CYCLOTRON RESONANCE IN A VARIABLE MAGNETIC FIELD

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Submitted to JETP editor May 19, 1961

J. Exptl. Theoret. Phys. (U.S.S.R.) 41, 1562-1565 (November, 1961)

The electrical conductivity tensor is calculated for an electron gas (plasma or semiconductor in the conduction band) situated in a magnetic field with a variable (harmonic) component parallel to the constant component. If the electric field induced by the variable magnetic field is comparable to the characteristic "plasma field" (1), the analysis will be valid only under the condition $p\tau \gtrsim 1$ (p is the cyclic frequency of the variation of the variable component of the magnetic field, τ is the effective mean time between electron collisions). It is shown that under these conditions the absorbing properties of the medium depend appreciably on the variable component of the magnetic field. Contrary to the opinion previously expressed[4,5] negative absorption is impossible.

RECENTLY discussions have taken place of the problem of the utilization of cyclotron resonance in semiconductors in order to obtain systems with negative absorption when the magnetic field has a variable component parallel to the constant component. It was supposed that negative absorption at the cyclotron resonance frequency $\omega_c = eH/mc$ can occur as a result of a parametric variation of the magnetic field at a frequency p close to the frequency $2\omega_c/n$ ($n = 1, 2, \ldots$).

In this paper we consider cyclotron (diamagnetic) resonance in a plasma or in a semiconductor in the case when the magnetic field **H** has a variable (harmonic) component parallel to the constant component. In this discussion we also take into account the electric field $\mathbf{E}_1$ induced by the variable magnetic field. If the field $\mathbf{E}_1$ is strong (cf., below), then the discussion is carried out subject to the condition (2) which is the condition under which separate maxima can be distinguished on the curve showing the absorption of energy in a weak external monochromatic field ($\mathbf{E}_2$). From subsequent discussion it follows that under these conditions the intensity of absorption (for $\mathbf{E}_2$) depends in an essential manner on the properties of the variable magnetic field (dependence on the properties of the field $\mathbf{E}_1$ is not significant). In particular, at the cyclotron resonance frequency ω_c this absorption can be reduced to zero. However, negative absorption is impossible.

We consider a certain volume of plasma or of a semiconductor situated in a homogeneous magnetic field **H** directed along the z axis ($H_z = H_0 - H_1 \cos pt$). Generally speaking, this volume will be simultaneously situated in a variable electric field $\mathbf{E}_1$ of frequency p. The plasma (or the semiconductor) under consideration situated in the field **H** (and also in the field $\mathbf{E}_1$ induced by it) represents a certain medium. We shall be interested in the electrical conductivity of this medium, and in the absorption in this medium of energy from a weak homogeneous electric field $\mathbf{E}_2 = \mathbf{E}_{20} \cos \omega t$ (external with respect to the medium under consideration, and having a general orientation with respect to the field **H**). The small contribution to the electrical conductivity (absorption) due to the heavy particles (molecules, ions) will not be taken into account, so that the given medium, in effect, represents an electron gas in the magnetic field **H** (and in the electric field $\mathbf{E}_1$).

The properties of this gas (in particular, the electrical conductivity and the absorptive power) depend on the fields **H** and $\mathbf{E}_1$. The field $\mathbf{E}_1$ must be regarded weak if its amplitude $\mathbf{E}_{10}$ satisfies the inequality

$$E_{10} \ll E_p = \sqrt{3kTme^{-2}\delta(p^2 + 1/\tau_0^2)}, \qquad (1)$$

where E_p is the characteristic "plasma field", e and m are the charge and the mass of the electron, k is the Boltzmann constant, T is the absolute temperature in the absence of $\mathbf{E}_1$, δ is the average relative fraction of the energy transferred in the collision of an electron with a heavy particle, and τ_0 is the effective time between collisions in the absence of $\mathbf{E}_1$. In this discussion the kinetic parameters of the electron gas under consideration (in particular τ) do not differ from their values in the absence of $\mathbf{E}_1$. However, if $E_{10} \gtrsim E_p$, then the field $\mathbf{E}_1$ is strong, and this, in the general case, determines the dependence of τ on $\mathbf{E}_1$ (and consequently, on the time). However, if the frequency p satisfies the condition

$$p\tau \gtrsim 1, \tag{2}$$

then the effective time between collisions τ "does not have time" to follow the variations of $\mathbf{E}_1$. In this case the quantity τ is a function only of the amplitude E_{10} [$\tau = \tau(E_{10})$] and is independent of the time.*

In order to determine the properties of interest to us for the medium under consideration we shall assume that it is situated in a weak external monochromatic field $\mathbf{E}_2 = \mathbf{E}_{20} \cos \omega t$ (we note right away that, in general, these properties can depend on the initial phase φ_0 if the field $\mathbf{E}_2$ is written in the form $\mathbf{E}_2 = \mathbf{E}_{20} \cos(\omega t + \varphi_0)$; however, in the present case this dependence is absent, as can be verified by a direct solution of the problem), and we shall utilize the equation for the average electron velocity[1-3]

$$m(d\mathbf{v}/dt + \mathbf{v}/\tau) = e(\mathbf{E} + c^{-1}[\mathbf{vH}]), \tag{3}†$$

where $\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2$. In future we shall consider only those cases when the quantity τ does not depend on the time. This is possible under the condition (2) if the field $\mathbf{E}_1$ is strong, and for any arbitrary frequency p if the field $\mathbf{E}_1$ is weak.

Since Eq. (3) is linear the total current density $\mathbf{J}$ ($\mathbf{J} = Ne\mathbf{v}$, N is the electron density) is the sum $\mathbf{J}_1 + \mathbf{J}_2$ of the currents corresponding to $\mathbf{E}_1$ and $\mathbf{E}_2$ [in both cases we have $\tau = \tau(E_{10})$]. It can be easily seen that in the steady state the time average of the energy of the total electric field $\mathbf{E}$ absorbed per unit time per unit volume of the medium can be decomposed into components which determine the absorption respectively for $\mathbf{E}_1$ and $\mathbf{E}_2$. In particular, for $\mathbf{E}_2$ this absorption is given by the expression

$$I_\omega = \lim_{T\to\infty} \frac{1}{T}\int_0^T \mathbf{J}_2\mathbf{E}_2\, dt. \tag{4}$$

The electrical conductivity tensor $\sigma_{ik}(\omega)$ for the medium under consideration determines in the usual manner the relation between the components of the harmonic component $\mathbf{J}_{2\omega}$ (of frequency ω) of the current $\mathbf{J}_2$ and the components of the field $\mathbf{E}_2$. At the same time

$$I_\omega = \tfrac{1}{2}(\mathrm{Re}\,\sigma_{xx}(\omega) E_{2x0}^2 + \mathrm{Re}\,\sigma_{yy}(\omega) E_{2y0}^2 + \mathrm{Re}\,\sigma_{zz}(\omega) E_{2z0}^2). \tag{5}$$

Thus, the problem reduces to finding the periodic solution of the equation for $\mathbf{J}_2$:

$$m(d\mathbf{J}_2/dt + \mathbf{J}_2/\tau(E_{10})) = e(Ne\,\mathbf{E}_2 + c^{-1}[\mathbf{J}_2\mathbf{H}]). \tag{6}$$

From this equation it follows immediately that in the coordinate system which we have selected $\sigma_{zx} \equiv \sigma_{xz} \equiv \sigma_{zy} \equiv \sigma_{yz} \equiv 0$, and that the usual expression holds for σ_{zz} in the absence of a magnetic field.[1,2] In order to determine the remaining components of the electrical conductivity tensor, we shall assume that the field $\mathbf{E}_2$ is directed perpendicular to the z axis. We obtain

$$\frac{dJ_{2x}}{dt} + \frac{J_{2x}}{\tau} = \frac{e}{mc}(H_0 - H_1\cos pt)\,J_{2y} + \frac{Ne^2}{m}E_{2x0}\cos\omega t,$$

$$\frac{dJ_{2y}}{dt} + \frac{J_{2y}}{\tau} = -\frac{e}{mc}(H_0 - H_1\cos pt)\,J_{2x} + \frac{Ne^2}{m}E_{2y0}\cos\omega t. \tag{7}$$

It can be easily shown that the fundamental system of solutions of (7) (for $E_{2x0} = E_{2y0} = 0$) has the form

$$Y_1 = e^{-t/\tau}\begin{pmatrix}\sin(\omega_c t - \Delta\sin pt)\\ \cos(\omega_c t - \Delta\sin pt)\end{pmatrix},$$

$$Y_2 = e^{-t/\tau}\begin{pmatrix}-\cos(\omega_c t - \Delta\sin pt)\\ \sin(\omega_c t - \Delta\sin pt)\end{pmatrix}. \tag{8}$$

Here $\omega_c = eH_0/mc$, $\Delta = eH_1/pmc$. On the basis of (8) we can easily find the desired solution of (7) in the presence of the field $\mathbf{E}_2$. As a final result we obtain

$$\mathrm{Re}\,\sigma_{xx}(\omega) = \frac{Ne^2}{2m}\sum_{n=-\infty}^{\infty} J_n^2(\Delta)\left\{\frac{1/\tau}{(\omega+\omega_c-np)^2+1/\tau^2} + \frac{1/\tau}{(\omega-\omega_c+np)^2+1/\tau^2}\right\},$$

$$\mathrm{Im}\,\sigma_{xx}(\omega) = \frac{Ne^2}{2m}\sum_{n=-\infty}^{\infty} J_n^2(\Delta)\left\{\frac{\omega+\omega_c-np}{(\omega+\omega_c-np)^2+1/\tau^2} + \frac{\omega-\omega_c+np}{(\omega-\omega_c+np)^2+1/\tau^2}\right\},$$

$$\mathrm{Re}\,\sigma_{yx}(\omega) = \frac{Ne^2}{2m}\sum_{n=-\infty}^{\infty} J_n^2(\Delta)\left\{\frac{\omega-\omega_c+np}{(\omega-\omega_c+np)^2+1/\tau^2} - \frac{\omega+\omega_c-np}{(\omega+\omega_c-np)^2+1/\tau^2}\right\},$$

$$\mathrm{Im}\,\sigma_{yx}(\omega) = \frac{Ne^2}{2m}\sum_{n=-\infty}^{\infty} J_n^2(\Delta)\left\{\frac{1/\tau}{(\omega+\omega_c-np)^2+1/\tau^2} - \frac{1/\tau}{(\omega-\omega_c+np)^2+1/\tau^2}\right\},$$

$$\sigma_{xy} = -\sigma_{yx}, \qquad \sigma_{yy} = \sigma_{xx}. \tag{9}$$

Here $J_n(\Delta)$ is a Bessel function of the n-th order.

From the equations obtained above it can be seen that the components of the electrical conductivity tensor depend both on the variable magnetic

*Strictly speaking, the condition for τ to be constant has the form[1,2] $p \gg \delta/\tau$, where δ – the mean relative fraction of the energy lost by an electron in a collision with a heavy particle – is always much smaller than unity.

†$[\mathbf{vH}] = \mathbf{v}\times\mathbf{H}$.

field (or Δ), and also on the electric field $\mathbf{E}_1$. The dependence on $\mathbf{E}_1$ consists of the fact that the effective time between collisions τ is a function of the amplitude E_{10}. If the field $\mathbf{E}_1$ is strong, then expressions (9) are valid under the condition (2) (in a weak field $\mathbf{E}_1$ they hold for any arbitrary value of p). In this case the intensity of absorption I_ω is determined by (5), from which it can be seen that only the absorption of energy of the component of the electric field $\mathbf{E}_2$ perpendicular to $\mathbf{H}$ depends on the variable magnetic field H_1, and that this absorption, in turn, is determined by the quantity $\mathrm{Re}\,\sigma_{xx}(\omega)$.

Equation (2) for the applicability of expressions (9) in a strong field $\mathbf{E}_1$ means in the case of $\mathrm{Re}\,\sigma_{xx}(\omega)$ that this function has separate maxima in the neighborhood of the frequencies $\pm\omega_c \pm kp$ $(k = 0, 1, 2, \ldots)$.

The dependence of the absorption curve on $\mathbf{E}_1$ is unimportant in the sense that it leads only to a change in the shape of these maxima, but does not qualitatively alter the absorption picture. However, the dependence of $\mathrm{Re}\,\sigma_{xx}(\omega)$ on the variable component of the magnetic field (or on Δ) is essential, and for all Δ the quantity $\mathrm{Re}\,\sigma_{xx}(\omega) > 0$.

For $H_1 = 0$ only $J_0(0) = 1$ is different from zero, and this gives the well-known expression for cyclotron resonance.[1-3] As H_1 increases the number of noticeable maxima increases. Moreover, from the properties of Bessel functions it follows that the height of each fixed maximum periodically falls to zero and increases again. In particular, the principal maximum (at the frequency ω_c) disappears after the field H_1 is introduced when Δ coincides with the first zero of $J_0(\Delta)$.

The author is deeply grateful to F. V. Bunkin for useful comments and discussion.

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Translated by G. Volkoff
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