 1.

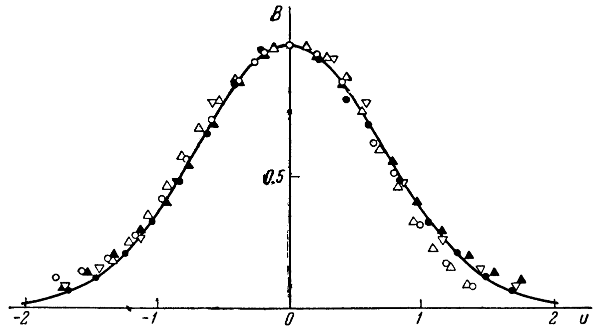

FIG. 1. Relative brightness for Rayleigh-Gauss distribution. Experimental axial distributions of the brightness $B(z/\sigma_z)$: △ – t = 0.146 sec; ▽ – t = 0.415 sec; ▲ – t = 0.605 sec. Radial distributions $B(r/\sigma_r)$: ○ – t = 0.144 sec, ● – t = 0.605 sec. Continuous curve – theoretical distribution B (u).

Formula (3) leads to the important conclusion that the half-width of the brightness distribution at the 1/e level is equal to the mean-square amplitude

$$0.5\,\Delta x_{0.368} = \sigma = (\overline{a^2})^{1/2}, \tag{4}$$

where Δx is the distribution width.

To check this conclusion, we compared the theoretical distribution B(u) with the experimental one. For this purpose we used the axial and radial distributions of the relative brightness, $B(z/\sigma_z)$ and $B(r/\sigma_r)$, obtained in investigations of electron motion in the C-60 synchrotron of the Academy of Sciences Physics Institute. The agreement between the experimental points and the theoretical curve was satisfactory over the entire investigated part of the acceleration cycle, except for the radial distributions of the brightness in the interval from the interception to the center of the acceleration cycle (see Fig. 1). This enabled us to conclude that the calculation is correct and the bunch electrons have an amplitude distribution close to the Rayleigh-Gauss distribution (except for the indicated interval). The detailed connection between the half-widths and the amplitudes can be written for our case in the form

$$0.5\,\Delta z_{0,368} = (\overline{a^2_{zb}} + \overline{a^2_{zb_q}})^{1/2}, \qquad (5)$$

$$0.5\,\Delta r_{0,368} = (\overline{a^2_{rb}} + \overline{a^2_{rs}} + \overline{a^2_{rb_q}} + \overline{a^2_{rs_q}})^{1/2}, \qquad (6)$$

where the index z stands for the amplitudes of the axial oscillations, r for the radial ones, b for the betatron oscillations, s for the synchrotron oscillations, and q for the oscillations due to quantum fluctuations of the radiation.

The brightness distribution for the second case of electron amplitude distribution was calculated for the simplified model of electron radial oscillations following interception. In addition to the foregoing assumptions (1) and (2) concerning the character of the electron oscillations, it is also assumed in this case that the Rayleigh-Gauss distribution is itself subject to harmonic oscillations given by

$$x_c = a_s \sin \Omega t, \qquad (7)$$

where x_c is the coordinate of the distribution center, a_s is the amplitude of the synchrotron oscillations of the bunch, which arise during interception, and Ω is the frequency of these oscillations.

The numerical results were used to plot the relative-brightness distribution for $a_s/\sigma_r = 0.2$, 0.4, . . ., 2.2. An analysis of the resultant curves enabled us to establish a connection between the brightness distributions observed after interception, the amplitudes a_s of the synchrotron oscillations, and the summary mean-square amplitude of the radial oscillations, σ_r. The principal results of the analysis are plotted in Fig. 2.

The values of a_s and σ_r are determined from these plots. Curves a, b, and c are first used to determine a_s/σ_r for the experimental brightness distribution, after which curves d and e are used to determine $a_s + \sigma_r$ and hence a_s and σ_r. To check the correctness of this method, the theoretical and experimental brightness distributions corresponding to the same ratios a_s/σ_r were compared and found to be in sufficiently good agreement. By way of an example, Fig. 3 shows one of the experimental radial brightness distributions after interception, compared with the corresponding theoretical curve.

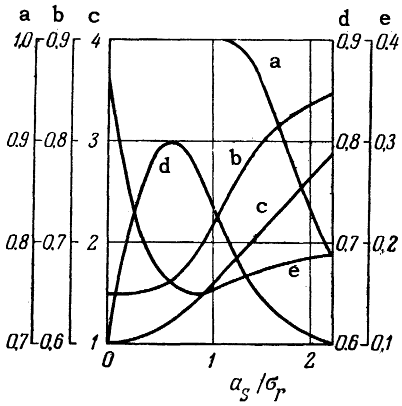

FIG. 2. Plots for determination of a_s and σ_r: a—B(0); b—$\Delta r_{0.6}/\Delta r_{0.3}$; c—0.5 $\Delta r_{0.368}/\sigma_r$; d—$(a_s + \sigma_r)/\Delta r_{0.5}$; e—$B(a_s + \sigma_r)$.

The procedure developed for determining the amplitudes was used to make more precise the experimental results obtained earlier in the investigation of electron motion in the C-60 synchrotron.[1] The shortcoming of the earlier results was that they did not yield the mean-square amplitudes necessary for a rigorous comparison with theory, nor did they make it possible to separate the synchrotron oscillations arising during interception from the previously existing betatron oscillations, so that the experimental damping of the radial betatron and synchrotron oscillations could not be determined in the interval between interception and the middle of the acceleration cycle. The use of the new procedure made it possible to overcome the foregoing difficulties.

By using the results of the new calculation method we can readily obtain the mean-square amplitudes of the axial betatron oscillations from earlier experimental results. To determine these from the plots of [1], one must multiply by 0.455 the ordinates of the plotted widths of the axial distributions at the 0.3 level. The course of these curves does not change in this case, and all the conclusions made in these investigations regarding the axial oscillations remain valid.

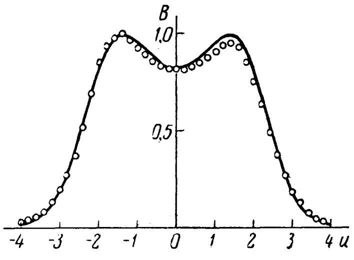

FIG. 3. Relative radial brightness for t = 0.215 sec. Points — experimental distribution from which a_s/σ_r = 2.0 was obtained. Curve — corresponding theoretical distribution.

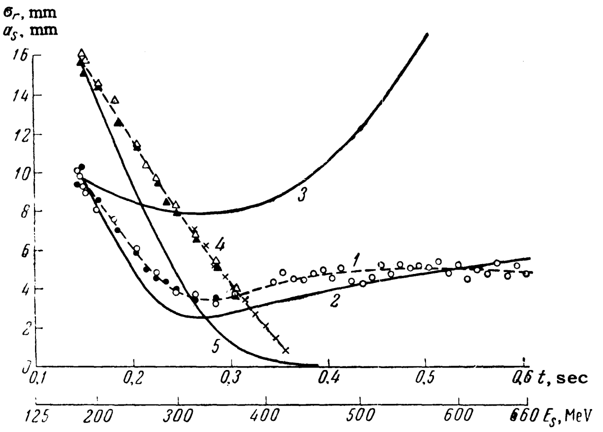

FIG. 4. Curves showing variation of the amplitudes of the radial oscillations: 1 – experimental curve of summary mean-square amplitude; 2 – theoretical curve of summary amplitude with account of radiation damping; 3 – curve without account of damping; 4 – experimental curve showing variation of the amplitudes of the synchrotron oscillations arising during interception; 5 – theoretical damping curve of the synchrotron oscillations.

The analysis of the radial brightness distributions yielded essentially new results. It became possible for the first time to plot experimentally the damping of the synchrotron oscillations arising in the bunch after interception (curve 4 of Fig. 4). Curve 5 of Fig. 4 shows the theoretical variation of these oscillations in accordance with the following formula (see [2])

$$a_{rs} \sim V^{1/4} E_s^{-3/4} \exp\left(-\frac{1}{2}\frac{3-4n}{1-n}\int\limits_0^t \frac{W_s}{E_s}\,dt\right). \qquad (8)$$

For the theoretical curve, the amplitude of the synchrotron oscillations at the instant of interception was chosen equal to the experimental value $a_s = 16.0$ mm. A comparison of the curves shows that the experimental curve attenuates more slowly than the theoretical one. A possible reason is that the theoretical curve corresponds to damping of synchrotron oscillations with low amplitudes, whereas the amplitudes of the bunch oscillations can apparently assume large values during interception, and their damping is not given by formula (8).

It was further possible to obtain a curve showing the variation of the summary mean-square amplitude of the radial oscillations, from which the experimental damping of the radial betatron oscillations after interception was obtained. It was also possible to determine the build-up of betatron and synchrotron oscillations at the end of the acceleration cycle (curve 1 of Fig. 4). A comparison of the experimental curve with the corresponding theoretical curve plotted with account of radial damping (curve 2, cf. [2]) and without account of the damping (curve 3, cf. [3,4]) has shown good agreement in the former case and great discrepancy in the latter.

The author is most grateful to A. N. Lebedev for valuable advice and useful discussions.

---

[1] Korolev, Ershov, and Kulikov, DAN SSSR **134**, 314 (1960), Soviet Phys.-Doklady **5**, 1011 (1961). Korolev, Ershov, and Kulikov, JETP **40**, 1644 (1961), Soviet Phys. JETP **13**, 1158 (1961).

[2] A. A. Kolomenskii and A. N. Lebedev, Uskoriteli élementarnykh chastits (Accelerators for Elementary Particles) Supplement 4 to Journal Atomnaya énergiya (Atomic Energy), 1957.

[3] A. A. Sokolov, Vvedenie v kvantovuyu élektrodinamiku (Introduction to Quantum Electrodynamics), Fizmatgiz, 1958.

[4] A. A. Sokolov and I. M. Ternov, DAN SSSR **117**, 967 (1957), Soviet Phys.-Doklady **2**, 573 (1958).

Translated by J. G. Adashko
92