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# *PARTICLE ACCELERATION BY PASSAGE OF A HYDROMAGNETIC SHOCK WAVE FRONT*

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An expression for the momentum increment of a high-energy particle (compared with the particles comprising the bulk of a plasma) due to passage of a hydromagnetic shock wave is derived and discussed.

We consider a plane hydromagnetic shock wave propagating in a direction perpendicular to a magnetic field (perpendicular shock wave). The velocity of the front relative to the medium 1 ahead of the front is $v_1$, and that relative to medium 2 behind the front is $v_2$. Let a charged particle of velocity $v \gg v_1$ be incident from medium 1 on the front of the shock wave. The particle-velocity component parallel to the magnetic field will not change on passing through the front. We therefore choose a system of coordinates in which the particle moves in a plane perpendicular to the magnetic field. We assume that the Larmor radius of the particle is much greater than the width of the front and that this width can be neglected. Moving alternately between regions 1 and 2, the particle will describe circular arcs in coordinate systems that are stationary with respect to media 1 and 2, respectively, the Larmor radius being determined by the magnetic field in region 1 or 2. Thus, a particle remains "tied" to the front for some time.

If the angle between the particle velocity and the normal to the front, directed from 1 into 2, is denoted by $\varphi$, then the central angle $\vartheta$ swept by the particle in medium 2 is connected with $\varphi$ by the relation $\vartheta = \pi + 2\varphi$. Here $-\pi/2 \le \varphi \le \pi/2$ and $0 \le \vartheta \le 2\pi$, and $\varphi$ is positive when $\vartheta > \pi$. If $v \gg v_1$ the displacement of the front $\Delta x$ relative to the medium 2 during the stay $\Delta t = \vartheta/\omega$ of the particle in medium 2 ($\omega$ is the Larmor frequency) is equal to $\Delta x = v_2 \Delta t = v_2 \vartheta/\omega$. On the other hand, $\Delta x = r\Delta\vartheta \cos\varphi$, where $r$ is the Larmor radius. Inasmuch as $\Delta\vartheta = 2\Delta\varphi$, and $\omega r = v = pc^2/\epsilon$ (where $p$ is the momentum of the relativistic particle, $\epsilon$ is the total energy, and $c$ is the velocity of light), we have for the change of the angle due to the finite stay of the particle in medium 2

$$\Delta\varphi_2 = (v_2 / 2v)(\pi + 2\varphi)/\cos\varphi. \quad (1)$$

We obtain similarly the change in the angle during the stay of the particle in medium 1. It is necessary to take account of the fact here that the front moves in medium 1 with velocity $v_1$, and that the same angle $\varphi$, which by definition is positive on going from medium 1 into 2, is negative on going from medium 2 into 1. Then

$$\Delta\varphi_1 = (v_1 / 2v)(\pi - 2\varphi)/\cos\varphi. \quad (2)$$

Along with the angle changes (1) and (2) due to the finite time of stay of the particle in each region (1 or 2), the angle will be increased by the change in momentum of the particle upon reflection from medium 2. The particle momentum component normal to the boundary, $p_\perp = p\cos\varphi$, is increased by $\Delta p_\perp = 2(v_1 - v_2)\epsilon/c^2$, while the increment of the parallel component $p_\| = p \sin\varphi$ is $\Delta p_\| = 0$. From the equations

$$\Delta p \sin\varphi + p\Delta\varphi' \cos\varphi = 0,$$

$$\Delta p \cos\varphi - p\Delta\varphi' \sin\varphi = 2(v_1 - v_2)\varepsilon / c^2$$

we obtain for the increment $\Delta\varphi'$ of the angle and for the change $\Delta p$ of the momentum the following expressions

$$\Delta\varphi' = -2(v_1 - v_2)\varepsilon / pc^2 = -2(v_1 - v_2)v^{-1}\sin\varphi, \quad (3)$$

$$\Delta p = 2(v_1 - v_2)\varepsilon c^{-2}\cos\varphi. \quad (4)$$

The increment in the angle over the cycle (within a complete revolution of the particle through regions 2 and 1) is made up of the increments* (1), (2), and (3), viz., $\Delta\varphi = \Delta\varphi_1 + \Delta\varphi_2 + \Delta\varphi'$. Dividing by the increment of momentum over the cycle as given by (4), and taking (1) and (2) into account, we obtain a differential equation relating p and $\varphi$:

$$dp / d\varphi = pf(\varphi), \tag{5}$$

where $f(\varphi)$ will be written out below.

The angle at which the particle can no longer leave region 2 is $\varphi = \pi/2$. We therefore, have for the complete increment of the particle-momentum component perpendicular to the magnetic field upon passage of the shock wave front

$$p = p_0 \exp I(\varphi_0), \tag{6}$$

$$I(\varphi_0) = \int_{\varphi_0}^{\pi/2} f(\varphi)\, d\varphi = \int_{\varphi_0}^{\pi/2} \frac{4(1 - v_2/v_1)\cos^2\varphi\, d\varphi}{(\pi + 2\varphi)\, v_2/v_1 + (\pi - 2\varphi) - 4(1 - v_2/v_1)\sin\varphi\cos\varphi}, \tag{7}$$

where $\varphi_0$ is the initial angle of entrance into region (2).

The particle will experience maximum acceleration when $\varphi_0 = -\pi/2$, i.e., when it travels parallel to the front in the same direction as the Lorentz force acting from medium 2 towards this front.†

It is characteristic that the magnetic field does not enter explicitly into this equation. The increment in momentum is determined only by the ratio of velocities of the front relative to the gas ahead and behind the shock-wave front, and increases with increasing density discontinuity, since

$$1 - v_2/v_1 = 1 - \rho_1/\rho_2 = (\rho_2 - \rho_1)/\rho_2,$$

where $\rho_1$ and $\rho_2$ are the densities of the medium in the regions 1 and 2. Since in a weak shock wave the density discontinuity decreases with increasing ratio of the magnetic pressure $H_1^2/8\pi$ to the gas pressure $P_1$ for equal wave intensity, the weak-field case is the most favorable from the point of view of particle acceleration on the front.

In the general case of hydromagnetic waves the expressions for $v_2/v_1$ are quite cumbersome. We shall consider two limiting cases, weak and strong waves.

In the case of weak waves we are interested only in the increment of the momentum of the particle under consideration, compared with the momentum increment of the average-energy particles making up the plasma in which the shock wave propagates. For simplicity we assume that the gas consists of particles with two degrees of freedom. Then the equation of state, which relates the pressure $P^* = P + H^2/8\pi$ and the energy $E^* = E + H^2/4\pi$, coincides with the equation for a perfect gas, and

$$v_2/v_1 = [(\gamma - 1) M_1^{*2} + 2] / [(\gamma + 1) M_1^{*2}], \tag{8}$$

where the generalized Mach number $M_1^* = v_1/c_1^*$ is connected with the ordinary Mach number $M_1 = v_1/c_1$ by the relation

$$M_1^{*2} = M_1^2 / (1 + H_1^2/8\pi P_1);$$

$\gamma$ is the adiabatic exponent, and in our case $\gamma = 2$; $c_1^{*2} = \gamma P_1/\rho_1$ is the magnetohydrodynamic speed of sound.

Let us assume that $M_1^* = 1 + \delta$, $\delta \ll 1$. This means that $M_1 \ll \sqrt{3}\,(1 + H_1^2/8\pi P_1)^{1/2}$ and, generally speaking, we can also have $M_1 \gg 1$. In first order in $\delta$ we find from (8) that $v_2/v_1 = 1 - 2\delta/(\gamma + 1)$. Expression (7) yields in this case

$$I(\varphi_0) = \int_{\varphi_0}^{\pi/2} \frac{4\delta\cos^2\varphi\, d\varphi}{\pi(\gamma + 1)} = \frac{2\delta}{\pi(\gamma + 1)}\left(\frac{\pi}{2} - \varphi_0 - \sin 2\varphi_0\right), \tag{9}$$

$$I(-\pi/2) = 2\delta/(\gamma + 1). \tag{10}$$

Substituting the maximum value of I as given by (10) in (6), we obtain for $\delta \ll 1$

$$\Delta p/p_0 = (p - p_0)/p_0 = 2\delta/(\gamma + 1). \tag{11}$$

Let us consider now the momentum increment of the particles of a medium in which a weak shock wave propagates. It can be shown that in the linear approximation in $\delta$ the temperature discontinuity in a gas with two degrees of freedom is equal to the

*Direct calculation of the increment of the angle in the coordinate system 1 over the cycle reduces to the following. We determine the increment $\Delta\varphi_1$ in medium 1 during the stay of the particle in this medium [formula (2)]. Then the angle $\varphi + \Delta\varphi_1$ is transformed to the coordinate system 2, using the relativistic transformation formulas.

We calculate the increment $\Delta(\varphi + \Delta\varphi_1)'$ during the stay of the particle in medium 2, and then the angle $(\varphi + \Delta\varphi_1)' + \Delta(\varphi + \Delta\varphi_1)'$ is again transformed to system 1 and the angle increment over the cycle is calculated, viz., $\Delta\varphi = \varphi - (\varphi + \Delta\varphi_1)'' - \Delta(\varphi + \Delta\varphi_1)''$. The momentum change in system 1, corresponding to this angle increment, is determined upon reflection from the medium with account of the fact that the particle energy does not change during its stay in medium 2. The results of this much more cumbersome calculation agrees with the data given in the text, apart from terms of higher order in the smallness parameter $v_1/v$. But even without this calculation it is seen from the physics of the problem that the angle increment is the result of the effects listed in the text. It does not matter in what system of coordinates we determine subsequently the momentum increment from the differential equation (5), since the relative difference between p in system 1 and p′ in system 2 is of order $v_1/c \ll 1$.

†It is easy to see that if the source of fast particles is far from the front (at a distance greater than the Larmor radius of the particle in the medium 1), then the initial angle of entry will always be $\varphi_0 = -\pi/2$, and the acceleration will be a maximum. This is indeed the real case.

discontinuity of total energy $E^*$, i.e., in our approximation almost all the energy on the front is dissipated into heat. Therefore, using the expressions for the discontinuity of $E^*$, we obtain

$$\frac{T_2}{T_1}=\frac{E_2^*}{E_1^*}=\frac{[2\gamma M_1^{*2}-(\gamma-1)]\,[(\gamma-1)\,M_1^{*2}+2]}{(\gamma+1)^2 M_1^{*2}}\approx 1+2\delta\frac{\gamma-1}{\gamma+1}. \quad (12)$$

The increment of the mean momentum (velocity) of the particles of the medium, $\bar{p}_2/\bar{p}_1=\sqrt{T_2/T_1}$, is

$$\Delta\bar{p}/\bar{p}_1=(\bar{p}_2-\bar{p}_1)/\bar{p}_1=\delta\,(\gamma-1)/(\gamma+1). \quad (13)$$

We see therefore that in our case $(\gamma = 2)$, which corresponds to a greater heating of the medium particles then in the real case of particles of a gas with three degrees of freedom, the increment (13) of the momentum of the particles of the medium is half as large as the maximum increment of the momentum of the particle under consideration (11), and is equal to the momentum increment of the accelerated particle if the latter is perpendicularly incident on the front $(\varphi_0 = 0)$.

The formulas given above are valid for a real particle gas with three degrees of freedom in the case when $H_1^2/8\pi P_1 \ll 1$. Then $M^* \approx M = 1+\delta$. The increment of the average momentum of the particles of the medium (13) is $\delta/4$, and that of the accelerated particle (11) is $3\delta/4$, i.e., three times greater.

Let us consider a strong shock wave $(M_1^* \gg 1)$. In this case $\rho_1/\rho_2 = v_2/v_1 = (\gamma-1)/(\gamma+1)$ and expression (7) reduces to

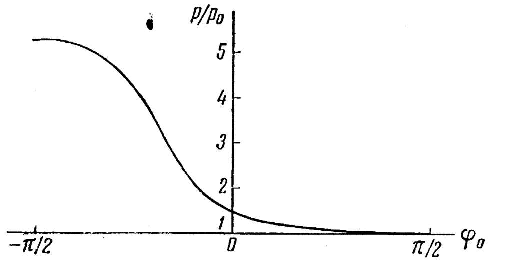
$$I(\varphi_0)=\int_{\varphi_0}^{\pi/2} f(\varphi)\,d\varphi=\int_{\varphi_0}^{\pi/2}\frac{4\cos^2\varphi\,d\varphi}{\gamma\pi-2\varphi-4\sin\varphi\cos\varphi}. \quad (14)$$

The results of the numerical integration of this expression with $\gamma = 5/3$ and substitution in (6) are shown in the figure. The maximum increment of the accelerated-particle momentum amounts to $p/p_0 = 5.23$ when $\varphi_0 = -\pi/2$. This means that the energy, say, of a nonrelativistic particle increases 27 times on passing through the front, while the energy of an ultrarelativistic particle increases 5.3 times.

When $M_1^* \gg 1$ we obtain instead of (12)

$$\frac{T_2}{T_1}<\frac{E_2^*}{E_1^*}=\frac{2\gamma(\gamma-1)}{(\gamma+1)^2}M_1^{*2}. \quad (15)$$

For a particle gas with two degrees of freedom $(\gamma = 2)$ (for which the heating is greatest), and when $M_1^{*2} < 60$, this quantity is less than the maximum increment in the energy of the accelerated particles. Therefore not only weak waves but also intense waves up to $M_1^* \lesssim 8$ will be most favorable for the acceleration of particles of the energy component of the plasma with minimum heating of the bulk of the gas, and can contribute to the decomposition of the plasma into two energy components, hot and cold.




Ratio of momentum p of accelerated particle after passage of the front to the initial momentum $p_0$, as a function of the angle of incidence $\varphi_0$

An attempt to determine the particle acceleration on the front of a hydrodynamic shock wave was made earlier by Dorman and Freĭdman.[1] However, the approximation used in [1] was too crude and not rigorously founded. The formula for the energy increment obtained in that paper did not reflect the dependence of the increment on the angle of incidence of the particle on the front* and had the form

$$\Delta\varepsilon=\varepsilon-\varepsilon_0=\frac{4}{\pi}\frac{(cp_0)^2}{\varepsilon_0}\frac{v_1/v_2-1}{v_1/v_2+1}.$$

For a strong wave $[v_1/v_2 \sim (\gamma+1)/(\gamma-1) \sim 4]$ we have

$$\Delta\varepsilon=\frac{4}{\pi}\frac{3}{5}\frac{(cp_0)^2}{\varepsilon_0}.$$

In the nonrelativistic case the total energy $\epsilon_0 \sim mc^2$ and we obtain for the increment in the kinetic energy

$$\frac{\Delta\varepsilon}{\varepsilon_k^0}=\frac{\varepsilon_k-\varepsilon_k^0}{\varepsilon_k^0}\sim\frac{4}{\pi}\frac{3\cdot 2}{5}\sim 1.6$$

$(\epsilon_k/\epsilon_k^0 \sim 2.6)$, where $\epsilon_k = mv_0^2/2$. In the ultrarelativistic case $\epsilon_0 \sim cp_0$ and $\Delta\epsilon/\epsilon_0 \sim 4\times 3/5\pi \sim 0.8$. The authors themselves state that their estimate of $\Delta\epsilon$ is approximate, accurate to a coefficient $\sim 2$–$3$ (as can be seen from the text of the present paper, the error is considerably greater). It is therefore difficult to judge on the basis of the paper by Dorman and Freĭdman whether particles can be accelerated on the front of a hydromagnetic shock wave.

[1] L. I. Dorman and G. I. Freĭdman, Voprosy magnitnoi gidrodinamiki i dinamiki plazmy (Problems of Magnetohydrodynamics and Plasma Dynamics), Riga, (1959).

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*By averaging expression (6) over the angles, assuming isotropic incidence of these fast particles on the front, we would obtain a formula which would coincide with the results of[1] in order of magnitude. But this must not be done since, as already noted, in the real case the fast particles are incident on the front at an angle $\varphi_0 = -\pi/2$.