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TRANSFORMATION OF SOUND AND ELECTROMAGNETIC WAVES AT THE BOUNDARY OF A CONDUCTOR IN A MAGNETIC FIELD

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The electromagnetic field induced when a sound wave strikes the boundary between a conducting and nonconducting medium in a weak magnetic field is calculated. The amplitude of the sound waves outgoing from an interface on which electromagnetic waves are incident is also determined.

1. INTRODUCTION

WHEN a sound wave strikes the boundary of a conductor situated in an external magnetic field, electromagnetic waves are induced by the oscillations of the conducting medium, along with the reflected and refracted sound waves. Accordingly, when an electromagnetic wave is incident on a conductor, sound waves that move away from the interface are produced. They are brought about by the Lorentz force acting in the external magnetic field on the current produced when the electromagnetic wave passes through the conductor.

This mutual convertibility of acoustic and electromagnetic waves [1] can be of interest in the research on magnetoacoustic effects in metals, [2] in sea water, [1] or in plasma, as a possible mechanism of radio emission from the sun and the stars, and can also be used to produce electromagnetic waves with large retardation (on the order of the ratio of the velocity of light to that of sound) and to generate ultrasound directly in a conducting medium (for example, in a metal).

We confine ourselves in this article to a case of a conducting liquid bounding on a non-conducting (liquid or gas) half-space ("air"), where the acoustic and electromagnetic wavelengths in the substance (skin depth) greatly exceed the characteristic mean free path (accordingly, the oscillation frequency ω is much less than the collision frequency $1/\tau$, i.e., $\omega\tau \ll 1$). This means that the hydrodynamic approach is applicable and normal skin effect takes place.

A highly conducting medium ($\sigma/\epsilon \gg \omega$) in a constant magnetic field **B** is characterized by two frequencies:

$$\omega_s = 4\pi\sigma s^2/c^2, \qquad \omega_u = 4\pi\sigma u^2/c^2. \tag{1}$$

Here σ — conductivity, s — velocity of sound, and $u = B/\sqrt{4\pi\rho_0}$ — Alfven velocity. When $\omega = \omega_s$ (or $\omega = \omega_u$) the electromagnetic wavelength in the substance $\lambda\!\!\!^{-}_{em} = c/\sqrt{4\pi\sigma\omega}$ is comparable with the acoustic wavelength $\lambda\!\!\!^{-}_{ac} = s/\omega$ (Alfven wavelength $\lambda\!\!\!^{-}_a = u/\omega$). Obviously $\sigma/\epsilon \gg \omega$; in addition, in relatively weak magnetic fields such as we are considering, we have

$$\omega_u/\omega_s = u^2/s^2 \ll 1. \tag{2}$$

In view of the fact that the magnetoacoustic effects, which are quite small by virtue of (2) and which manifest themselves in movements of sufficiently large scale, decrease with increasing ω when $\omega > \omega_s$, we confine ourselves to the frequency region

$$\sigma/\varepsilon \gg \omega \gg \omega_u. \tag{3}$$

We note that the medium can be regarded as infinitely conducting only for frequencies $\omega \ll \sigma/\epsilon$, ω_s, ω_u, i.e., when the skin depth is the smallest of all the characteristic lengths. For liquid metals $\sigma \sim 10^{17}$ sec^{-1}, $\omega_s/2\pi \sim 10^6$ cps, $\omega_u \sim 10^3$ sec^{-1} in a field $B \sim 10^4$ oe; in sea water with $\sigma = 4 \times 10^{10}$ sec^{-1} we have $\omega_s/2\pi \sim 2$ cps and $\omega_u \sim 10^{-12}$ sec^{-1} in the earth's magnetic field ($B \sim 1$ oe).

Subject to limitations (2) and (3), the transformation of acoustic and electromagnetic perturbations can be regarded as particular cases of reflection and refraction of waves in magnetohydrodynamics (more accurately, the conversion of various types of waves into each other). Reflection and refraction in magnetohydrodynamics have been analyzed only for Alfven waves in a perfectly conducting medium. [3] We likewise disregard the general case of arbitrary frequencies and magnetic fields and confine ourselves to the condition of weak hydromagnetic coupling (2) with finite conductivity of the medium (3).

In the magnetohydrodynamic approximation there exist in the conducting liquid, as is well known, three types of waves (Alfven and fast or slow magnetic sound), in which both "acoustic" (the velocity **v** or the pressure p) and "electromagnetic" quantities (the current **j**, the electric and magnetic field **E** and **H**) oscillate. Under conditions (2) and (3), the Alfven wave and the slow magnetic-sound wave are electromagnetic waves in the medium, while the fast magnetic-sound wave is a sound wave. These waves being modified as follows: the sound wave carries also induced oscillating electromagnetic quantities, while the electromagnetic waves contain acoustic quantities whose reaction on the fields that generate them can be neglected. Since the electromagnetic and acoustic quantities are interrelated in a conducting medium situated in a magnetic field, incidence of any one of the three waves (sound or one of the two electromagnetic waves, which have different polarizations) gives rise to six waves moving away from the boundary (four electromagnetic and two sound). In particular, when a sound wave incident from air is reflected from a liquid surface, electromagnetic wave are produced. The conversion coefficients for these waves are calculated in Secs. 4 and 5, where we also consider the incidence of sound on a conducting medium and determine the coefficients for the conversion of electromagnetic waves into sound waves.

2. WAVES IN A CONDUCTING LIQUID

If we neglect the viscosity and heat conduction of the medium, and also the displacement current, then the dispersion equation connecting the frequency ω with the wave vector **k** of a plane wave in a conducting liquid[4] reduces to

$$[X-(\Omega_u \cos^2\theta - i\Omega)^{-1}]\,\{X^2(\Omega_u\cos^2\theta - i\Omega) - X(1-i\Omega+\Omega_u)+1\}=0. \tag{4}$$

Here

$$X=(ks/\omega)^2,\quad \Omega=\omega/\omega_s,\quad \Omega_u=\omega_u/\omega_s,$$
$$\cos\theta=\varkappa\mathbf{h},\quad \varkappa=\mathbf{k}/k,\quad \mathbf{h}=\mathbf{B}/B. \tag{5}$$

The electrodynamic and acoustic quantities are expressed in terms of the current by means of the following chain of equations

$$\mathbf{f}=\frac{B}{c}[\mathbf{jh}],\quad \mathbf{v}=\frac{i}{\rho_0\omega}\left\{\frac{X}{1-X}(\varkappa\mathbf{f})\,\varkappa+\mathbf{f}\right\},$$
$$p=\rho_0 s^2\frac{k}{\omega}(\varkappa\mathbf{v}),\quad \mathbf{E}=\frac{\mathbf{j}}{\sigma}-\frac{B}{c}[\mathbf{vh}],\quad \mathbf{H}=\frac{kc}{\omega}[\varkappa\mathbf{E}]. \tag{6}^*$$

*$[\mathbf{jh}]=\mathbf{j}\times\mathbf{h}$; $(\varkappa\mathbf{v})=\varkappa\cdot\mathbf{v}$.

Under conditions (2) and (3), the dispersion laws become

$$X_a=X_{sl}=i/\Omega,\quad X_a=(sk_a/\omega)^2,\quad X_{sl}=(sk_{sl}/\omega)^2, \tag{7}$$
$$X_{ac}=1-\Omega_u\sin^2\theta/(1+i\Omega),\quad X_{ac}=(sk_{ac}/\omega)^2, \tag{8}$$

where $\mathbf{k}_a$, $\mathbf{k}_{sl}$, and $\mathbf{k}_{ac}$ are the wave vectors of the Alfven, slow magnetic-sound and sound waves. The connection between the fields amplitudes (wave "polarization") in a frame having unit vectors

$$\varkappa=\mathbf{k}/k,\quad \boldsymbol{\eta}=[[\varkappa\mathbf{h}]\,\varkappa]/\sin\theta,\quad \boldsymbol{\zeta}=[\varkappa\mathbf{h}]/\sin\theta, \tag{9}$$

which are fixed in the wave, is as follows:

in the modified sound wave

$$\mathbf{E}_{ac}=\mathscr{E}_{ac}\boldsymbol{\zeta}_{ac}p_{ac},\quad \mathbf{H}_{ac}=\mathscr{H}_{ac}\boldsymbol{\eta}_{ac}p_{ac},\quad \mathbf{j}_{ac}=J_{ac}\boldsymbol{\zeta}_{ac}p_{ac},$$
$$\mathbf{v}_{ac}=\mathbf{a}p_{ac},\quad \mathbf{a}=\frac{\varkappa_{ac}}{\rho_0 s},\quad \mathscr{E}_{ac}=-\frac{B}{\rho_0 sc}\frac{\sin\theta_{ac}}{1+i\Omega},$$
$$\mathscr{H}_{ac}=-\frac{c}{s}\mathscr{E}_{ac},\quad J_{ac}=i\frac{\omega c}{Bs}\frac{\Omega_u\sin\theta_{ac}}{1+i\Omega}; \tag{10}$$

in the Alfven wave

$$\mathbf{E}_a=\boldsymbol{\eta}_a j_a/\sigma,\quad \mathbf{H}_a=\mathscr{H}_a\boldsymbol{\zeta}_a j_a,\quad \mathbf{j}_a=\boldsymbol{\eta}_a j_a,\quad p_a=0,$$
$$\mathbf{v}_a=\mathbf{V}_a j_a,\quad \mathbf{V}_a=-(iB/\rho_0\omega c)\,\boldsymbol{\zeta}_a\cos\theta_a,$$
$$\mathscr{H}_a=(c/\sigma s)\sqrt{i/\Omega},\quad \operatorname{Re}\sqrt{i}>0; \tag{11}$$

in the slow magnetic-sound wave

$$\mathbf{E}_{sl}=\boldsymbol{\zeta}_{sl}j_{sl}/\sigma,\quad \mathbf{H}_{sl}=\mathscr{H}_{sl}\boldsymbol{\eta}_{sl}j_{sl},\quad \mathbf{j}_{sl}=\boldsymbol{\zeta}_{sl}j_{sl},$$
$$\mathbf{v}_{sl}=\mathbf{V}_{sl}j_{sl},\quad \mathbf{V}_{sl}=\frac{iB}{\rho_0\omega c}\left(\varkappa_{sl}\frac{\sin\theta_{sl}}{i/\Omega-1}+\boldsymbol{\eta}_{sl}\cos\theta_{sl}\right),$$
$$p_{sl}=P_{sl}j_{sl},\quad P_{sl}=B\sqrt{\frac{i}{4\pi\sigma\omega}}\frac{\sin\theta_{sl}}{1+\Omega i},\quad \mathscr{H}_{sl}=-\mathscr{H}_a. \tag{12}$$

The difference between (8) and the dispersion law for sound

$$X_{ac}=(sk_{ac}/\omega)^2=1 \tag{8'}$$

is responsible for the appearance of electric and magnetic fields in the modified wave (10), and also for the additional anisotropic attenuation of the sound, connected with the Joule losses. It will be sufficient in what follows to use the dispersion law (8′), and resort to the more exact expression (8) only in the investigation of the singularities of the conversion coefficient.

We now introduce a coordinate frame fixed to the interface (plane x = 0) and to the external magnetic field **B**. We direct the unit vector $\mathbf{e}_x$ from the liquid into the air. If **q** is the wave-vector component parallel to the interface, φ the angle of incidence, γ the angle between **q** and the plane z = 0 in which the magnetic field **B** is located, and ϵ the angle of inclination of the magnetic field, we obtain

$$\varkappa=\mathbf{e}_x\cos\varphi+\frac{\mathbf{q}}{q}\sin\varphi,\qquad \frac{\mathbf{q}}{q}=\mathbf{e}_y\cos\gamma+\mathbf{e}_z\sin\gamma,$$
$$\mathbf{h}=-\mathbf{e}_x\sin\varepsilon+\mathbf{e}_y\cos\varepsilon,$$
$$\cos\theta=-\cos\varphi\sin\varepsilon+\sin\varphi\cos\gamma\cos\varepsilon. \tag{13}$$

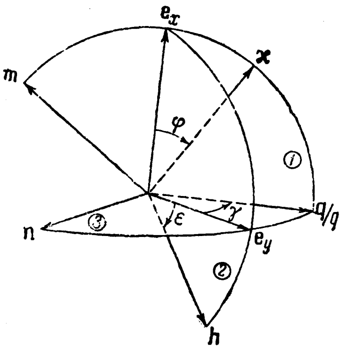

FIG. 1. System of unit vectors fixed in the interface; 1 – plane of incidence; 2 – plane in which the normal to the interface and the constant magnetic field lie; 3 – interface.

To describe the electromagnetic waves in air we introduce the unit vectors (Fig. 1)

$$\mathbf{n} = \left[\frac{\mathbf{q}}{q}\mathbf{e}_x\right], \qquad \mathbf{m} = [\mathbf{n}\varkappa_{el}];$$

$$\mathbf{n} = \mathbf{e}_y \sin\gamma - \mathbf{e}_z \cos\gamma, \qquad \mathbf{m} = \mathbf{e}_x \sin\varphi_{el} - \frac{\mathbf{q}}{q}\cos\varphi_{el}. \quad (14)$$

They are orthogonal to κ_{el}, with **n** lying in the interface and **m** in the plane of incidence. We choose as the two independent waves with specified value of κ_{el} TE and TM waves with electric or magnetic field respectively parallel to **n**:

$$\mathbf{E} = E_n\mathbf{n} + H_n\mathbf{m}, \qquad \mathbf{H} = H_n\mathbf{n} - E_n\mathbf{m};$$
$$\omega = ck_{el}, \qquad \overline{\mathbf{v}_{el}} = 0, \qquad \overline{p_{el}} = 0, \qquad \overline{\mathbf{j}_{el}} = 0. \quad (15)$$

The electric field in the TM wave and the magnetic field in the TE wave are parallel to **m**.

For sound in air we obviously have

$$\omega = \bar{s}\bar{k}_{ac}, \quad \bar{\mathbf{v}}_{ac} = \bar{\mathbf{a}}\bar{p}_{ac}, \quad \bar{\mathbf{a}} = \bar{\varkappa}_{ac}/\bar{\rho}_0\bar{s},$$
$$\bar{\mathbf{H}}_{ac} = \bar{\mathbf{E}}_{ac} = \bar{\mathbf{j}}_{ac} = 0. \quad (16)$$

The quantities pertaining to air are designated by a bar (except for **E** and **H** in the electromagnetic wave).

3. BOUNDARY CONDITIONS. CONVERSION OF WAVES

To be specific, we discuss first a sound wave incident from a conducting medium onto the interface. The connection between the angles of reflection, refraction, and conversion (the definition of the latter is obvious) and the angle of incidence φ'_{ac} follows from the equality of the frequencies ω and of the tangential wave-vector components q in the incident plane wave and in all the outgoing waves: $\sqrt{X_i}\,\sin\varphi_i = \sqrt{X'_{ac}}\,\sin\varphi_{ac}$. The principal terms of these equations have the form

$$\sin\varphi'_{ac} = \sin\varphi_{ac} = \frac{s}{\bar{s}}\,\sin\bar{\varphi}_{ac} = \frac{s}{c}\sin\varphi_{el}$$
$$= \sqrt{\frac{i}{\Omega}}\sin\varphi_a = \sqrt{\frac{i}{\Omega}}\sin\varphi_{sl}. \quad (17)$$

If the angles are complex, the following condition must be satisfied for the outgoing waves

$$\begin{matrix} \operatorname{Im}\cos\varphi < 0 & x < 0, \\ \operatorname{Im}\cos\varphi > 0 & x > 0, \end{matrix} \quad (18)$$

so as to make the amplitude decrease with increasing distance from the boundary.

The electromagnetic wave produced in air by the incidence of sound, as follows from "Snell's law" (17), is a surface wave if $\sin\varphi'_{ac} > s/c$, i.e., at all angles of incidence with the exception of the narrow region of angles $\sim 10^{-5}$, corresponding to normal incidence.

When $\varphi'_{ac} \gg s/c$ the "depth of penetration" of the electromagnetic field in air is $1/\operatorname{Im} k_{el}x = \lambda'_{ac}/2\pi\sin\varphi'_{ac}$, where λ'_{ac} is the length of the incident sound wave. The electromagnetic wave propagates along the surface with a velocity equal to the horizontal component of the velocity of light, i.e., at $\varphi'_{ac} \sim 1$, we obtain a slowing-down ratio on the order of $c/s \sim 10^5$. For such a surface wave, both $\sin\varphi_{el}$ and $|\cos\varphi_{el}|$ are on the order of c/s:

$$-i\cos\varphi_{el} \approx \sin\varphi_{el} = (c/s)\sin\varphi'_{ac}.$$

According to (14) and (15), it follows therefore that the electric field in a surface TM wave parallel to **m** is circularly polarized and is c/s times greater than the magnetic field. Accordingly, in a surface TE wave the magnetic field is c/s times greater than the electric field and is circularly polarized:

$$H_x/H_q = E_x/E_q \approx i, \qquad H_q \equiv (\mathbf{H}\mathbf{q}/q),$$
$$E_q \equiv (\mathbf{E}\mathbf{q}/q). \quad (19)$$

To determine the amplitudes of the outgoing waves from the known amplitudes of the incident waves, we use the boundary conditions on the interface (x = 0), namely the continuity of the pressure, of the tangential components of the magnetic and electric fields, and of the normal velocity component:

$$[\mathbf{H}_t] = [\mathbf{E}_t] = [\mathbf{v}_x] = [p] = 0. \quad (20)$$

Here [A] denotes the jump in the quantity A on the interface. Expanding the quantities in the boundary condition in normal modes (sound and electromagnetic), we can rewrite the boundary conditions (20) with the aid of (10)—(12), (15), and (16) in the form of equations for the amplitudes p_{ac}, j_{sl}, j_a, H_n, E_n, and $\bar{p}$ of the outgoing waves:

$$T_{i1}p_{ac} + T_{i2}j_{sl} + T_{i3}j_a + T_{i4}(-H_n) + T_{i5}(-E_n)$$
$$+ T_{i6}(-\bar{p}) = U_i, \qquad i = 1, 2, \ldots, 6. \quad (21)$$

The matrix T_{ik} has the form

$$\left\| \begin{matrix} \mathscr{H}_{ac}\eta_{acy} - \mathscr{H}_a\eta_{ay} & \mathscr{H}_a\zeta_{ay} & n_y & -m_y & 0 \\ \mathscr{H}_{ac}\eta_{acz} - \mathscr{H}_a\eta_{az} & \mathscr{H}_a\zeta_{az} & n_z & -m_z & 0 \\ \mathscr{E}_{ac}\zeta_{acy} \quad \zeta_{ay}/\sigma & \eta_{ay}/\sigma & m_y & n_y & 0 \\ \mathscr{E}_{ac}\zeta_{acz} \quad \zeta_{az}/\sigma & \eta_{az}/\sigma & m_z & n_z & 0 \\ a_x \quad V_{s1x} & V_{ax} & 0 & 0 & \bar{a}_x \\ 1 \quad P_{s1} & 0 & 0 & 0 & 1 \end{matrix} \right\| . \qquad (22)$$

The right halves U_i of the system (21) are determined by the incident perturbations.

4. CONVERSION COEFFICIENTS

Let us write down the principal terms of the expansion of the coefficients of conversion of sound waves into electromagnetic waves and electromagnetic waves into sound waves in powers of the small Ω_u, Ω_u/Ω, and s/c. The principal term of the determinant of the system (21) is

$$\text{Det}\,|T_{ik}| = -d_1(1-Z)\,\bar{a}_x, \qquad (23)$$

where

$$Z = (\bar{\rho}_0\bar{s}/\rho_0 s)(\cos\varphi_{ac}/\cos\bar{\varphi}_{ac}), \qquad (24)$$

$$d_1 = \frac{1}{\sigma^2}\left(\frac{c}{s}\sqrt{\frac{i}{\Omega}}\cos\varphi_{el} - \cos\varphi_a\right)\left(\cos\varphi_{el} - \frac{c}{s}\sqrt{\frac{i}{\Omega}}\cos\varphi_a\right). \qquad (25)$$

The term cos φ_a in the first parentheses becomes significant when cos $\varphi_{el} \to 0$, and it can be shown that (23) in this range of angles is again the principal term of the determinent of the system.

Conversion of Sound Waves into Electromagnetic Waves in Air

When sound is incident from air, the conversion coefficients are

$$H_n/\bar{p}' = 2d_4/d_1(1-Z), \qquad E_n/\bar{p}' = 2d_5/d_1(1-Z). \qquad (26)$$

Here $\bar{p}'$ — amplitude of the incident sound wave; H_n and E_n — amplitudes of the fields in the TM and TE waves, respectively;

$$d_4 = \frac{B\zeta_{acx}\sin\theta_{ac}}{\rho_0\sigma^2 s^2(1+i\Omega)\sin\varphi_{ac}}\left(\cos\varphi_a - \sqrt{\frac{i}{\Omega}}\cos\varphi_{ac}\right) \times\left(-\frac{c}{s}\sqrt{\frac{i}{\Omega}}\cos\varphi_a + \cos\varphi_{el}\right), \qquad (27)$$

$$d_5 = \frac{B\eta_{acx}\sin\theta_{ac}}{\rho_0\sigma^2 s^2(1+i\Omega)\sin\varphi_{ac}}\left(\cos\varphi_{ac} - \sqrt{\frac{i}{\Omega}}\cos\varphi_a\right) \times\left(\frac{c}{s}\sqrt{\frac{i}{\Omega}}\cos\varphi_{el} - \cos\varphi_a\right). \qquad (28)$$

When sound is incident from a conducting medium, we obtain the following expressions for the conversion coefficients:

$$\frac{H_n}{p'_{ac}} = \frac{d_1^{pH}}{d_1} - \frac{1+Z}{1-Z}\frac{d_4}{d_1}, \qquad \frac{E_n}{p'_{ac}} = \frac{d_1^{pE}}{d_1} - \frac{1+Z}{1-Z}\frac{d_5}{d_1}. \qquad (29)$$

Here p'_{ac} is the amplitude of the sound wave incident from the conductor; d_1^{pH} and d_1^{pE} are obtained respectively from d_4 and $(-1)d_5$ by replacing the parameters of the outgoing sound wave by the parameters of the incident wave ($\varphi_{ac} \to \varphi'_{ac}$ etc).

Conversion of Electromagnetic Waves into Sound Waves in a Conductor

The coefficients of conversion of electromagnetic waves (incident from air) into sound waves (propagating in the conductor) are

$$p_{ac}/H'_n = -P_{s1}\{(1-Z_{s1})\,d_2^{pH} + Z_a d_3^{pH}\}/d_1(1-Z),$$

$$p_{ac}/E'_n = -P_{s1}\{(1-Z_{s1})\,d_2^{pE} + Z_a d_3^{pE}\}/d_1(1-Z). \qquad (30)$$

Here E'_n and H'_n are the amplitudes of the incoming TE and TM waves;

$$Z_{s1} = V_{s1x}/P_{s1}\bar{a}_x, \qquad Z_a = V_{ax}/P_{s1}\bar{a}_x;$$

$$d_2^{pH} = \frac{2c}{\sigma s}\zeta_{ax}\frac{\cos\varphi'_{el}}{\sin\varphi_a}\left(\sqrt{\frac{i}{\Omega}}\cos\varphi_a + \frac{s}{c}\cos\varphi'_{el}\right), \qquad (31)$$

$$d_2^{pE} = \frac{2c}{\sigma s}\eta_{ax}\frac{\cos\varphi'_{el}}{\sin\varphi_a}\left(\sqrt{\frac{i}{\Omega}}\cos\varphi'_{el} + \frac{s}{c}\cos\varphi_a\right); \qquad (32)$$

d_3^{pH} and d_3^{pE} are obtained from d_2^{pH} and d_2^{pE} by making the substitution

$$\zeta_a \to -\eta_a, \qquad \eta_a \to \zeta_a.$$

Conversion of Sound into Electromagnetic Waves in a Conductor

When sound is incident from air, the conversion coefficients are

$$\frac{j_{s1}}{\bar{p}'} = \frac{-2d_2}{d_1(1-Z)}, \qquad \frac{j_a}{\bar{p}'} = \frac{2d_3}{d_1(1-Z)}; \qquad (33)$$

$$d_2 = \frac{cB\cos\varphi_{el}\sin\theta_{ac}}{\sigma\rho s^3(1+i\Omega)}\left\{\sqrt{\frac{i}{\Omega}}[\eta_{ac}\zeta_a]_x - \frac{s}{c}\frac{\cos\varphi_{el}}{\sin\varphi_{ac}\sin\varphi_a}\left(\zeta_{acx}\zeta_{ax} + \sqrt{\frac{i}{\Omega}}\eta_{acx}\eta_{ax}\right)\right\}. \qquad (34)$$

d_3 is obtained from d_2 by making the substitutions $\zeta_a \to -\eta_a$ and $\eta_a \to \zeta_a$. When the sound is incident from the conductor we have

$$j_{s1}/p'_{ac} = \{(1+Z)\,d_2 - (1-Z)\,d_1^{s1p}\}/d_1(1-Z),$$
$$j_a/p'_{ac} = -\{(1+Z)\,d_3 - (1-Z)\,d_1^{ap}\}/d_1(1-Z). \qquad (35)$$

Here d_1^{slp} and d_1^{ap} are obtained from d_2 and d_3 by replacing the parameters of the outgoing sound wave by the parameters of the incoming wave.

5. INVESTIGATION OF THE CONVERSION COEFFICIENTS

The general formulas become somewhat simpler if we recognize that the acoustic impedance of air is small compared with the acoustic impedance of the conducting medium. Except for a small range of angles

$$\cos\bar{\varphi}_{ac} \lesssim \bar{\rho}_0\bar{s}/\rho_0 s \ll 1, \tag{36}$$

corresponding to gliding propagation of sound in air, we can neglect Z, Z_a and Z_{sl} compared with unity. When sound is incident from air, the range of angles (36) is attained but is of no interest, for the sound does not penetrate into the conductor in this case[5] and no electromagnetic waves are produced. On the other hand, when plane homogeneous electromagnetic waves are incident from air or sound waves are incident from a conducting medium, we get

$$Z \sim Z_a \sim Z_{sl} \ll 1 \tag{37}$$

for all real angles of incidence.

Conversion of Sound Waves into Electromagnetic Waves

With (37) taken into account, the conversion coefficients assume the following form: for incidence from air

$$\frac{E_n}{\bar{p}'} = \frac{2B(\sin\varepsilon\sin\varphi_{ac} + \cos\varepsilon\cos\gamma\cos\varphi_{ac})(\sqrt{i/\Omega}\cos\varphi_a - \cos\varphi_{ac})}{\rho_0 sc(1+i\Omega)(1-Z)\left(\sqrt{i/\Omega}\cos\varphi_a - \frac{s}{c}\cos\varphi_{el}\right)}, \tag{38}$$

$$\frac{H_n}{\bar{p}'} = \frac{2B\sin\gamma\cos\varepsilon(\cos\varphi_a - \sqrt{i/\Omega}\cos\varphi_{ac})}{\rho_0 s^2(1+i\Omega)(1-Z)(\cos\varphi_a - (c/s)\sqrt{i/\Omega}\cos\varphi_{el})}; \tag{39}$$

for incidence from a conductor

$$\frac{E_n}{p'_{ac}} = \frac{2B(\sin\varepsilon\sin\varphi'_{ac} - \sqrt{i/\Omega}\cos\varepsilon\cos\gamma\cos\varphi_a)\cos\varphi'_{ac}}{\rho_0 sc((s/c)\cos\varphi_{el} - \sqrt{i/\Omega}\cos\varphi_a)(1+i\Omega)}, \tag{40}$$

$$\frac{H_n}{p'_{ac}} = \frac{2B\sin\gamma\cos\varepsilon\cos\varphi'_{ac}}{\rho_0 s^2(1+i\Omega)((c/s)\cos\varphi_{el} - \sqrt{\Omega/i}\cos\varphi_a)}. \tag{41}$$

The frequency dependence of the conversion coefficients is determined principally by the frequency dependence of the electromagnetic field in the modified sound wave, i.e., when $\omega \gg \omega_s$ the coefficient decreases as ω_s/ω with increasing frequency. An increase in frequency also brings about a modification in the angular dependence, owing to a certain redistribution of the contributions of the various types of waves, connected with the change in the phase velocities of the electromagnetic waves in the conductor. We confine ourselves in the following estimates to the case $\omega \sim \omega_s$.

In different ranges of sound-incidence angles, the angular dependences and the orders of magnitude of the electric and magnetic fields of TE waves differ appreciably from those of TM waves. At normal incidence ($\varphi'_{ac} < s/c$) the electric and

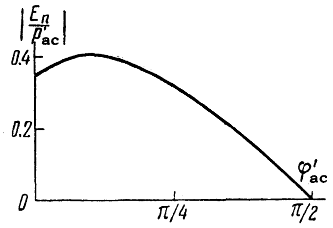

FIG. 2. Dependence of the amplitude of the TE wave on the angle of incidence of a sound wave from a conducting liquid when $\varepsilon = \pi/4$, $\gamma = \pi/3$, and $\Omega = 1$. The scale unit along the ordinate axis is $B\sqrt{2}/\rho_0 sc$.

magnetic fields of electromagnetic waves are equal to each other in air, but in a conducting medium the electric field is c/s times smaller than the magnetic field. Since the electric field of a TE wave lies in the interface, it follows from the continuity of $\mathbf{E}_t$ that the magnetic field of the TE wave is c/s times smaller than the magnetic field of the modified sound. When $\varphi'_{ac} \gg s/c$, the rotating magnetic field is c/s times greater than the electric field, and as a result both fields have the same order of magnitude as in the conductor:

$$E \sim (B/\rho_0 sc)\,p'_{ac}, \qquad H \sim (B/\rho_0 s^2)\,p'_{ac}. \tag{42}$$

The dependence of E and H on the angle of incidence φ'_{ac} is a smooth one (Fig. 2).

In a TM wave, as in a TE wave, the magnetic field at normal incidence is c/s times smaller than in the modified sound. However, when $\sin\varphi'_{ac} = s/c$ the tangential component $E_q = -H_n\cos\varphi_{el}$ vanishes with $\cos\varphi_{el}$ and the electric field in air, which is orthogonal to the interface, can greatly exceed the field in the conductor. As can be seen from (41), the coefficient of conversion of sound in a TM wave has at this point a very sharp maximum. The magnetic field at the maximum is of the same order of magnitude as the field in the sound wave ($H \sim Bp'_{ac}/\rho_0 s^2$), c/s times greater than the value of H for $\varphi'_{ac} \ll s/c$, and $(c/s)^2$ times greater than the value of H in the surface TM wave (Fig. 3). The half-

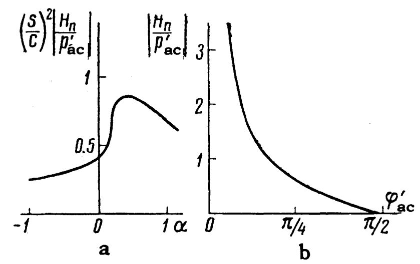

FIG. 3. Dependence of the amplitude of the TM wave on the angle of incidence of a sound wave from a conducting medium when $\varepsilon = \pi/4$, $\gamma = \pi/3$, and $\Omega = 1$. The scale unit along the ordinate axis is $B\sqrt{2}/\rho_0 c^2$. a – region of maximum, $\alpha = (c/s)^3(\varphi'_{ac} - s/c)$; b – surface-wave region ($\sin\varphi'_s > s/c$).

width of this exceedingly narrow maximum, as follows from (41), is of the order of $(s/c)^3$. It follows, however, from the dispersion law (8) that when the angle of incidence is real, $\cos\varphi_{el}$ does not vanish:

$$\cos\varphi_{el} = \sqrt{\Omega_u}\sin\theta_{ac}/\sqrt{1+i\Omega} \quad \text{for } \sin\varphi_{ac} = \frac{s}{c} \qquad (43)$$

(we put $\Omega_u = 0$ in $\sin\theta_{ac}$). Therefore when $\sqrt{\Omega_u} \gg s/c$ we can neglect $\cos\varphi_a$ in the denominator; the height and the width of the maximum are respectively $1/\sqrt{\Omega_u}$ and $(s/c)^2\sqrt{\Omega_u}$.

It is obvious that a similar dependence of the conversion coefficient on the angle of incidence should occur also when the conducting medium is a crystalline solid.

It is seen from (38)—(41) that the conversion coefficients are anisotropic. We can point out the singular cases when there is no conversion of acoustic oscillations into electromagnetic ones. For this purpose it is obviously necessary that the sound propagate in the conducting medium along the magnetic field, for in this case neither fields nor currents are induced. When the sound is incident from the air, the refracted sound wave propagates along the magnetic field without interacting with it if $\gamma = 0$ and $\varphi_{ac} = \epsilon + \pi/2$. As can be seen from (38) and (39), the conversion coefficients for both electromagnetic waves vanish in this case. When incidence is from a conductor, in view of the fact that both the incident and the reflected sound wave must propagate in the liquid in order to have no induction along the magnetic field, there is no connection between the acoustic and electromagnetic oscillations only if the magnetic field is perpendicular to the interface ($\epsilon = \pi/2$) and the wave is normally incident ($\varphi'_{ac} = 0$). We note also that when $\epsilon = \pi/2$ or $\epsilon = 0$ and $\gamma = 0$, only a TE wave is produced in the air, and when $\epsilon = 0$ and $\gamma = \pi/2$ only a TM wave is produced.

Conversion of Electromagnetic Waves into Sound Waves

The coefficients of conversion of electromagnetic waves incident from air into sound waves in a conductor, subject to condition (37), are

$$\frac{p_{ac}}{H'_n} = \frac{B\sin\gamma\cos\varepsilon\cos\varphi'_{el}}{2\pi(1+i\Omega)(\cos\varphi'_{el} + (s/c)\sqrt{\Omega/i}\cos\varphi_a)}, \qquad (44)$$

$$\frac{p_{ac}}{E'_n} = \frac{B\cos\varphi'_{el}(\sin\varepsilon\sin\varphi_a + \cos\varepsilon\cos\gamma\cos\varphi_a)}{2\pi(1+i\Omega)(\cos\varphi_a + (s/c)\sqrt{\Omega/i}\cos\varphi'_{el})}. \qquad (45)$$

Upon incidence of homogeneous plane waves (φ'_{el} real), by virtue of Snell's law (17), the sound and electromagnetic waves propagate in the conductor normally to the interface. This makes it possible to simplify expressions (44) and (45) further, by putting $\sin\varphi_{ac} = \sin\bar{\varphi}_{ac} = \sin\varphi_a = 0$:

$$\frac{p_{ac}}{H'_n} = \frac{B\sin\gamma\cos\varepsilon\cos\varphi'_{el}}{2\pi(1+i\Omega)(\cos\varphi'_{el} - (s/c)\sqrt{\Omega/i})}, \qquad (44')$$

$$\frac{p_{ac}}{E'_n} = \frac{B\cos\varphi'_{el}\cos\varepsilon\cos\gamma}{2\pi(1+i\Omega)(1 - (s/c)\sqrt{\Omega/i}\cos\varphi'_{el})}. \qquad (45')$$

We note that in the calculation of the principal terms of the coefficients of conversion of sound into electromagnetic waves we neglect in this approximation the presence of the acoustic field in the modified electromagnetic wave. Accordingly, only the acoustic field of the electromagnetic wave plays a role in the conversion of electromagnetic waves into sound.

The conversion of electromagnetic waves into a sound wave and vice versa is also of interest in investigations of the properties of solids. In this case the angle relationships differ from those obtained for a liquid and will be calculated separately, but the orders of magnitude of the conversion coefficients remain the same:

$$p \sim BH'/4\pi(1+i\Omega), \qquad H \sim Bp'/\rho_0 s^2(1+i\Omega).$$

These estimates are valid down to low temperatures such that the mean free path l of the carriers (electrons) is still considerably less than the mean free path of sound $\lambda\!\!\!^{-}$ and the skin depth δ, i.e., when

$$\sigma \ll \frac{s}{4\pi v_0}\frac{\omega_p^2}{\omega} \sim \frac{10^{26}}{\omega}, \qquad \lambda\!\!\!^{-} \gg l, \qquad (46)$$

$$\sigma^{3/2} \ll \frac{1}{(4\pi)^{3/2}}\frac{c}{v_0}\frac{\omega_p^2}{\sqrt{\omega}} \sim \frac{10^{31}}{\sqrt{\omega}}, \qquad \delta \gg l. \qquad (47)$$

Here $v_0 \sim 10^8$ cm/sec is the Fermi velocity and $\omega_p \sim 10^{15}$ sec^{-1} is the plasma frequency of the electrons. Since $v_0/s \gg 1$, this automatically guarantees the inequality $\omega\tau \ll 1$. Let us also rewrite these conditions in different form

$$\Omega \gg v_0 c^2 s^{-3}(\omega/\omega_p)^2 \sim 10^{-17}\omega^2, \qquad (46')$$

$$\Omega^{3/2} \gg v_0 c^2 s^{-3}(\omega/\omega_p)^2. \qquad (47')$$

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APPENDIX

In connection with the vector character of the electrodynamic boundary conditions, the use of the Laplace theorem leads to appreciable simplifications in the calculation of the conversion coefficients.

Let us examine the fourth-order minor obtained by crossing out the last two lines and any columns from the determinant of the system (21) (or from determinants derived from it by Cramer's rule). These determinants are made up of vector components and have the form

$$d = \begin{vmatrix} \mathbf{a}_1 & \mathbf{a}_2 & \mathbf{a}_3 & \mathbf{a}_4 \\ \mathbf{b}_1 & \mathbf{b}_2 & \mathbf{b}_3 & \mathbf{b}_4 \end{vmatrix}, \qquad \text{(A.1)}$$

where $\mathbf{a}_k$ stands for the column $\begin{pmatrix} a_{ky} \\ a_{kz} \end{pmatrix}$, and for d we obtain

$$d = \sum_{i<k,\ l<m} \varepsilon_{iklm} [\mathbf{a}_i \mathbf{a}_k]_x [\mathbf{b}_l \mathbf{b}_m]_x. \qquad \text{(A.2)}$$

We note that the sum contains only six non-vanishing terms (ϵ_{iklm} is an anti-symmetrical unit tensor). In the calculation of determinants with the aid of (A.2) we use the following corollaries of (13) and (14):

$$[\boldsymbol{\zeta}_i \mathbf{m}]_x = -\eta_{ix} \cos\varphi_{e1}/\sin\varphi_i, \qquad [\boldsymbol{\eta}_i \mathbf{m}]_x = \zeta_{ix} \cos\varphi_{e1}/\sin\varphi_i,$$
$$[\boldsymbol{\zeta}_i \mathbf{n}]_x = \zeta_{ix} \cot\varphi_i, \qquad [\boldsymbol{\eta}_i \mathbf{n}]_x = \eta_{ix} \cot\varphi_i. \qquad \text{(A.3)}$$

The text uses the following notation for the minors d: the lower index denotes the number of the column, crossed out together with the sixth column; the upper pair of indices denotes in which column and for what incident wave the change was made in accordance with Cramer's rule.

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