THEORY OF FLUCTUATIONS OF THE PARTICLE DISTRIBUTIONS IN A PLASMA

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We obtain expressions for the correlations in the phase densities in different points in phase space and at different times for a non-equilibrium plasma. We use the general formulae to obtain expressions for the field correlations, the charge density, the particle-distribution correlations, and the charge-density correlations. We consider the case where the plasma is in a constant uniform magnetic field.

INTRODUCTION

RECENTLY attention has been drawn to the problem of constructing a theory of fluctuations in a plasma which is not in thermodynamic equilibrium. Hubbard,[1] Rostoker,[2] and one of the authors[3] have considered the theory of fluctuations in the electromagnetic field in a collisionless plasma. In [1,2] only the Coulomb interaction was taken into account, while in [3] the fluctuations in the total electromagnetic field were considered.*

The correlations of currents and the effective temperature for stationary states of a non-equilibrium plasma were evaluated by Bunkin,[4] who assumed that the collisions were the decisive factor.

In the present paper we evaluate the correlations

$$\overline{\delta N_\alpha(\mathbf{r}_\alpha, \mathbf{p}_\alpha, t)\, \delta N_\beta(\mathbf{r}_\beta, \mathbf{p}_\beta, t')} \tag{1}$$

of the phase density functions

$$N_\alpha = \sum_i \delta(\mathbf{r}_\alpha - \mathbf{r}_{\alpha i}(t))\, \delta(\mathbf{p}_\alpha - \mathbf{p}_{\alpha i}(t))$$

for quasi-equilibrium states of a collision-free plasma. The quasi-equilibrium condition means that the averages of the functions N_α change little over distances of the order of the correlation radius and times of the order of the correlation time.

We solve this problem by a method developed in the papers of one of the authors[5] in connection with the analogous problem for equilibrium states of a plasma. Kadomtsev[6] has also used equations for the phase density to study fluctuations in gases.

The general formulae obtained here are used to determine density correlations, the field, and correlations in the particle distribution and in the charge density distribution. We consider both the case of a plasma without strong fields and the case where the plasma is in a constant, uniform magnetic field.

*In [3] there was also given a quantum theory of electromagnetic fluctuations; this is important for fine-grained correlations.

1. SOLUTION OF THE SET OF EQUATIONS FOR THE FUNCTIONS $N_\alpha(\mathbf{r}_\alpha, \mathbf{p}_\alpha, t)$

We use as initial equations the set of equations for the functions

$$N_\alpha(\mathbf{r}_\alpha, \mathbf{p}_\alpha, t) = \sum_i \delta(\mathbf{r}_\alpha - \mathbf{r}_{\alpha i}(t))\, \delta(\mathbf{p}_\alpha - \mathbf{p}_{\alpha i}(t))$$

which are the charged-particle densities in phase space. The index α corresponds to the different kinds of charged particles in the plasma. When there are no average fields and we are dealing with charged particles with Coulomb interactions the equations for the functions N_α are of the form

$$\frac{\partial N_\alpha}{\partial t} + \mathbf{v}_\alpha \frac{\partial N_\alpha}{\partial \mathbf{r}_\alpha} - \sum_\beta \int d\mathbf{r}_\beta d\mathbf{p}_\beta \frac{\partial U_{\alpha\beta}(|\mathbf{r}_\alpha - \mathbf{r}_\beta|)}{\partial \mathbf{r}_\alpha} N_\beta \frac{\partial N_\alpha}{\partial \mathbf{p}_\alpha} = 0. \tag{1.1}$$

The averages of the functions N_α are proportional to the first distribution functions f_α. Denoting an average by a bar on top, we write

$$\overline{N}_\alpha(\mathbf{r}_\alpha, \mathbf{p}_\alpha, t) = n_\alpha f_\alpha(\mathbf{r}_\alpha, \mathbf{p}_\alpha, t).$$

Here n_α is the average number of particles of kind α per unit volume.

We define the deviations δN_α of the functions from their averages

$$\delta N_\alpha = N_\alpha - \overline{N}_\alpha.$$

The average of a product of functions $\delta N_\alpha \delta N_\beta$ at the same instant of time is connected with the single-time correlation functions $g_{\alpha\beta}(\mathbf{r}_\alpha, \mathbf{r}_\beta, \mathbf{p}_\alpha, \mathbf{p}_\beta, t)$ by the relations

$$\overline{\delta N_\alpha \delta N_\beta} = \delta_{\alpha\beta}\delta(\mathbf{r}_\alpha - \mathbf{r}_\beta)\,\delta(\mathbf{p}_\alpha - \mathbf{p}_\beta)\, n_\beta f_\beta + n_\alpha n_\beta g_{\alpha\beta}. \quad (1.2)$$

Using Eqs. (1.1) we find an equation for the functions δN_α.

By virtue of the condition $e^2 n^{1/3} \ll \overline{mv^2}/2$ the functions $\delta N_\alpha \delta N_\beta$ will be small compared to the product of the functions $\overline{N}_\alpha \overline{N}_\beta$. This enables us to break off the chain of equations for the functions

$$\overline{\delta N_\alpha \delta N_\beta},\ \overline{\delta N_\alpha \delta N_\beta \delta N_\gamma},\ \ldots$$

or the corresponding chain for the correlation functions. The neglect of triple products corresponds then to the neglect of triple correlation functions. One must note that in this approximation we do not assume that the other "energy" parameter $(4\pi e^2 n_\alpha/k^2)/(\overline{mv^2}/2)$ is small. This enables us to take polarization effects in the plasma into account in the framework of the approximation used here.

If we can neglect triple products in the equation for the function $\overline{\delta N_\alpha \delta N_\beta}$, we can drop terms containing double products in the equations for the functions δN_α themselves. As a result we get the following equations for the functions δN_α:

$$\frac{\partial \delta N_\alpha}{\partial t} + \mathbf{v}_\alpha \frac{\partial \delta N_\alpha}{\partial \mathbf{r}_\alpha} - n_\alpha \sum_\beta \int d\mathbf{p}_\beta d\mathbf{r}_\beta \frac{\partial U_{\alpha\beta}(|\mathbf{r}_\alpha - \mathbf{r}_\beta|)}{\partial \mathbf{r}_\alpha} \delta N_\beta \frac{\partial f_\alpha}{\partial \mathbf{p}_\alpha} = 0. \quad (1.3)$$

We have dropped in Eqs. (1.3) the term that accounts for the average electrical field; this is possible because the functions f_α are assumed to be slowly varying functions of space and time.

The system (1.3) is solved with the boundary condition

$$\delta N_\alpha(\mathbf{r}_\alpha, \mathbf{p}_\alpha, t) = \delta N_\alpha(\mathbf{p}_\alpha, \mathbf{r}_\alpha, 0) \text{ for } t = 0. \quad (1.4)$$

It is expedient to use in the solution a Fourier transformation over positive times and a Fourier transformation over the coordinates. We define the Fourier components of the functions δN_α by the equations

$$\delta N_\alpha(\omega, \mathbf{k}, \mathbf{p}_\alpha) = \int_0^\infty \int dt\, d\mathbf{r}_\alpha\, \delta N_\alpha(\mathbf{r}_\alpha, \mathbf{p}_\alpha, t)\, e^{i(\omega t - \mathbf{k}\mathbf{r}_\alpha)},$$

$$\omega = \omega' + i\omega'', \quad \omega'' > 0,$$

from which it follows that the functions $\delta N_\alpha(\omega, \mathbf{k}, \mathbf{p}_\alpha)$ are analytical in the upper half-plane of the complex variable ω.

We first find an expression for the Fourier components of the total charge density

$$\delta\rho(\mathbf{r}_\alpha, t) = \sum_\alpha e_\alpha \int \delta N_\alpha(\mathbf{r}_\alpha, \mathbf{p}_\alpha, t)\, d\mathbf{p}_\alpha.$$

From the system (1.3) we get the following expression for the function $\delta\rho(\omega, \mathbf{k})$:

$$\delta\rho(\omega, \mathbf{k}) = i \sum_\alpha e_\alpha \int \frac{\delta N_\alpha(\mathbf{k}, \mathbf{p}_\alpha, 0)}{(\omega - \mathbf{k}\mathbf{v}_\alpha + i\Delta)} d\mathbf{p}_\alpha \frac{1}{\varepsilon^{(+)}(\omega, \mathbf{k})}. \quad (1.5)$$

In this expression and henceforth $\omega = \omega'$, $\Delta = \omega''$, $\Delta > 0$, and

$$\varepsilon^{(+)}(\omega, \mathbf{k}) = 1 + \sum_\beta \frac{4\pi e_\beta^2 n_\beta}{k^2} \int \mathbf{k} \frac{\partial f_\beta}{\partial \mathbf{p}_\beta} \frac{d\mathbf{p}_\beta}{\omega - \mathbf{k}\mathbf{v}_\beta + i\Delta} \quad (1.6)$$

is the dielectric constant of the plasma.

Using Eq. (1.5) we find a solution of Eq. (1.3)

$$\delta N_\alpha(\mathbf{p}_\alpha, \mathbf{r}_\alpha, t) = \delta N_\alpha(\mathbf{r}_\alpha - \mathbf{v}_\alpha t, \mathbf{p}_\alpha, 0)$$

$$- \frac{i}{(2\pi)^4} \sum_\gamma \int d\omega \int d\mathbf{k} e^{-i(\omega t - \mathbf{k}(\mathbf{r}_\alpha - \mathbf{r}_\gamma))}$$

$$\times \int d\mathbf{p}_\gamma d\mathbf{r}_\gamma \frac{4\pi e_\alpha e_\gamma n_\alpha \delta N_\gamma(\mathbf{r}_\gamma, \mathbf{p}_\gamma, 0)}{k^2(\omega - \mathbf{k}\mathbf{v}_\alpha + i\Delta)(\omega - \mathbf{k}\mathbf{v}_\gamma + i\Delta)} \mathbf{k} \frac{\partial f_\alpha}{\partial \mathbf{p}_\alpha} \frac{1}{\varepsilon^{(+)}(\omega, \mathbf{k})}. \quad (1.7)$$

The solution obtained here enables us to find the functions δN_α at time t, provided we know the functions δN_α at time $t = 0$.

Using the solution (1.7) we can express the double-time functions $\overline{\delta N_\alpha(\mathbf{r}_\alpha, \mathbf{p}_\alpha, t)\, \delta N_\beta(\mathbf{r}_\beta, \mathbf{p}_\beta, 0)}$ in terms of the first distribution functions and thus solve the problem of describing fluctuation processes in a quasi-equilibrium plasma.

We now get the solution of the corresponding equations for the case where the plasma is in a constant magnetic field. In the approximation considered by us we must when there is a magnetic field present impose still one more condition upon the functions f_α. We must, namely, assume that these functions depend merely on the longitudinal and transverse momentum components $(p_\alpha^{\|}, p_\alpha^{\perp})$ defined with respect to the vector **B**, but should not depend on the corresponding angular variable. We assume thereby that the functions f_α are also in a magnetic field slowly varying functions. Under those conditions there occurs in Eqs. (1.3) only the additional term $(e_\alpha/c)[\mathbf{v}_\alpha \times \mathbf{B}]\,\partial\delta N_\alpha/\partial \mathbf{p}_\alpha$.

The expression for $\delta\rho(\omega, \mathbf{k})$ is now of the form

$$\delta\rho(\omega, \mathbf{k}) = \sum_\alpha e_\alpha \int_0^\infty dt \int d\mathbf{p}_\alpha \frac{\delta N_\alpha(\mathbf{P}_\alpha(0, t, \mathbf{p}_\alpha), \mathbf{k}, 0)}{\varepsilon^{(+)}(\omega, \mathbf{k})}$$

$$\times \exp\{i(\omega t + \mathbf{k}\mathbf{R}_\alpha(0, t, \mathbf{p}_\alpha, 0))\}. \quad (1.8)$$

We have introduced here the following notation:

$$\mathbf{P}(t', t, \mathbf{p}_\alpha) = \frac{(\mathbf{B}\mathbf{p}_\alpha)}{B^2}\mathbf{B} - \sin\Omega_\alpha(t' - t)\frac{[\mathbf{B}\mathbf{p}_\alpha]}{B}$$

$$+ \cos\Omega_\alpha(t' - t)\frac{[[\mathbf{B}\mathbf{p}_\alpha]\,\mathbf{B}]}{B^2}, \quad (1.9)^*$$

*$[\mathbf{Bp}] = \mathbf{B} \times \mathbf{p}$.

$$\mathbf{R}(t', t, \mathbf{p}_\alpha, \mathbf{r}_\alpha) = \mathbf{r}_\alpha + \mathbf{B}\frac{(\mathbf{B}\mathbf{v}_\alpha)}{B^2}(t'-t) - \frac{[1-\cos\Omega_\alpha(t'-t)]\,[\mathbf{B}\mathbf{v}_\alpha]}{\Omega_\alpha B} + \frac{\sin\Omega_\alpha(t'-t)}{\Omega_\alpha}\frac{[[\mathbf{B}\mathbf{v}_\alpha]\,\mathbf{B}]}{B^2}, \qquad \Omega_\alpha = \frac{e_\alpha B}{m_\alpha c}. \tag{1.10}$$

The dielectric constant in a magnetic field is now defined by the equation

$$\varepsilon^{(+)}(\omega, \mathbf{k}) = 1 - i\sum_\beta \frac{4\pi e_\beta^2 n_\beta}{k^2}\int_0^\infty dt \int d\mathbf{p}_\beta \exp\{i(\omega t + \mathbf{k}\mathbf{R}_\beta(0, t, \mathbf{p}_\beta, 0))\} \times \mathbf{k}\frac{\partial f_\beta(\mathbf{P}_\beta(0, t, \mathbf{p}_\beta))}{\partial \mathbf{P}_\beta} = 1 + \sum_\beta \frac{4\pi e_\beta^2 n_\beta}{k^2}\int^\infty 2\pi p_\beta^\perp dp_\beta^\perp \int_{-\infty}^\infty dp_\beta^\parallel \times \sum_{n=-\infty}^\infty \frac{J_n^2(k_\perp v_\beta^\perp/\Omega_\beta)}{(\omega - k_\parallel v_\beta^\parallel - n\Omega_\beta + i\Delta)}\left[k_\parallel\frac{\partial}{\partial p_\beta^\parallel} + \frac{n\Omega_\beta}{v_\beta^\perp}\frac{\partial}{\partial p_\beta^\perp}\right] f_\beta. \tag{1.11}$$

The J_n in this equation are Bessel functions.

Using Eq. (1.8) we find easily a solution of the set of equations for the functions δN_α when a magnetic field is present

$$\delta N_\alpha(\mathbf{r}_\alpha, \mathbf{p}_\alpha, t) = \delta N_\alpha(\mathbf{P}_\alpha(0, t, \mathbf{p}_\alpha), \mathbf{R}_\alpha(0, t, \mathbf{p}_\alpha, 0), 0) + \frac{i}{(2\pi)^4}\sum_\gamma \frac{4\pi e_\alpha e_\gamma n_\alpha}{k^2}\int d\omega e^{-i\omega t}\int d\mathbf{k} e^{i\mathbf{k}(\mathbf{r}_\gamma - \mathbf{r}_\alpha)}\int d\mathbf{p}_\gamma d\mathbf{r}_\gamma \frac{1}{\varepsilon^{(+)}(\omega, \mathbf{k})} \times \int_0^\infty d\tau \exp\{i(\omega\tau + \mathbf{k}\mathbf{R}_\alpha(0, \tau, \mathbf{p}_\alpha, 0))\}\,\mathbf{k}\frac{\partial f_\alpha(\mathbf{P}_\alpha(0, \tau, \mathbf{p}_\alpha))}{\partial \mathbf{P}_\alpha} \times \int_0^\infty d\tau' \exp\{i(\omega\tau' - \mathbf{k}\mathbf{R}_\gamma(\tau', 0, \mathbf{p}_\gamma, 0))\}\,\delta N_\gamma(\mathbf{p}_\gamma, \mathbf{r}_\gamma, 0). \tag{1.12}$$

We can use the solutions obtained here to determine averages of product of any number of functions δN_α. The correlations of the functions δN_α of any order can thus in final reckoning be expressed in terms of the first distribution functions f_α.

2. THE DETERMINATION OF THE FUNCTIONS $\overline{\delta N_\alpha(\mathbf{r}_\alpha, \mathbf{p}_\alpha, t)\,\delta N_\beta(\mathbf{r}_\beta, \mathbf{p}_\beta, 0)}$ FOR A PLASMA WITHOUT STRONG FIELDS

We can obtain a solution of the problem mentioned in the heading of this section by using Eqs. (1.2) and (1.7). Indeed, multiplying Eq. (1.7) by $\delta N_\beta(\mathbf{r}_\beta, \mathbf{p}_\beta, 0)$ we get after averaging and using Eq. (1.2)

$$\overline{\delta N_\alpha(\mathbf{r}_\alpha, \mathbf{p}_\alpha, t)\,\delta N_\beta(\mathbf{r}_\beta, \mathbf{p}_\beta, 0)} = \frac{1}{(2\pi)^4}\int d\omega \int d\mathbf{k} \exp\{-i(\omega t - \mathbf{k}(\mathbf{r}_\alpha - \mathbf{r}_\beta))\} \times (\delta N_\alpha(\mathbf{p}_\alpha)\,\delta N_\beta(\mathbf{p}_\beta))_{\omega,\mathbf{k}}^{(+)}, \tag{2.1}$$

$$(\delta N_\alpha(\mathbf{p}_\alpha)\,\delta N_\beta(\mathbf{p}_\beta))_{\omega,\mathbf{k}}^{(+)} = \frac{i}{(\omega - \mathbf{k}\mathbf{v}_\alpha + i\Delta)} \times \Big\{\delta_{\alpha\beta}\delta(\mathbf{p}_\alpha - \mathbf{p}_\beta)\, n_\beta f_\beta(\mathbf{p}_\beta) + n_\alpha n_\beta G_{\alpha\beta}(\mathbf{k}, \mathbf{p}_\alpha, \mathbf{p}_\beta) - \frac{4\pi e_\alpha}{k^2} n_\alpha \mathbf{k}\frac{\partial f_\alpha}{\partial \mathbf{p}_\alpha}\frac{1}{\varepsilon^{(+)}(\omega, \mathbf{k})}\Big[\frac{e_\beta n_\beta f_\beta(\mathbf{p}_\beta)}{\omega - \mathbf{k}\mathbf{v}_\beta + i\Delta} + \sum_\gamma \int d\mathbf{p}_\gamma \frac{e_\gamma n_\gamma n_\beta G_{\gamma\beta}(\mathbf{k}, \mathbf{p}_\gamma, \mathbf{p}_\beta)}{\omega - \mathbf{k}\mathbf{v}_\gamma + i\Delta}\Big]\Big\}. \tag{2.2}$$

It is clear from the form of the right-hand side of Eq. (2.2) that the function defined by this formula is analytical in the upper half-plane of the complex variable ω. This corresponds to the fact that $t \geq 0$ in Eq. (2.1). In Eq. (2.2) $G_{\alpha\beta}$ is the Fourier component of the pair correlation function

$$G_{\alpha\beta}(\mathbf{k}, \mathbf{p}_\alpha, \mathbf{p}_\beta) = \int d(\mathbf{r}_\alpha - \mathbf{r}_\beta)\, e^{-i\mathbf{k}(\mathbf{r}_\alpha - \mathbf{r}_\beta)}\, g_{\alpha\beta}(\mathbf{r}_\alpha - \mathbf{r}_\beta, \mathbf{p}_\alpha, \mathbf{p}_\beta)$$

and is of the form (see [7,8])

$$G_{\alpha\beta}(\mathbf{k}, \mathbf{p}_\alpha, \mathbf{p}_\beta) = \frac{1}{\mathbf{k}(\mathbf{v}_\alpha - \mathbf{v}_\beta) - i\Delta}\frac{4\pi e_\alpha e_\beta}{k^2}\Big\{\mathbf{k}\frac{\partial f_\alpha}{\partial \mathbf{p}_\alpha}\frac{f_\beta}{\varepsilon^{(+)}(\mathbf{k}\mathbf{v}_\beta, \mathbf{k})} - \mathbf{k}\frac{\partial f_\beta}{\partial \mathbf{p}_\beta}\frac{f_\alpha}{\varepsilon^{(-)}(\mathbf{k}\mathbf{v}_\alpha, \mathbf{k})} + \Big(\mathbf{k}\frac{\partial f_\alpha}{\partial \mathbf{p}_\alpha}\Big)\Big(\mathbf{k}\frac{\partial f_\beta}{\partial \mathbf{p}_\beta}\Big)\sum_\gamma \frac{4\pi e_\gamma^2 n_\gamma}{k^2}\int d\mathbf{p}_\gamma \frac{f_\gamma(\mathbf{p}_\gamma)}{|\varepsilon(\mathbf{k}\mathbf{v}_\gamma)|^2} \times \Big[\frac{1}{\mathbf{k}(\mathbf{v}_\beta - \mathbf{v}_\gamma) + i\Delta} - \frac{1}{\mathbf{k}(\mathbf{v}_\alpha - \mathbf{v}_\gamma) - i\Delta}\Big]\Big\}. \tag{2.3}$$

Apart from Eq. (2.1), which determines the correlation in the distributions of the coordinates and momenta of a pair of particles at different times, it is expedient also to obtain equations determining the correlations between the coordinate and momentum distributions of a particle and the charge density in the plasma, and also the correlations between the charge density in the plasma at different times and at different points in space:

$$\overline{\delta N_\alpha(\mathbf{r}_\alpha, \mathbf{p}_\alpha, t)\,\delta\rho(\mathbf{r}, 0)} = \frac{1}{(2\pi)^4}\int d\omega \int d\mathbf{k} e^{-i(\omega t - \mathbf{k}(\mathbf{r}_\alpha - \mathbf{r}))}(\delta N_\alpha(\mathbf{p}_\alpha)\,\delta\rho)_{\omega,\mathbf{k}}^{(+)}, \tag{2.4}$$

$$\overline{\delta\rho(\mathbf{r}, t)\,\delta N_\beta(\mathbf{r}_\beta, \mathbf{p}_\beta, 0)} = \frac{1}{(2\pi)^4}\int d\omega \int d\mathbf{k} e^{-i(\omega t - \mathbf{k}(\mathbf{r} - \mathbf{r}_\beta))}(\delta\rho\,\delta N_\beta(\mathbf{p}_\beta))_{\omega,\mathbf{k}}^{(+)}, \tag{2.5}$$

$$\overline{\delta\rho(\mathbf{r}, t)\,\delta\rho(\mathbf{r}', 0)} = \frac{1}{(2\pi)^4}\int d\omega \int d\mathbf{k} e^{-i(\omega t - \mathbf{k}(\mathbf{r} - \mathbf{r}'))}(\delta\rho\,\delta\rho)_{\omega,\mathbf{k}}^{(+)}. \tag{2.6}$$

The corresponding expressions can be obtained from Eq. (2.2) by integration over the momenta, multiplication by the charge, and summation over the different kinds of particles. For instance, we have

$$(\delta\rho\,\delta N_\beta(\mathbf{p}_\beta))_{\omega,\mathbf{k}}^{(+)} = \frac{1}{\varepsilon^{(+)}(\omega, \mathbf{k})}\Big[\frac{i e_\beta n_\beta f_\beta}{\omega - \mathbf{k}\mathbf{v}_\beta + i\Delta} + \sum_\alpha \int d\mathbf{p}_\alpha \frac{i e_\alpha n_\alpha n_\beta}{\omega - \mathbf{k}\mathbf{v}_\alpha + i\Delta} G_{\alpha\beta}(\mathbf{k}, \mathbf{p}_\alpha, \mathbf{p}_\beta)\Big]. \tag{2.7}$$

This last formula enables us to write, in particular,

$$(\delta N_\alpha(\mathbf{p}_\alpha)\,\delta N_\beta(\mathbf{p}_\beta))^{(+)}_{\omega,\mathbf{k}} = \frac{i}{\omega - \mathbf{k}\mathbf{v}_\alpha + i\Delta}\Big\{\delta_{\alpha\beta}\delta(\mathbf{p}_\alpha - \mathbf{p}_\beta)\,n_\beta f_\beta + n_\alpha n_\beta G_{\alpha\beta}(\mathbf{k}, \mathbf{p}_\alpha, \mathbf{p}_\beta) + \frac{4\pi e_\alpha n_\alpha}{k^2}\, i\mathbf{k}\frac{\partial f_\alpha}{\partial \mathbf{p}_\alpha}(\delta\rho\,\delta N_\beta(\mathbf{p}_\beta))^{(+)}_{\omega,\mathbf{k}}\Big\}. \quad (2.8)$$

Using the explicit expression for the correlation function (2.3) we get

$$(\delta\rho\delta\rho)^{(\pm)}_{\omega,\mathbf{k}} = i\int \frac{d\omega'}{\omega - \omega' \pm i\Delta}(\delta\rho\delta\rho)_{\omega',\mathbf{k}}, \quad (2.9)$$

$$(\delta\rho\delta\rho)_{\omega,\mathbf{k}} = \sum_\alpha e_\alpha^2 n_\alpha \times \int d\mathbf{p}_\alpha f_\alpha(\mathbf{p}_\alpha)\,\delta(\omega - \mathbf{k}\mathbf{v}_\alpha) \Big/ \varepsilon^{(+)}(\omega, \mathbf{k})\,\varepsilon^{(-)}(\omega, \mathbf{k}). \quad (2.10)$$

We note that since the correlation function for the Coulomb field is connected with the correlation function for the charge density through the relation

$$(\mathbf{E}\mathbf{E})_{\omega,\mathbf{k}} = (4\pi/k)^2(\delta\rho\delta\rho)_{\omega,\mathbf{k}}, \quad (2.11)$$

Equation (2.10) corresponds to the one obtained in [1-3].

We can use Eqs. (2.9) and (2.10) to express the correlation function (2.3) in terms of the charge fluctuations

$$G_{\alpha\beta}(\mathbf{k}, \mathbf{p}_\alpha, \mathbf{p}_\beta) = \frac{1}{\mathbf{k}(\mathbf{v}_\alpha - \mathbf{v}_\beta) - i\Delta}\frac{4\pi e_\alpha e_\beta}{k^2}\Big\{\mathbf{k}\frac{\partial f_\alpha}{\partial \mathbf{p}_\alpha}\frac{f_\beta}{\varepsilon^{(+)}(\mathbf{k}\mathbf{v}_\beta, \mathbf{k})} - \mathbf{k}\frac{\partial f_\beta}{\partial \mathbf{p}_\beta}\frac{f_\alpha}{\varepsilon^{(-)}(\mathbf{k}\mathbf{v}_\alpha, \mathbf{k})} - i\frac{4\pi}{k^2}\Big(\mathbf{k}\frac{\partial f_\alpha}{\partial \mathbf{p}_\alpha}\Big)\Big(\mathbf{k}\frac{\partial f_\beta}{\partial \mathbf{p}_\beta}\Big) \times [(\delta\rho\delta\rho)^{(+)}_{\mathbf{k}\mathbf{v}_\beta,\mathbf{k}} - (\delta\rho\delta\rho)^{(-)}_{\mathbf{k}\mathbf{v}_\alpha,\mathbf{k}}]\Big\}. \quad (2.12)$$

We can also express the other correlation functions in terms of the charge fluctuations:

$$(\delta N_\alpha(\mathbf{p}_\alpha)\,\delta\rho)^{(+)}_{\omega,\mathbf{k}} = \frac{ie_\alpha n_\alpha}{\omega - \mathbf{k}\mathbf{v}_\alpha + i\Delta}\Big\{\frac{f_\alpha}{\varepsilon^{(-)}(\mathbf{k}\mathbf{v}_\alpha, \mathbf{k})} + i\frac{4\pi}{k^2}\Big(\mathbf{k}\frac{\partial f_\alpha}{\partial \mathbf{p}_\alpha}\Big)[(\delta\rho\delta\rho)^{(+)}_{\omega,\mathbf{k}} - (\delta\rho\delta\rho)^{(-)}_{\mathbf{k}\mathbf{v}_\alpha,\mathbf{k}}]\Big\}, \quad (2.13)$$

$$(\delta\rho\,\delta N_\beta(\mathbf{p}_\beta))^{(+)}_{\omega,\mathbf{k}} = \frac{ie_\beta n_\beta}{\omega - \mathbf{k}\mathbf{v}_\beta + i\Delta}\Big\{\frac{f_\beta}{\varepsilon^{(+)}(\mathbf{k}\mathbf{v}_\beta, \mathbf{k})} + i\frac{4\pi}{k^2}\Big(\mathbf{k}\frac{\partial f_\beta}{\partial \mathbf{p}_\beta}\Big)[(\delta\rho\delta\rho)^{(+)}_{\omega,\mathbf{k}} - (\delta\rho\delta\rho)^{(+)}_{\mathbf{k}\mathbf{v}_\beta,\mathbf{k}}]\Big\}, \quad (2.14)$$

$$(\delta N_\alpha(\mathbf{p}_\alpha)\,\delta N_\beta(\mathbf{p}_\beta))^{(+)}_{\omega,\mathbf{k}} = \frac{i}{\omega - \mathbf{k}\mathbf{v}_\alpha + i\Delta}\delta_{\alpha\beta}\delta(\mathbf{p}_\alpha - \mathbf{p}_\beta)\,n_\beta f_\beta + \frac{4\pi e_\alpha}{k^2}n_\alpha\frac{4\pi e_\beta}{k^2}n_\beta\Big(\mathbf{k}\frac{\partial f_\alpha}{\partial \mathbf{p}_\alpha}\Big)\frac{1}{\omega - \mathbf{k}\mathbf{v}_\alpha + i\Delta}\Big(\mathbf{k}\frac{\partial f_\beta}{\partial \mathbf{p}_\beta}\Big)\frac{1}{\omega - \mathbf{k}\mathbf{v}_\beta + i\Delta} \times (\delta\rho\delta\rho)^{(+)}_{\omega,\mathbf{k}} - \frac{4\pi e_\alpha e_\beta}{k^2}n_\alpha n_\beta\frac{1}{\mathbf{k}\mathbf{v}_\alpha - \mathbf{k}\mathbf{v}_\beta - i\Delta} \times \Big\{\Big(\mathbf{k}\frac{\partial f_\beta}{\partial \mathbf{p}_\beta}\Big)\frac{1}{\omega - \mathbf{k}\mathbf{v}_\alpha + i\Delta}\Big[i\frac{f_\alpha}{\varepsilon^{(-)}(\mathbf{k}\mathbf{v}_\alpha, \mathbf{k})} + \frac{4\pi}{k^2}\Big(\mathbf{k}\frac{\partial f_\alpha}{\partial \mathbf{p}_\alpha}\Big)(\delta\rho\delta\rho)^{(-)}_{\mathbf{k}\mathbf{v}_\alpha,\mathbf{k}}\Big] - \Big(\mathbf{k}\frac{\partial f_\alpha}{\partial \mathbf{p}_\alpha}\Big)\frac{1}{\omega - \mathbf{k}\mathbf{v}_\beta + i\Delta} \times \Big[i\frac{f_\beta}{\varepsilon^{(+)}(\mathbf{k}\mathbf{v}_\beta, \mathbf{k})} + \frac{4\pi}{k^2}\Big(\mathbf{k}\frac{\partial f_\beta}{\partial \mathbf{p}_\beta}\Big)(\delta\rho\delta\rho)^{(+)}_{\mathbf{k}\mathbf{v}_\beta,\mathbf{k}}\Big]\Big\}. \quad (2.15)$$

The fact that the function $(\delta\rho\delta\rho)$ occurs in all these formulae corresponds to our taking into account the fluctuations in the force exerted by the Coulomb field on a particle [cf. Eq. (2.11)]. The additional occurrence of expressions such as

$$4\pi e_\alpha / k^2\varepsilon(\mathbf{k}\mathbf{v}_\alpha, \mathbf{k}) \quad (2.16)$$

corresponds to the modification of the Coulomb field on the α-th particle in the plasma by the polarization, which is characterized by a complex dielectric constant $\epsilon(\omega, \mathbf{k})$.

We note, finally, that by using Eqs. (2.13) and (2.14) we can write Eq. (2.15) in the form

$$(\delta N_\alpha(\mathbf{p}_\alpha)\,\delta N_\beta(\mathbf{p}_\beta))^{(+)}_{\omega,\mathbf{k}} = \frac{i}{\omega - \mathbf{k}\mathbf{v}_\alpha + i\Delta}\delta_{\alpha\beta}\delta(\mathbf{p}_\alpha - \mathbf{p}_\beta)\,n_\beta f_\beta(\mathbf{p}_\beta) - \frac{1}{\mathbf{k}\mathbf{v}_\alpha - \mathbf{k}\mathbf{v}_\beta - i\Delta}\Big\{\frac{4\pi e_\beta n_\beta}{k^2}\Big(\mathbf{k}\frac{\partial f_\beta}{\partial \mathbf{p}_\beta}\Big)(\delta N_\alpha(\mathbf{p}_\alpha)\delta\rho)^{(+)}_{\omega,\mathbf{k}} - \frac{4\pi e_\alpha n_\alpha}{k^2}\Big(\mathbf{k}\frac{\partial f_\alpha}{\partial \mathbf{p}_\alpha}\Big)(\delta\rho\,\delta N_\beta(\mathbf{p}_\beta))^{(+)}_{\omega,\mathbf{k}}\Big\}. \quad (2.17)$$

3. THE DETERMINATION OF THE FUNCTIONS $\overline{\delta N_\alpha(\mathbf{r}_\alpha, \mathbf{p}_\alpha, t)\,\delta N_\beta(\mathbf{r}_\beta, \mathbf{p}_\beta, 0)}$ FOR A PLASMA IN A CONSTANT AND UNIFORM MAGNETIC FIELD

Assuming that there is a constant and uniform magnetic field **B** in the plasma and using Eqs. (1.12) and (1.2) we can write down the following formula, which is in a certain sense the analog of Eq. (2.8) of the preceding section:

$$(\delta N_\alpha(\mathbf{p}_\alpha)\,\delta N_\beta(\mathbf{p}_\beta))^{(+)}_{\omega,\mathbf{k}} = \int_0^\infty dt\,\exp\{i(\omega t + \mathbf{k}\mathbf{R}_\alpha(0, t, \mathbf{p}_\alpha, 0))\} \times \Big\{\delta_{\alpha\beta}\delta(\mathbf{p}_\beta - \mathbf{P}_\alpha(0, t, \mathbf{p}_\alpha))\,n_\beta f_\beta(\mathbf{p}_\beta) + n_\alpha n_\beta G_{\alpha\beta}(\mathbf{k}, \mathbf{P}_\alpha(0, t, \mathbf{p}_\alpha), \mathbf{p}_\beta) + (\delta\rho\,\delta N_\beta(\mathbf{p}_\beta))^{(+)}_{\omega,\mathbf{k}}\frac{4\pi e_\alpha i}{k^2}n_\alpha\mathbf{k}\frac{\partial f_\alpha(\mathbf{P}_\alpha(0, t, \mathbf{p}_\alpha))}{\partial \mathbf{P}_\alpha}\Big\}; \quad (3.1)$$

$$(\delta\rho\,\delta N_\beta(\mathbf{p}_\beta))^{(+)}_{\omega,\mathbf{k}} = \frac{1}{\varepsilon^{(+)}(\omega, \mathbf{k})}\int_0^\infty dt e^{i\omega t}\sum_\alpha e_\alpha\int d\mathbf{p}_\alpha\exp\{-i\mathbf{k}\mathbf{R}_\alpha(t, 0, \mathbf{p}_\alpha, 0)\} \times \{\delta_{\alpha\beta}n_\beta f_\beta(\mathbf{p}_\beta)\,\delta(\mathbf{p}_\alpha - \mathbf{p}_\beta) + n_\alpha n_\beta G_{\alpha\beta}(\mathbf{k}, \mathbf{p}_\alpha, \mathbf{p}_\beta)\}. \quad (3.2)$$

Here $G_{\alpha\beta}(\mathbf{k}, \mathbf{p}_\alpha, \mathbf{p}_\beta)$ is, as in the preceding section, the Fourier component of the correlation function of a pair of particles in the plasma. We derive an explicit expression for such a function in the Appendix.

We can also easily obtain the formulae

$$(\delta\rho\delta\rho)^{(+)}_{\omega,\mathbf{k}} = \frac{1}{\varepsilon^{(+)}(\omega, \mathbf{k})}\int_0^\infty dt e^{i\omega t}\sum_{\alpha\beta}e_\alpha e_\beta\int d\mathbf{p}_\alpha d\mathbf{p}_\beta\exp\{-i\mathbf{k}\mathbf{R}_\alpha(t, 0, \mathbf{p}_\alpha, 0)\} \times \{\delta_{\alpha\beta}n_\beta f_\beta(\mathbf{p}_\beta)\,\delta(\mathbf{p}_\beta - \mathbf{p}_\alpha) + n_\alpha n_\beta G_{\alpha\beta}(\mathbf{k}, \mathbf{p}_\alpha, \mathbf{p}_\beta)\}, \quad (3.3)$$

$$(\delta N_\alpha(\mathbf{p}_\alpha)\delta\rho)^{(+)}_{\omega,\mathbf{k}} = \int_0^\infty dt \exp\{i(\omega t + \mathbf{k}\mathbf{R}_\alpha(0,t,\mathbf{p}_\alpha,0))\}$$
$$\times\Big\{e_\alpha n_\alpha f_\alpha(\mathbf{p}_\alpha) + \sum_\beta e_\beta n_\beta n_\alpha \int d\mathbf{p}_\beta G_{\alpha\beta}(\mathbf{k},\mathbf{P}_\alpha(0,t,\mathbf{p}_\alpha),\mathbf{p}_\beta)$$
$$+ (\delta\rho\delta\rho)^{(+)}_{\omega,\mathbf{k}} \frac{4\pi e_\alpha n_\alpha}{k^2} i\mathbf{k} \frac{\partial f_\alpha(\mathbf{P}_\alpha(0,t,\mathbf{p}_\alpha))}{\partial \mathbf{P}_\alpha}\Big\}. \quad (3.4)$$

Using Eq. (A.3), which determines the correlation function of a pair of particles in the plasma when there is a constant magnetic field present, we can write Eq. (3.3) in the form

$$(\delta\rho\delta\rho)^{(\pm)}_{\omega,\mathbf{k}} = i\int \frac{d\omega'}{\omega-\omega'\pm i\Delta}(\delta\rho\delta\rho)_{\omega',\mathbf{k}}; \quad (3.5)$$

$$(\delta\rho\delta\rho)_{\omega,\mathbf{k}} = \frac{1}{\varepsilon^{(+)}(\omega,\mathbf{k})\,\varepsilon^{(-)}(\omega,\mathbf{k})}\sum_\beta e_\beta^2 n_\beta \int d\mathbf{p}_\beta f_\beta(\mathbf{p}_\beta)\int_{-\infty}^{\infty}\frac{dt}{2\pi}$$
$$\times\cos(\omega t - \mathbf{k}\mathbf{R}_\beta(t,0,\mathbf{p}_\beta,0)) = \sum_\beta e_\beta^2 n_\beta \int_{-\infty}^{\infty} dp_\beta^{\parallel}\int_0^\infty 2\pi p_\beta^{\perp}\,dp_\beta^{\perp} f_\beta$$
$$\times\sum_{n=-\infty}^{\infty} J_n^2\Big(\frac{k_\perp v_\beta^\perp}{\Omega_\beta}\Big)\frac{\delta(\omega - n\Omega_\beta - k_\parallel v_\beta^\parallel)}{\varepsilon^{(+)}(\omega,\mathbf{k})\,\varepsilon^{(-)}(\omega,\mathbf{k})}. \quad (3.6)$$

Taking Eq. (2.11) into account we can easily show that Eq. (3.6) corresponds to the one obtained in [1-3] for the Coulomb field.

We can also use Eqs. (A.11) and (A.14) to express all the other spectral functions in terms of the charge density fluctuations. We get thus for the functions (3.4) and (3.2) the expressions

$$(\delta N_\alpha(\mathbf{p}_\alpha)\,\delta\rho)^{(+)}_{\omega,\mathbf{k}} = e_\alpha n_\alpha \int_0^\infty d\tau \exp\{i(\omega\tau + \mathbf{k}\mathbf{R}_\alpha(0,\tau,\mathbf{p}_\alpha,0))\}$$
$$\times\Big\{i\frac{4\pi}{k^2}\mathbf{k}\frac{\partial f_\alpha(\mathbf{P}_\alpha(0,\tau,\mathbf{p}_\alpha))}{\partial\mathbf{P}_\alpha}(\delta\rho\delta\rho)^{(+)}_{\omega,\mathbf{k}}$$
$$+\int_0^\infty\int_{-\infty}^\infty dt\,d\omega' \exp\{i(\omega' t + k[\mathbf{R}_\alpha(0,t+\tau,\mathbf{p}_\alpha,0)$$
$$-\mathbf{R}_\alpha(0,\tau,\mathbf{p}_\alpha,0)])\}$$
$$\times\Big[\frac{4\pi}{k^2}\mathbf{k}\frac{\partial f_\alpha(\mathbf{P}_\alpha(0,t+\tau,\mathbf{p}_\alpha))}{\partial\mathbf{P}_\alpha} i(\delta\rho\delta\rho)_{\omega',\mathbf{k}} + \frac{1}{2\pi}\frac{f_\alpha}{\varepsilon^{(-)}(\omega',\mathbf{k})}\Big]\Big\}, \quad (3.7)$$

$$(\delta\rho\delta N_\beta(\mathbf{p}_\beta))^{(+)}_{\omega,\mathbf{k}} = e_\beta n_\beta \int_0^\infty d\tau \exp\{i(\omega\tau + \mathbf{k}\mathbf{R}_\beta(0,-\tau,\mathbf{p}_\beta,0))\}$$
$$\times\Big\{i\frac{4\pi}{k^2}\mathbf{k}\frac{\partial f_\beta(\mathbf{P}_\beta(0,-\tau,\mathbf{p}_\beta)}{\partial\mathbf{P}_\beta}(\delta\rho\delta\rho)^{(+)}_{\omega,\mathbf{k}}$$
$$+\int_0^\infty\int_{-\infty}^\infty dt\,d\omega' \exp\{-i(\omega' t + \mathbf{k}[\mathbf{R}_\beta(0,t-\tau,\mathbf{p}_\beta,0)$$
$$-\mathbf{R}_\beta(0,-\tau,\mathbf{p}_\beta,0)])\}\Big[-i\frac{4\pi}{k^2}\mathbf{k}\frac{\partial f_\beta(\mathbf{P}_\beta(0,t-\tau,\mathbf{p}_\beta))}{\partial\mathbf{P}_\beta}$$
$$\times(\delta\rho\delta\rho)_{\omega',\mathbf{k}} + \frac{1}{2\pi}\frac{f_\beta}{\varepsilon^{(+)}(\omega',\mathbf{k})}\Big]\Big\}. \quad (3.8)$$

We can use Eq. (A.14) to obtain from (3.1) the corresponding expression for the function $(\delta N_\alpha(\mathbf{p}_\alpha)\,\delta N_\beta(\mathbf{p}_\beta))^{(+)}_{\omega,\mathbf{k}}$

$$(\delta N_\alpha(\mathbf{p}_\alpha)\,\delta N_\beta(\mathbf{p}_\beta))^{(+)}_{\omega,\mathbf{k}}$$
$$= \delta_{\alpha\beta} n_\alpha n_\beta \int_0^\infty dt \exp\{i(\omega t - \mathbf{k}\mathbf{R}_\beta(0,t,\mathbf{p}_\beta,0))\}$$
$$\times\delta(\mathbf{p}_\beta - \mathbf{P}_\alpha(0,t,\mathbf{p}_\alpha))$$
$$-(\delta\rho\delta\rho)^{(+)}_{\omega,\mathbf{k}} L_\alpha^{(+)}(\omega,\mathbf{k},\mathbf{p}_\alpha)\,L_\beta^{(-)}(-\omega,-\mathbf{k},\mathbf{p}_\beta)$$
$$+\frac{1}{2\pi i}\int_{-\infty}^\infty \frac{d\omega'}{\omega'-\omega-i\Delta}\{L_\beta^{(+)}(-\omega',-\mathbf{k},\mathbf{p}_\beta)$$
$$\times[M_\alpha(\omega',\mathbf{k},\mathbf{p}_\alpha)/\varepsilon^{(-)}(\omega',\mathbf{k})$$
$$-(\delta\rho\delta\rho)^{(-)}_{\omega',\mathbf{k}}(L_\alpha^{(+)}(\omega',\mathbf{k},\mathbf{p}_\alpha) + L_\alpha^{(-)}(\omega',\mathbf{k},\mathbf{p}_\alpha))]$$
$$+L_\alpha^{(+)}(\omega',\mathbf{k},\mathbf{p}_\alpha)[M_\beta(-\omega',-\mathbf{k},\mathbf{p}_\beta)/\varepsilon^{(+)}(\omega',\mathbf{k})$$
$$+(\delta\rho\delta\rho)^{(+)}_{\omega',\mathbf{k}}(L_\beta^{(+)}(-\omega',-\mathbf{k},\mathbf{p}_\beta) + L_\beta^{(-)}(-\omega',-\mathbf{k},\mathbf{p}_\beta))]\}. \quad (3.9)$$

Here

$$M_\alpha(\omega,\mathbf{k},\mathbf{p}_\alpha) = \int_{-\infty}^\infty dt \exp\{i(\omega t + \mathbf{k}\mathbf{R}_\alpha(0,t,\mathbf{p}_\alpha,0))\}e_\alpha n_\alpha f_\alpha,$$

$$L^{(\pm)}(\omega,\mathbf{k},\mathbf{p}_\alpha) = \pm\int_0^{\pm\infty} dt \exp\{i(\omega t + \mathbf{k}\mathbf{R}_\alpha(0,t,\mathbf{p}_\alpha,0))\}$$
$$\times\frac{4\pi e_\alpha n_\alpha}{k^2} i\mathbf{k}\frac{\partial f_\alpha(\mathbf{P}_\alpha(0,t,\mathbf{p}_\alpha))}{\partial\mathbf{P}_\alpha}.$$

The formulae obtained here fully describe all pair correlations in a plasma with Coulomb interactions, situated in a constant magnetic field.

In the case of a strong magnetic field, interest attaches to the spectral functions averaged over the fast changing angular momentum-space variable corresponding to the Larmor rotation. The function $(\delta\rho\delta\rho)^{(+)}_{\omega,\mathbf{k}}$ does, of course, not change when averaged. The averaged functions $(\delta N_\alpha(\mathbf{p}_\alpha)\,\delta\rho)^{(+)}_{\omega,\mathbf{k}}$ and $(\delta\rho\delta N_\beta(\mathbf{p}_\beta))^{(+)}_{\omega,\mathbf{k}}$ are of the form

$$\langle(\delta N_\alpha(\mathbf{p}_\alpha)\,\delta\rho)^{(+)}_{\omega,\mathbf{k}}\rangle = ie_\alpha n_\alpha \sum_n \frac{J_n^2(k_\perp v_\alpha^\perp/\Omega_\alpha)}{\omega - k_\parallel v_\alpha^\parallel - n\Omega_\alpha + i\Delta}$$
$$\times\Big\{\frac{f_\alpha}{\varepsilon^{(-)}(k_\parallel v_\alpha^\parallel + n\Omega_\alpha,\mathbf{k})} + i\frac{4\pi}{k^2}\Big[k_\parallel\frac{\partial}{\partial p_\alpha^\parallel} + \frac{n\Omega_\alpha\partial}{v_\alpha^\perp\partial p_\alpha^\perp}\Big]f_\alpha$$
$$\times\Big[(\delta\rho\delta\rho)^{(+)}_{\omega,\mathbf{k}} - (\delta\rho\delta\rho)^{(-)}_{k_\parallel v_\alpha^\parallel + n\Omega_\alpha,\mathbf{k}}\Big]\Big\}, \quad (3.10)$$

$$\langle(\delta\rho\,\delta N_\beta(\mathbf{p}_\beta))^{(+)}_{\omega,\mathbf{k}}\rangle = ie_\beta n_\beta \sum_n \frac{J_n^2(k_\perp v_\beta^\perp/\Omega_\beta)}{\omega - k_\parallel v_\beta^\parallel - n\Omega_\beta + i\Delta}$$
$$\times\Big\{\frac{f_\beta}{\varepsilon^{(+)}(k_\parallel v_\beta^\parallel + n\Omega_\beta,\mathbf{k})} + i\frac{4\pi}{k^2}\Big[k_\parallel\frac{\partial}{\partial p_\beta^\parallel} + \frac{n\Omega_\beta\partial}{v_\beta^\perp\partial p_\beta^\perp}\Big]f_\beta$$
$$\times\Big[(\delta\rho\delta\rho)^{(+)}_{\omega,\mathbf{k}} - (\delta\rho\delta\rho)^{(+)}_{k_\parallel v_\beta^\parallel + n\Omega_\beta,\mathbf{k}}\Big]\Big\}. \quad (3.11)$$

When averaging Eq. (3.1) we must bear in mind that

$$\left\langle \int_0^\infty dt \exp\{i(\omega t - \mathbf{k}\mathbf{R}_\beta(t, 0, \mathbf{p}_\beta, 0))\}\, \delta(\mathbf{p}_\beta - \mathbf{P}_\alpha(0, t, \mathbf{p}_\alpha))\right\rangle$$

$$= \delta(p_\alpha^\parallel - p_\beta^\parallel)\, \delta(p_\beta^\perp - p_\alpha^\perp)\,(p_\alpha^\perp p_\beta^\perp)^{-1/2}$$

$$\times \sum_{n=-\infty}^{\infty} J_n^2\left(\frac{k_\perp v_\alpha^\perp}{\Omega_\alpha}\right) \frac{i}{\omega - k_\parallel v_\alpha^\parallel - n\Omega_\alpha + i\Delta}.$$

4. SPECIFIC EXAMPLES. DISCUSSION OF THE RESULTS

As an application of the general formulae obtained in the foregoing we consider first the example of an equilibrium plasma when the functions $f_\alpha(\mathbf{p}_\alpha)$ are Maxwell distributions with the same plasma temperatures for all components. One can in that case write Eq. (2.14) in the form

$$(\delta N_\alpha(\mathbf{p}_\alpha)\,\delta N_\beta(\mathbf{p}_\beta))^{(+)}_{\omega,\mathbf{k}} = \frac{i}{\omega - \mathbf{k}\mathbf{v}_\alpha + i\Delta}\,\delta_{\alpha\beta}\,\delta(\mathbf{p}_\alpha - \mathbf{p}_\beta)\, n_\beta f_\beta$$

$$+ \frac{i n_\alpha n_\beta}{\omega + i\Delta} f_\alpha f_\beta$$

$$\times \left[g_{\alpha\beta}(\mathbf{k}) + \frac{4\pi e_\alpha e_\beta}{k^2 \varkappa T} \frac{(\mathbf{k}\mathbf{v}_\alpha)(\mathbf{k}\mathbf{v}_\beta)}{(\omega - \mathbf{k}\mathbf{v}_\alpha + i\Delta)(\omega - \mathbf{k}\mathbf{v}_\beta + i\Delta)} \frac{1}{\varepsilon^{(+)}(\omega, \mathbf{k})}\right], \tag{4.1}$$

where

$$g_{\alpha\beta}(\mathbf{k}) = -\frac{e_\alpha e_\beta}{\sum_\gamma e_\gamma^2 n_\gamma} \frac{1}{r_d^2 k^2 + 1}, \quad r_d^2 = \frac{\varkappa T}{\sum_\gamma 4\pi e_\gamma^2 n_\gamma} \tag{4.2}$$

are the single-time correlation functions of an equilibrium plasma. One can also obtain Eq. (4.1) from Eq. (23) of Rostoker's paper [2] by generalizing that equation to the case of a multicomponent plasma.

However, there is considerably more interest in the corresponding equations for a non-equilibrium plasma, of which a non-isothermal plasma is a typical example. In that case we get the following expression:

$$(\delta N_\alpha(\mathbf{p}_\alpha)\,\delta N_\beta(\mathbf{p}_\beta))^{(+)}_{\omega,\mathbf{k}} = \frac{i}{\omega - \mathbf{k}\mathbf{v}_\alpha + i\Delta}\,\delta_{\alpha\beta}\,\delta(\mathbf{p}_\alpha - \mathbf{p}_\beta)\, n_\beta f_\beta$$

$$+ \frac{4\pi e_\alpha n_\alpha}{k^2 \varkappa T_\alpha} \frac{4\pi e_\beta n_\beta}{k^2 \varkappa T_\beta} \frac{(\mathbf{k}\mathbf{v}_\alpha)(\mathbf{k}\mathbf{v}_\beta)}{(\omega - \mathbf{k}\mathbf{v}_\alpha + i\Delta)(\omega - \mathbf{k}\mathbf{v}_\beta + i\Delta)} f_\alpha f_\beta\, (\delta\rho\delta\rho)^{(+)}_{\omega,\mathbf{k}}$$

$$+ \frac{4\pi e_\alpha e_\beta}{k^2} \frac{n_\alpha n_\beta}{\mathbf{k}\mathbf{v}_\alpha - \mathbf{k}\mathbf{v}_\beta - i\Delta} \left\{ \frac{(\mathbf{k}\mathbf{v}_\beta)/\varkappa T_\beta}{\omega - \mathbf{k}\mathbf{v}_\alpha + i\Delta} \right.$$

$$\times \left[\frac{i}{\varepsilon^{(-)}(\mathbf{k}\mathbf{v}_\alpha, \mathbf{k})} - \frac{4\pi(\mathbf{k}\mathbf{v}_\alpha)}{k^2 \varkappa T_\alpha} (\delta\rho\delta\rho)^{(-)}_{\mathbf{k}\mathbf{v}_\alpha,\mathbf{k}}\right]$$

$$\left. - \frac{(\mathbf{k}\mathbf{v}_\alpha)/\varkappa T_\alpha}{\omega - \mathbf{k}\mathbf{v}_\beta + i\Delta} \left[\frac{i}{\varepsilon^{(+)}(\mathbf{k}\mathbf{v}_\beta, \mathbf{k})} - \frac{4\pi(\mathbf{k}\mathbf{v}_\beta)}{k^2 \varkappa T_\beta} (\delta\rho\delta\rho)^{(+)}_{\mathbf{k}\mathbf{v}_\beta,\mathbf{k}}\right]\right\}. \tag{4.3}$$

Here, as above, $(\delta\rho\delta\rho)^{(\pm)}_{\omega,\mathbf{k}}$ are the spectral functions of the charge densities. They are defined by Eqs. (2.9) and (2.10), where one must substitute the functions

$$f_\alpha(\mathbf{p}_\alpha) = (2\pi m_\alpha \varkappa T_\alpha)^{-3/2} \exp(-\mathbf{p}_\alpha^2 / 2m_\alpha \varkappa T_\alpha).$$

We now consider a plasma in a strong magnetic field when the longitudinal temperature is considerably larger than the transverse one ($T_\parallel \gg T_\perp$) so that we can neglect the latter. In that case

$$f_\alpha = F_\alpha(p_\alpha^\parallel)\,\delta(p_\alpha^\perp)/2\pi p_\alpha^\perp;$$

$$F_\alpha(p_\alpha^\parallel) = (2\pi m_\alpha \varkappa T_\parallel)^{-1/2} \exp[-p_\alpha^{\parallel 2}/2m_\alpha \varkappa T_\parallel].$$

Substituting these expression into Eq. (3.9) we get

$$(\delta N_\alpha(\mathbf{p}_\alpha)\,\delta N_\beta(\mathbf{p}_\beta))^{(+)}_{\omega,\mathbf{k}}$$

$$= \frac{\delta(p_\alpha^\perp)\,\delta(p_\beta^\perp)}{2\pi p_\alpha^\perp\, 2\pi p_\beta^\perp} \left\{ \frac{i n_\beta}{\omega - k_\parallel v_\alpha^\parallel + i\Delta}\,\delta_{\alpha\beta}\,\delta(p_\alpha^\parallel - p_\beta^\parallel)\, F_\alpha(p_\alpha^\parallel) \right.$$

$$- \frac{i}{\omega + i\Delta}\left[e_\alpha e_\beta n_\alpha n_\beta F_\alpha(p_\alpha^\parallel)\, F_\beta(p_\beta^\parallel) \frac{1}{\sum_\gamma e_\gamma^2 n_\gamma (1 + r_\parallel^2 k^2)}\right.$$

$$\left.\left. - \frac{4\pi e_\alpha e_\beta n_\alpha n_\beta}{k^2 \varkappa T_\parallel} \frac{1}{\varepsilon^{(+)}(\omega, k_\parallel)} \frac{(k_\parallel v_\alpha^\parallel)(k_\parallel v_\beta^\parallel)\, F_\alpha(p_\alpha^\parallel)\, F_\beta(p_\beta^\parallel)}{(\omega - k_\parallel v_\alpha^\parallel + i\Delta)(\omega - k_\parallel v_\beta^\parallel + i\Delta)}\right]\right\};$$

$$\varepsilon^{(+)}(\omega, \mathbf{k}) = 1 + \sum_\gamma \frac{4\pi e_\gamma^2 n_\gamma}{k^2} \int dp_\gamma^\parallel \frac{k_\parallel \partial F_\gamma(p_\gamma^\parallel)/\partial p_\gamma^\parallel}{\omega - k_\parallel v_\gamma^\parallel + i\Delta},$$

$$r_\parallel^2 = \varkappa T_\parallel \Big/ \sum_\gamma 4\pi e_\gamma^2 n_\gamma. \tag{4.4}$$

Using the formulae given here we can find the space-time correlations of the electrodynamic functions $\delta\rho$, $\mathbf{j}$, and $\mathbf{E}$. Such correlations have been considered before (see [1-4]). However, for a complete characterization of the non-equilibrium state of a multicomponent plasma it is also necessary to be able to determine the correlations of gas-dynamic functions such as the density, velocity, temperature, velocity moments, and so on, and also the mutual correlations of the electrodynamic and gas-dynamic functions. The space-time correlations of all these functions can under the conditions mentioned in the introduction be found by using the formulae obtained in the present paper for the correlations of phase densities.

It is important that, as shown in the present paper, such complicated functions as the correlations of the phase densities can be expressed in terms of the simple functions $(\delta\rho\delta\rho)_{\omega,\mathbf{k}}$ and $\epsilon(\omega, \mathbf{k})$, the spectral density of the charges and the dielectric constant of the plasma, which have been well studied for many cases. This means that any arbitrary correlations of electrodynamic and gas-dynamic functions can also be expressed in terms of these two functions.

APPENDIX

THE CORRELATION FUNCTION FOR A PLASMA IN A MAGNETIC FIELD

The integral equation for the correlation function of a plasma when there is a magnetic field present can be written in the form

$$e_\alpha e_\beta n_\alpha n_\beta G_{\alpha\beta}(\mathbf{k}, \mathbf{p}_\alpha, \mathbf{p}_\beta)$$
$$= i\int_0^\infty d\tau \exp\{i\mathbf{k}(\mathbf{R}_\alpha(0, \tau, \mathbf{p}_\alpha, 0) - \mathbf{R}_\beta(0, \tau, \mathbf{p}_\beta, 0)\}$$
$$\times\left\{\frac{4\pi e_\alpha^2 n_\alpha}{k^2}\mathbf{k}\frac{\partial f_\alpha(\mathbf{P}_\alpha)}{\partial \mathbf{P}_\alpha}h_\beta(-\mathbf{k}, \mathbf{P}_\beta) - \frac{4\pi e_\beta^2 n_\beta}{k^2}\mathbf{k}\frac{\partial f_\beta(\mathbf{P}_\beta)}{\partial \mathbf{P}_\beta}h_\alpha(\mathbf{k}, \mathbf{P}_\alpha)\right.$$
$$\left. + \frac{4\pi e_\alpha^2 e_\beta^2}{k^2}n_\alpha n_\beta\left[\mathbf{k}\frac{\partial f_\alpha(\mathbf{P}_\alpha)}{\partial \mathbf{P}_\alpha}f_\beta(\mathbf{P}_\beta) - \mathbf{k}\frac{\partial f_\beta(\mathbf{P}_\beta)}{\partial \mathbf{P}_\beta}f_\alpha(\mathbf{P}_\alpha)\right]\right\};$$
$$\mathbf{P}_\alpha = \mathbf{P}_\alpha(0, \tau, \mathbf{p}_\alpha), \quad \mathbf{P}_\beta = \mathbf{P}_\beta(0, \tau, \mathbf{p}_\beta),$$
$$h_\alpha(\mathbf{k}, \mathbf{p}_\alpha) = \sum_\beta e_\alpha e_\beta n_\alpha n_\beta \int G_{\alpha\beta}(\mathbf{k}, \mathbf{p}_\alpha, \mathbf{p}_\beta)\, d\mathbf{p}_\beta. \quad (A.1)$$

The functions $\mathbf{P}_\alpha$ and $\mathbf{R}_\alpha$ are defined by Eqs. (1.9) and (1.10). From (A.1) we find an integral equation for $h_\alpha(\mathbf{k}, \mathbf{p}_\alpha)$:

$$h_\alpha(\mathbf{k}, \mathbf{p}_\alpha) = i\sum_\beta\int_0^\infty d\tau\int d\mathbf{p}_\beta \exp\{i\mathbf{k}\,[\mathbf{R}_\alpha(0, \tau, \mathbf{p}_\alpha, 0)$$
$$- \mathbf{R}_\beta(0, \tau, \mathbf{p}_\beta, 0)]\}$$
$$\times\left\{\frac{4\pi e_\alpha^2 e_\beta^2}{k^2}n_\alpha n_\beta\left[\mathbf{k}\frac{\partial f_\alpha(\mathbf{P}_\alpha)}{\partial \mathbf{P}_\alpha}f_\beta(\mathbf{P}_\beta) - \mathbf{k}\frac{\partial f_\beta(\mathbf{P}_\beta)}{\partial \mathbf{P}_\beta}f_\alpha(\mathbf{P}_\alpha)\right]\right.$$
$$+\left[\frac{4\pi e_\alpha^2 n_\alpha}{k^2}\mathbf{k}\frac{\partial f_\alpha(\mathbf{P}_\alpha)}{\partial \mathbf{P}_\alpha}h_\beta(-\mathbf{k}, \mathbf{P}_\beta)\right.$$
$$\left.\left. - \frac{4\pi e_\beta^2 n_\beta}{k^2}\mathbf{k}\frac{\partial f_\beta(\mathbf{P}_\beta)}{\partial \mathbf{P}_\beta}h_\alpha(\mathbf{k}, \mathbf{P}_\alpha)\right]\right\}. \quad (A.2)$$

We solve (A.2) assuming that the functions $f_\alpha(\mathbf{p}_\alpha)$ depend only on the longitudinal and transverse momentum components, but are independent of the angle variable φ_α. Expanding the function $h_\alpha(\mathbf{k}, p_\alpha^\perp, p_\alpha^\parallel, \varphi_\alpha)$ in a Fourier series in φ_α:

$$h_\alpha = \exp\left(-i\frac{k_\perp v_\alpha^\perp}{\Omega_\alpha}\sin\varphi_\alpha\right)\sum_{n=-\infty}^{\infty} e^{-in\varphi_\alpha}h_\alpha^{(n)}(\mathbf{k}, p_\alpha^\parallel, p_\alpha^\perp),$$

we get the following equation for the Fourier component $h_\alpha^{(n)}$:

$$h_\alpha^{(n)}\varepsilon^{(-)}(k_\parallel v_\alpha^\parallel + n\Omega_\alpha, \mathbf{k}) = 2\pi i\frac{4\pi e_\alpha^2 n_\alpha}{k^2}J_n\left(\frac{k_\perp v_\alpha^\perp}{\Omega_\alpha}\right)$$
$$\times\left[k_\parallel\frac{\partial f_\alpha}{\partial p_\alpha^\parallel} + \frac{n\Omega_\alpha\partial f_\alpha}{v_\alpha^\perp\partial p_\alpha^\perp}\right]H^{(-)}(k_\parallel v_\alpha^\parallel + n\Omega_\alpha, -\mathbf{k}) + 2\pi i\frac{4\pi e_\alpha^2 n_\alpha}{k^2}$$
$$\times\left[k_\parallel\frac{\partial f_\alpha}{\partial p_\alpha^\parallel} + \frac{n\Omega_\alpha}{v_\alpha^\perp}\frac{\partial f_\alpha}{\partial p_\alpha^\perp}\right]F^{(-)}(k_\parallel v_\alpha^\parallel + n\Omega_\alpha, \mathbf{k})J_n\left(\frac{k_\perp v_\alpha^\perp}{\Omega_\alpha}\right)$$
$$+ n_\alpha e_\alpha^2(1 - \varepsilon^{(-)}(k_\parallel v_\alpha^\parallel + n\Omega_\alpha, \mathbf{k}))f_\alpha J_n\left(\frac{k_\perp v_\alpha^\perp}{\Omega_\alpha}\right). \quad (A.3)$$

Here $\epsilon^{(-)}(\omega, \mathbf{k}) = \epsilon^{(+)*}(\omega, \mathbf{k})$ is the dielectric constant of the plasma in a magnetic field. It is defined by Eq. (1.11). The functions $F^{(\pm)}$ are defined by the formulae

$$F^{(\pm)}(\omega, \mathbf{k}) = \frac{1}{2\pi i}\sum_\beta n_\beta e_\beta^2$$
$$\times\sum_{m=-\infty}^{\infty}\int_0^\infty\int_{-\infty}^\infty J_m^2\left(\frac{k_\perp v_\beta^\perp}{\Omega_\beta}\right)\frac{f_\beta(p_\beta^\perp, p_\beta^\parallel)}{\omega - k_\parallel v_\beta^\parallel - m\Omega_\alpha + i\Delta}2\pi p_\beta^\perp dp_\beta^\perp dp_\beta^\parallel. \quad (A.4)$$

The functions $H_{1,2}^{(\pm)}$ are defined in terms of the functions $h_\alpha^{(n)}$:

$$H_1^{(\mp)}(\omega, \mathbf{k}) = \frac{1}{2\pi i}\sum_\beta\int_0^\infty\int_{-\infty}^\infty\sum_{m=-\infty}^{\infty}J_m\left(\frac{k_\perp v_\beta^\perp}{\Omega_\beta}\right)\frac{h_\beta^{(m)}(k, p_\beta^\parallel, p_\beta^\perp)}{\omega - k_\parallel v_\beta^\parallel - m\Omega_\beta \pm i\Delta}$$
$$\times 2\pi p_\beta^\perp dp_\beta^\perp dp_\beta^\parallel. \quad (A.5)$$

The functions $H_2^{(\pm)}$ are obtained by replacing $h_\beta^{(n)}(\mathbf{k}, \mathbf{p}_\beta)$ by $h_\beta^{(n)}(-\mathbf{k}, \mathbf{p}_\beta)$.

By a method similar to the one used by Balescu and Taylor [7] we can show that

$$H_1^{(\pm)}(\omega, \mathbf{k}) = H_2^{(\pm)}(\omega, \mathbf{k}). \quad (A.6)$$

It is thus sufficient to know the piecewise-analytic function $H(\omega, \mathbf{k})$ to determine the functions $h_\alpha^{(n)}$. Multiplying Eq. (A.3) by

$$J_n(k_\perp v_\alpha^\perp/\Omega_\alpha)\,\delta(\omega - k_\parallel v_\alpha^\parallel - n\Omega_\alpha),$$

summing over α and n, integrating over p_α, and using the definition (A.5) and the property (A.6), we get the following expression for the discontinuity in the piecewise-analytic function:

$$\left(\frac{H^{(-)}}{\varepsilon^{(-)}} - \frac{H^{(+)}}{\varepsilon^{(+)}}\right)_{\omega,\mathbf{k}} = \left(\frac{F^{(-)}(1 - \varepsilon^{(+)}) - F^{(+)}(1 - \varepsilon^{(-)})}{\varepsilon^{(+)}\varepsilon^{(-)}}\right)_{\omega,\mathbf{k}}. \quad (A.7)$$

From the given discontinuity we find the functions $H^{(\pm)}/\epsilon^{(\pm)}$ themselves:

$$\frac{H^{(\pm)}(\omega, \mathbf{k})}{\varepsilon^{(\pm)}(\omega, \mathbf{k})} = \frac{1}{2\pi i}\int\frac{d\omega'}{\omega - \omega' \pm i\Delta}\left(\frac{F^{(-)}(1 - \varepsilon^{(+)}) - F^{(+)}(1 - \varepsilon^{(-)})}{\varepsilon^{(+)}\varepsilon^{(-)}}\right)_{\omega',\mathbf{k}}. \quad (A.8)$$

Substituting the functions $H_{1,2}^{(\pm)}$ found here into Eq. (A.3) we find the functions $h_\alpha^{(n)}$:

$$h_\alpha^{(n)}(\mathbf{k}, p_\alpha^\parallel, p_\alpha^\perp) = \frac{4\pi e_\alpha^2 n_\alpha}{k^2}J_n\left(\frac{k_\perp v_\alpha^\perp}{\Omega_\alpha}\right)\left[k_\parallel\frac{\partial f_\alpha}{\partial p_\alpha^\parallel} + \frac{n\Omega_\alpha}{v_\alpha^\perp}\frac{\partial f_\alpha}{\partial p_\alpha^\perp}\right]$$
$$\times\int d\omega\frac{(\delta\rho\delta\rho)_{\omega,\mathbf{k}}}{k_\parallel v_\alpha^\parallel + n\Omega_\alpha - \omega - i\Delta}$$
$$+ f_\alpha n_\alpha e_\alpha^2 J_n\left(\frac{k_\perp v_\alpha^\perp}{\Omega_\alpha}\right)\left(\frac{1 - \varepsilon^{(-)}(\omega, \mathbf{k})}{\varepsilon^{(-)}(\omega, \mathbf{k})}\right)_{\omega = k_\parallel v_\alpha^\parallel + n\Omega_\alpha}. \quad (A.9)$$

We have used in Eq. (A.9) Eq. (3.6) for $(\delta\rho\delta\rho)_{\omega,\mathbf{k}}$.

Substituting the expression we have found for $h_\alpha^{(n)}$ into the Fourier series we find the function $h_\alpha(\mathbf{k}, \mathbf{p}_\alpha)$ itself:

$$h_\alpha(k, \mathbf{p}_\alpha) = i\int_0^\infty \int_{-\infty}^\infty dt\, d\omega \exp\{i(\omega t + \mathbf{R}_\alpha(0, t, \mathbf{p}_\alpha, 0))\}$$
$$\times \left\{\frac{4\pi e_\alpha^2}{k^2} n_\alpha \mathbf{k} \frac{\partial f_\alpha(\mathbf{P}_\alpha(0, t, \mathbf{p}_\alpha))}{\partial \mathbf{P}_\alpha} (\delta\rho\delta\rho)_{\omega,\mathbf{k}} + \frac{n_\alpha e_\alpha^2}{2\pi} \frac{f_\alpha}{\varepsilon^{(-)}(\omega, \mathbf{k})}\right\}$$
$$- n_\alpha e_\alpha^2 f_\alpha. \qquad \text{(A.10)}$$

To obtain an expression for the correlation function we must substitute the function $h_\alpha(\mathbf{k}, \mathbf{P}_\alpha(0, \tau, \mathbf{p}_\alpha))$ into Eq. (A.1). Using Eqs. (1.9) and (1.10) we find that

$$\mathbf{P}_\alpha(0, t, \mathbf{P}_\alpha(0, \tau, \mathbf{p}_\alpha)) = \mathbf{P}_\alpha(0, t + \tau, \mathbf{p}_\alpha), \qquad \text{(A.11)}$$

$$\mathbf{R}_\alpha(0, t, \mathbf{P}_\alpha, (0, \tau, \mathbf{p}_\alpha))$$
$$= \mathbf{R}_\alpha(0, t + \tau, \mathbf{p}_\alpha, 0) - \mathbf{R}_\alpha(0, \tau, \mathbf{p}_\alpha, 0). \qquad \text{(A.12)}$$

Using these formulae and Eq. (A.10) we get an expression for the correlation function of a plasma in a magnetic field:

$$G_{\alpha\beta}(\mathbf{k}, \mathbf{p}_\alpha, \mathbf{p}_\beta) = \int_0^\infty dt \int_0^\infty d\tau \int_{-\infty}^\infty d\omega \left\{\left[\frac{4\pi e_\alpha 4\pi e_\beta}{k^4}\left(\mathbf{k}\frac{\partial f_\alpha(\mathbf{P}_\alpha(0, \tau, \mathbf{p}_\alpha))}{\partial \mathbf{P}_\alpha}\right)\right.\right.$$
$$\times \left(\mathbf{k}\frac{\partial f_\beta(\mathbf{P}_\beta(0, t+\tau, \mathbf{p}_\beta))}{\partial \mathbf{P}_\beta}\right)(\delta\rho\delta\rho)_{\omega,\mathbf{k}}$$
$$\left. + \frac{i}{2\pi}\frac{4\pi e_\alpha e_\beta}{k^2}\mathbf{k}\frac{\partial f_\alpha(\mathbf{P}_\alpha(0, \tau, \mathbf{p}_\alpha))}{\partial \mathbf{P}_\alpha}\frac{f_\beta(\mathbf{p}_\beta)}{\varepsilon^{(+)}(\omega, \mathbf{k})}\right]$$
$$\times \exp\{-i(\omega t + \mathbf{k}[\mathbf{R}_\beta(0, t + \tau, \mathbf{p}_\beta, 0)$$
$$- \mathbf{R}_\alpha(0, \tau, \mathbf{p}_\alpha, 0)])\} + \left[\frac{4\pi e_\alpha 4\pi e_\beta}{k^2}\left(\mathbf{k}\frac{\partial f_\beta(\mathbf{P}_\beta(0, \tau, \mathbf{p}_\beta))}{\partial \mathbf{P}_\beta}\right)\right.$$
$$\times \left(\mathbf{k}\frac{\partial f_\alpha(\mathbf{P}_\alpha(0, t+\tau, \mathbf{p}_\alpha))}{\partial \mathbf{P}_\alpha}\right)(\delta\rho\delta\rho)_{\omega,\mathbf{k}}$$
$$\left. - \frac{i}{2\pi}\frac{4\pi e_\alpha e_\beta}{k^2}\mathbf{k}\frac{\partial f_\beta(\mathbf{P}_\beta(0, \tau, \mathbf{p}_\beta))}{\partial \mathbf{P}_\beta}\frac{f_\alpha(\mathbf{p}_\alpha)}{\varepsilon^{(-)}(\omega, \mathbf{k})}\right]$$
$$\times \exp\{i(\omega t + \mathbf{k}[\mathbf{R}_\alpha(0, t + \tau, \mathbf{p}_\alpha, 0)$$
$$\left. - \mathbf{R}_\beta(0, \tau, \mathbf{p}_\beta, 0)])\}\right\}. \qquad \text{(A.13)}$$

The average over the angle variables of Eq. (A.13) is of the form

$$\langle G_{\alpha\beta}\rangle = \frac{4\pi e_\alpha e_\beta}{k^2}\sum_{m,n}\frac{J_n^2(k_\perp v_\alpha^\perp/\Omega_\alpha)\,J_m^2(k_\perp v_\beta^\perp/\Omega_\beta)}{k_\| v_\alpha^\| - k_\| v_\beta^\| + n\Omega_\alpha - m\Omega_\beta - i\Delta}$$
$$\times\left\{\left[k_\|\frac{\partial}{\partial p_\alpha^\|} + \frac{n\Omega_\alpha}{v_\alpha^\perp}\frac{\partial}{\partial p_\alpha^\perp}\right]f_\alpha\frac{f_\beta}{\varepsilon^{(+)}(k_\| v_\beta^\| + m\Omega_\beta)}\right.$$
$$-\left[k_\|\frac{\partial}{\partial p_\beta^\|} + \frac{m\Omega_\beta}{v_\beta^\perp}\frac{\partial}{\partial p_\beta^\perp}\right]f_\beta\frac{f_\alpha}{\varepsilon^{(-)}(k_\| v_\alpha^\| + n\Omega_\alpha)}$$
$$- i\frac{4\pi}{k^2}\left[k_\|\frac{\partial f_\alpha}{\partial p_\alpha^\|} + \frac{n\Omega_\alpha}{v_\alpha^\perp}\frac{\partial f_\alpha}{\partial p_\alpha^\perp}\right]\left[k_\|\frac{\partial f_\beta}{\partial p_\beta^\|} + \frac{m\Omega_\beta}{v_\beta^\perp}\frac{\partial f_\beta}{\partial p_\beta^\perp}\right]$$
$$\left.\times\left[(\delta\rho\delta\rho)^{(+)}_{k_\| v_\beta^\| + m\Omega_\beta} - (\delta\rho\delta\rho)^{(-)}_{k_\| v_\alpha^\| + n\Omega_\alpha}\right]\right\}.$$

Here

$$f_\alpha = f_\alpha(p_\alpha^\perp, p_\alpha^\|), \qquad f_\beta = f_\beta(p_\beta^\perp, p_\beta^\|).$$

Note added in proof (December 7, 1961): Equation (4.3) of our paper which is an example of an application of our general formulae to the case of a non-isothermal plasma is, in fact, the same as Eq. (22) in a paper by A. I. Akhiezer, I. A. Akhiezer, and A. G. Sitenko [JETP 41, 644 (1961), Soviet Phys. JETP 14, 462 (1962)]. In that paper the spectral functions of the particle distributions in a non-isothermal plasma are found by using a transport equation in which a random source with a given spectral function is included. Our paper contains essentially a proof that one can use a transport equation with a random source to evaluate the fluctuations in arbitrary steady states of collisionless plasma.

[1] J. Hubbard, Proc. Roy. Soc. (London) **A260**, 114 (1961).

[2] N. Rostoker, Yadernyĭ sintez (Nuclear Synthesis) **1**, 101 (1961).

[3] V. P. Silin, JETP **41**, 969 (1961), Soviet Phys. JETP **14**, 685 (1962).

[4] F. V. Bunkin, JETP **41**, 288 (1961), Soviet Phys. JETP **14**, 206 (1962).

[5] Yu. L. Klimontovich, DAN SSSR **96**, 43 (1954); JETP **33**, 982 (1957), Soviet Phys. JETP **6**, 753 (1958).

[6] B. B. Kadomtsev, JETP **32**, 943 (1957), Soviet Phys. JETP **5**, 771 (1957).

[7] R. Balescu and H. S. Taylor, Phys. Fluids **4**, 85 (1961).

[8] V. P. Silin, JETP **40**, 1768 (1961), Soviet Phys. JETP **13**, 1244 (1961).

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