EFFECT OF POLARIZATION OF THE MEDIUM ON THE DEVELOPMENT OF ELECTRON-PHOTON SHOWERS

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A one-dimensional cascade theory of electron-photon showers is developed in the A-approximation with account of polarization for a dense medium. The integral energy spectra of shower particles at depths $t = 1$ and $t = 2$ in lead and air are computed. The results of the calculations are compared with the data of the usual cascade theory.

HIGH-ENERGY electron-photon showers in a dense medium exhibit many properties that are due to the character of the electromagnetic interaction in the medium. If the particle energy exceeds 10^{12} ev, then the cross sections for bremsstrahlung and pair production in the dense medium are greatly influenced by multiple scattering of the shower particles by the atoms of the medium. The theory of this phenomenon was developed by Landau and Pomeranchuk[1] and also by Migdal.[2] The differential cross sections for bremsstrahlung and pair production were calculated by Migdal[3] with account of multiple scattering. These proved to be not homogeneous functions of the energy, as were the Bethe-Heitler cross sections[4] that describe the interaction between the particles and the individual atoms. The cascade-theory formalism is therefore not suitable for the analysis of high-energy showers in a condensed medium. High-energy showers in the first few t-units of matter were investigated by the Monte Carlo method.[5-8]

The polarization of the medium plays an important role in the interaction between charged particles and matter. The differential cross section of the bremsstrahlung of an electron in the medium, with account of polarization, was found by Ter-Mikaélyan[9] to be

$$W_c(E, E') = E^{-2}E'^{-1}\left(\tfrac{4}{3}E^2 - \tfrac{4}{3}EE' + E'^2\right)\left[1 + \omega^2 (E/E')^2\right]^{-1}. \quad (1)$$

Here $\omega = \sqrt{4\pi NZe^2\hbar^2/m^3c^4}$, N—number of atoms per cm^3 of the medium, Z—charge of the nuclei of the medium, m and e—mass and charge of the electron, ħ—Planck's constant, c—velocity of light.

Ter-Mikaélyan's cross section (1) differs from the Bethe-Heitler cross section by a factor $[1 + \omega^2(E/E')^2]^{-1}$. If the ratio E'/E of the emitted photon energy to the electron energy is of the same order as or less than ω, this factor is appreciably smaller than unity. This means that the probability of an electron emitting a low-energy photon in the medium is much less than in the interaction with the individual atom. We note that the cross section for electron bremsstrahlung in the medium tends to zero if the energy of the emitted photon tends to zero.

Our problem is to obtain the integral spectra of the electrons and photons in showers that develop in lead and air. The value of ω is 1.9×10^{-4} for lead and 7.5×10^{-5} for air. When $E'/E < 10^{-4}$, the integral number of shower particles in these media should be less than usual.[10] The development of a shower in a dense medium is described in the A approximation by the usual cascade equations. The bremsstrahlung cross section is given by expression (1), and the pair-production cross section is taken from the book by Belen'kii.[10] Since the cross sections of all the processes are functions of the ratio E'/E, we can use the method of functional transformations[10] to solve the problem.

Going from the transforms $P(t, s)$ and $\Gamma(t, s)$ to the originals $P(t, E)$ and $\Gamma(t, E)$[5] and integrating with respect to the energy we obtain the expressions for the integral spectrum of the electrons and photons

$$N_P(t, E) = \frac{1}{2\pi i}\int_{\delta - i\infty}^{\delta + i\infty}\left[\left(\frac{E_0}{E}\right)^s - 1\right]\left\{\frac{\sigma_0 + \lambda_1(s)}{\lambda_1(s) - \lambda_2(s)}e^{\lambda_1(s)t} - \frac{\sigma_0 + \lambda_2(s)}{\lambda_1(s) - \lambda_2(s)}e^{\lambda_2(s)t}\right\}ds,$$

$$N_\Gamma(t, E) = \frac{1}{2\pi i}\int_{\delta - i\infty}^{\delta + i\infty}\left[\left(\frac{E_0}{E}\right)^s - 1\right]C(s)\left\{\frac{\sigma_0 + \lambda_1(s)}{\lambda_1(s) - \lambda_2(s)}e^{\lambda_1(s)t} - \frac{\sigma_0 + \lambda_2(s)}{\lambda_1(s) - \lambda_2(s)}e^{\lambda_2(s)t}\right\}ds. \quad (2)$$

s	$A(s)$	$B(s)$	Air					Lead				
			$C(s)$	$\lambda_1(s)$	$\lambda_1'(s)$	$-\lambda_2(s)$	$H_1(s)\cdot 10$	$C(s)$	$\lambda_1(s)$	$-\lambda_1'(s)$	$-\lambda_2(s)$	$H_1(s)\cdot 10$
−1.0	−∞	∞	$2.06\cdot 10^6$	∞	∞	∞	5.00	$1.75\cdot 10^5$	∞	∞	∞	5.00
−0.9	−9.73	18.8	$5.24\cdot 10^5$	3150	32800	3140	5.01	7170	372	3600	360	5.07
−0.8	−4.63	8.97	$1.37\cdot 10^5$	1110	10500	1100	5.01	3010	166	1160	163	5.08
−0.7	−2.86	5.76	$3.68\cdot 10^4$	461	3840	459	5.02	579	87.6	522	85.6	5.11
−0.6	−1.94	4.19	$1.02\cdot 10^4$	207	1690	206	5.03	266	49.9	267	48.7	5.14
−0.5	−1.36	3.27	$2.93\cdot 10^3$	98.3	711	97.7	5.05	127	28.8	147	29.2	5.18
−0.4	−0.950	2.68	884	48.8	331	48.6	5.09	63.5	18.5	84.8	18.4	5.23
−0.3	−0.637	2.26	284	25.3	160	26.4	5.14	33.5	11.9	63.1	12.1	5.29
−0.2	−0.389	1.96	99.3	13.8	80.1	14.2	5.21	18.7	7.93	31.3	8.31	5.36
−0.1	−0.185	1.73	38.9	7.92	41.4	8.51	5.29	11.2	5.42	23.4	6.01	5.42
0	0	1.55	17.6	4.84	22.2	5.61	5.37	11.2	3.79	12.9	4.57	5.46
0.1	0.162	1.40	9.23	3.15	12.6	4.07	5.43	7.17	2.72	8.77	3.65	5.49
0.2	0.286	1.28	5.58	2.15	7.69	3.21	5.45	4.90	1.99	6.12	3.05	5.48
0.3	0.407	1.18	3.78	1.53	5.17	2.71	5.43	3.55	1.46	4.42	2.64	5.45
0.4	0.515	1.10	2.78	1.11	3.55	2.39	5.37	2.70	1.08	3.31	2.37	5.37
0.5	0.615	1.02	2.17	0.803	2.65	2.19	5.27	2. 4	0.79	2.55	2.18	5.27

The poles of the Mellin transforms $P(t, s)$ and $\Gamma(t, s)$ coincide with the poles of the functions $A(s)$, $B(s)$, $C(s)$, and σ_0. Thus, the region of existence $P(t, s)$ and $\Gamma(t, s)$ can be determined if we know which of the functions $A(s)$, $B(s)$, $C(s)$ and σ_0 has the pole with the largest real part.

In the case of the Bethe-Heitler cross section, the pole with the largest real part $s = 0$ belongs to $C(s)$. This means that $P(t, s)$ and $\Gamma(t, s)$ exist when $s > 0$ and when $\delta > 0$ in (2). The numerical values of $A(s)$, $B(s)$, $C(s)$, and σ_0 are given by Belen'kii.[10] The use of bremsstrahlung cross sections in the form (1) results in different expressions for $A(s)$ and $C(s)$, while $B(s)$ and σ_0 remain unchanged:

$$A(s) = \int_0^1 [\tfrac{4}{3}(1 - x) + x^2]\,[1 - (1 - x)^s]\,\frac{x}{x^2 + \omega^2}\,dx,$$

$$C(s) = \int_0^1 [\tfrac{4}{3}(1 - x) + x^2]\,\frac{x^{s+1}}{x^2 + \omega^2}\,dx. \qquad (3)$$

Using the smallness of ω, we can show that $A(s)$ coincides within 1% with the value obtained from the Bethe-Heitler cross sections. The integral (3) can be reduced to a sum of integrals of the type

$$\int_0^{\omega^{-2}} u^{\nu}\,\frac{1}{1+u}\,du, \qquad 0 < \operatorname{Re}\nu < 1.$$

In the evaluation of this integral it is convenient to divide the integration region into two parts, $0 < u < 1$ and $1 < u < \omega^{-2}$. Since ω^{-2} is of the order of 10^8, the upper limit of integration in the second region can be set equal to infinity. Then the integrals in both regions are expressed in terms of the Euler β function. The approximate expressions obtained in this fashion

$$C(s) = \begin{cases} \frac{4}{3}\,\omega^s\,\frac{\pi(s+2)}{2}\,\sin^{-1}\frac{\pi(s+2)}{2}\,\frac{1}{s+2} + \frac{1}{s+2} \\ \quad\times\left[1 - \omega^{s+2}\,\frac{\pi(s+2)}{2}\,\sin^{-1}\frac{\pi(s+2)}{2}\right] \\ \quad - \frac{4}{3(s+1)}\left[1 - \omega^{s+1}\,\frac{\pi(s+1)}{2}\,\sin^{-1}\frac{\pi(s+1)}{2}\right] + \frac{4}{3s}, \\ \quad -1 \geqslant \operatorname{Re}s > -2 \\ \frac{1}{s+2} - \frac{4}{3(s+1)}\left[1 - \omega^{s+1}\,\frac{\pi(s+1)}{2}\,\sin^{-1}\frac{\pi(s+1)}{2}\right] \\ \quad + \frac{4}{3s}\left[1 - \omega^s\,\frac{\pi s}{2}\,\sin^{-1}\frac{\pi s}{2}\right], \qquad 0 \geqslant \operatorname{Re}s \geqslant -1 \\ \frac{1}{s+2} - \frac{4}{3(s+1)} + \frac{4}{3s}\left[1 - \omega^s\,\frac{\pi s}{2}\,\sin^{-1}\frac{\pi s}{2}\right], \\ \qquad 1 \geqslant \operatorname{Re}s \geqslant 0 \\ \frac{1}{s+2} - \frac{4}{3(s+1)} + \frac{4}{3s}, \qquad \operatorname{Re}s \geqslant 0 \end{cases}$$

differ from the exact ones by terms of order ω^2. The pole having the largest real part $s = -1$ belongs to the functions $A(s)$ and $B(s)$, and the Mellin transforms exist when $s > -1$. The table lists the values of the functions, necessary for numerical calculations, for real $-1 < s \leq 0.5$ in the case of lead and air.

We note that when $0 < \operatorname{Re} s < 0.5$, $C(s)$ (and hence $\lambda_1(s)$) is smaller than the corresponding functions obtained by Belen'kii[10] with Bethe-Heitler cross sections. When $s \geq 0.5$ the functions $C(s)$ and $\lambda_1(s)$, which pertain to cross sections, are practically identical.

When $t > 1$ we can neglect the second term in the intergrand of (2). We then have approximately

$$N_P(t, E) = \frac{1}{2\pi i}\int_{\delta-i\infty}^{\delta+i\infty}\left[\left(\frac{E_0}{E}\right)^s - 1\right]H_1(s)\,e^{\lambda_1(s)\,t}\,ds,$$

$$N_\Gamma(t, E) = \frac{1}{2\pi i}\int_{\delta-i\infty}^{\delta+i\infty}\left[\left(\frac{E_0}{E}\right)^s - 1\right]C(s)\,H_1(s)\,e^{\lambda_1(s)\,t}\,ds. \qquad (4)$$

We have evaluated the integrals in (4) by the saddle-point method, using for the exponent the function

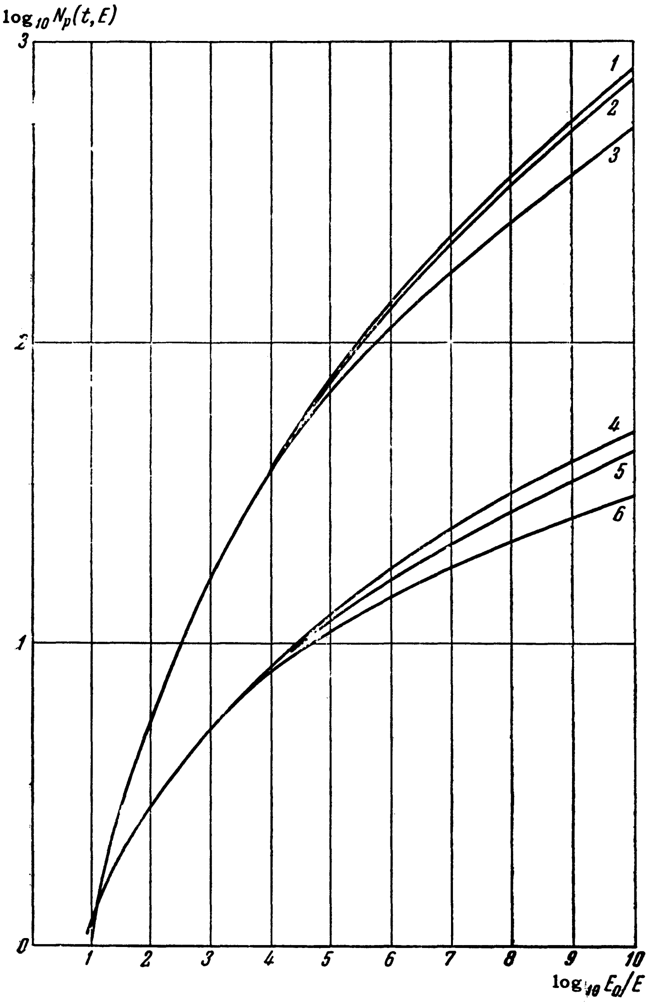

FIG. 1. Integral spectrum of electrons: 1 – t = 2, Bethe-Heitler cross section; 2 – t = 2, Ter-Mikaélyan cross sections (in air); 3 – t = 2, Ter-Mikaélyan cross sections (in lead); 4 – t = 1, Bethe-Heitler cross sections; 5 – t = 1, Ter-Mikaélyan cross sections (in air); 6 – t = 1, Ter-Mikaélyan cross sections (in lead).

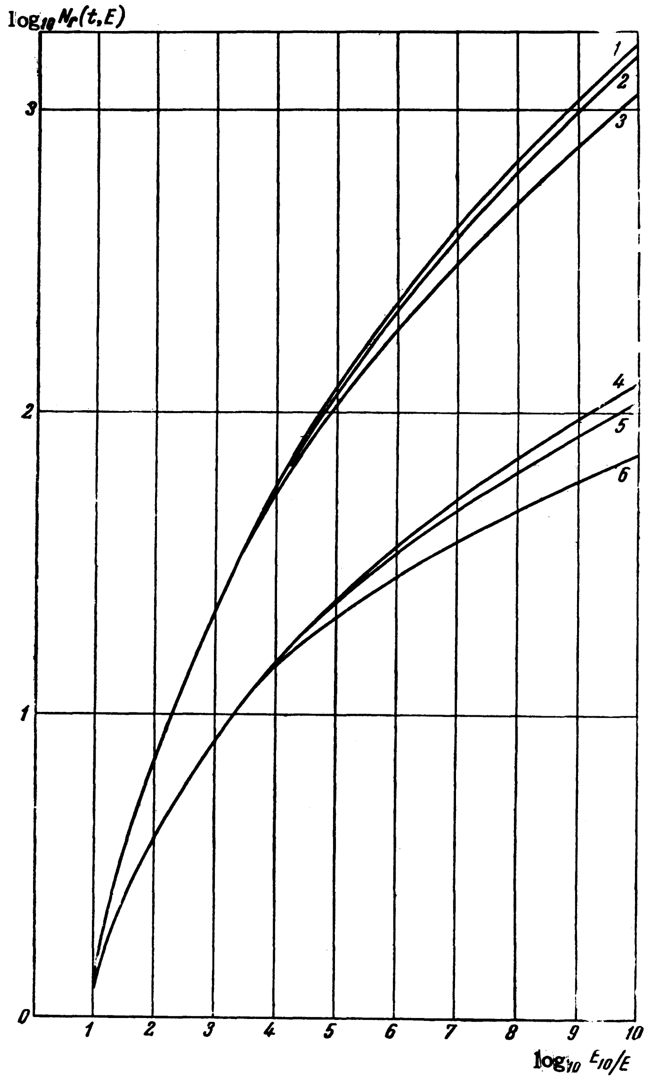

FIG. 2. Integral spectrum of photons: 1 – t = 2, Bethe-Heitler cross section; 2 – t = 2, Ter-Mikaélyan cross sections (in air); 3 – t = 2, Ter-Mikaélyan cross sections (in lead); 4 – t = 1, Bethe-Heitler cross section; 5 – t = 1, Ter-Mikaélyan cross sections (in air); 6 – t = 1, Ter-Mikaélyan cross sections (in lead).

$$\varphi(s) = \lambda_1(s)\, t + \ln \frac{(E_0/E)^s - 1}{s}.$$

The calculated formulas are

$$\begin{aligned} N_P(t, E) &= H_1(s)\, e^{\lambda_1(s)\, t} (u - 1)/s\sqrt{2\pi d^2\varphi/ds^2}, \\ N_\Gamma(t, E) &= C(s)\, H_1(s)\, e^{\lambda_1(s)\, t} (u - 1)/s\sqrt{2\pi d^2\varphi/ds^2}, \end{aligned} \qquad (5)$$

where

$$H_1(s) = \frac{\sigma_0 + \lambda_1(s)}{\lambda_1(s) - \lambda_2(s)}, \quad \frac{d^2\varphi}{ds^2} = \frac{1}{s^2}\left\{1 - \frac{u \ln^2 u}{(u-1)^2}\right\} + t\frac{d^2\lambda_1(s)}{ds^2},$$

and $u = (E_0/E)^s$ is determined from the equation

$$\frac{u \ln u}{u - 1} = 1 - ts\frac{d\lambda_1(s)}{ds}.$$

We have calculated the integral spectrum of the electrons and photons due to a primary electron in lead or air at depths t = 1 and t = 2. The results are illustrated in Figs. 1 and 2, which also show (curves 1 and 4) the results obtained by Messel et al[11] using Bethe-Heitler cross sections.

The integral spectra of the sharp particles become different when $E_0/E > 10^{-4}$, the difference increasing with the ratio E_0/E. This difference, however, is quite small. Even when $E_0/E = 10$ the shortage of shower particles in lead, for the spectra calculated by us, amounts to 10% of Messel's spectrum. By virtue of the multiple scattering of the particles we should confine ourselves to an examination of showers generated by a primary of energy $E_0 < 10^{12}$. It is therefore advantageous to consider only the results pertaining to $E_0/E \leq 10^6$. In the case of lead, the difference in the spectra is only 5% of the Messel spectrum when $E_0/E = 10^6$. We can therefore state that the polarization of the medium hardly influences the integral spectrum of the shower particles. Polarization in a less dense medium, such as in air, is even less effective. As the depth of shower observation increases, the integral spectrum in the dense medium becomes equal to the ordinary spectrum. We have disregarded the fact that the electrons lose energy to ionization. This process will decrease the polarization effect for the observed showers. Our results pertain to integral spectra. The influence of polarization on the differential spectra is apparently more appreciable.

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Note added in proof (October 30, 1961). **Supplementing the table, we give the following values of $\lambda_1''(s)$ for several values of s:**

s	—0.9	—0.7	—0,5	—0,3	—0.2	—0.1	0	0,1	0.2	0.3
λ_1'' (air)	$5.47\cdot 10^5$	35500	5580	1140	541	267	132	68,1	35,9	19,7
λ_1'' (lead)	54400	3760	842	254	147	75.6	52,5	33,5	21,3	13.9

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