

*THRESHOLD SINGULARITIES IN THE TOTAL PION SCATTERING CROSS SECTION  
OF PROTONS*

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An analysis of the experimental data indicates the possible existence of singularities in the total cross section for the scattering of positive pions by protons near the thresholds for production of an additional pion.

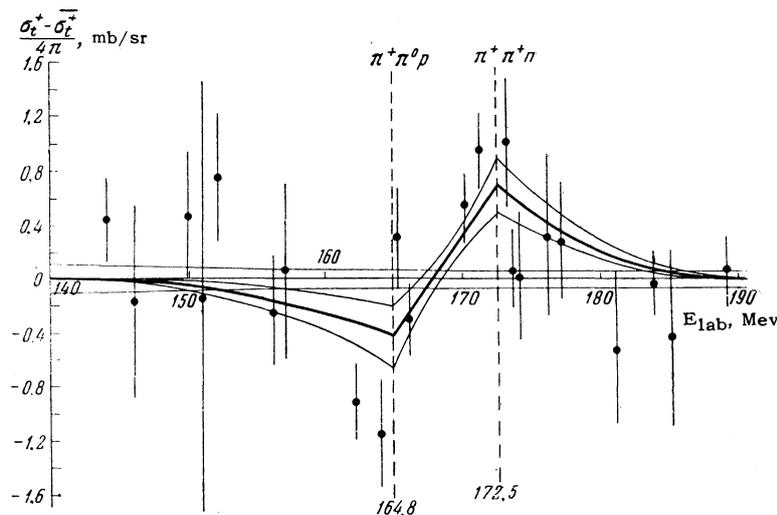
QUANTUM-MECHANICAL calculations<sup>[1]</sup> show that the elastic-scattering cross section can have singularities at the thresholds for inelastic reactions. This paper describes an attempt to find such singularities in the total cross section for the scattering of positive pions by protons. We consider the energy region from 140 to 190 Mev, which contains the thresholds for the reactions  $\pi^+p \rightarrow \pi^+\pi^0p$  and  $\pi^+p \rightarrow \pi^+\pi^+n$  at 164.8 and 172.5 Mev, respectively.

On the basis of more than 250 experimental cross sections for scattering of positive and negative pions, an interpolation formula with 15 parameters has been found<sup>[2]</sup> which describes satisfactorily the total cross sections for interactions of pions with protons at all energies. In particular, the cross section in the region under consideration is essentially described by a Breit-Wigner resonance formula.

In order to study the threshold singularities, the values of the interpolation formula of<sup>[2]</sup> were subtracted from all the experimental points in this

region (literature references for these are given in<sup>[2]</sup>). The results of this subtraction are shown in Fig. 1. The error bars shown on Fig. 1 include the effect of errors in the average energy of the pion beams. The two fine lines nearly parallel to the energy level corresponding to the average cross section show the "error corridor" due to the general interpolation. The experimental points in the region under consideration are much more widely scattered about the average cross section than in neighboring intervals, and they are displaced from it by distances exceeding their errors. For the 22 points (the number of degrees of freedom is  $f = 22$ ) we find  $\chi^2/f = 2.12$ , which gives a random scatter probability of merely 0.16%. It is also evident that the points lie mainly below the average cross section near the lower threshold and above it near the upper one.

Therefore we performed an additional interpolation of the total cross section in this region with functions of the form ( $\sigma$  and  $\bar{\sigma}$  are in mb and energies are in Mev):



$$(\sigma - \bar{\sigma})/4\pi = a(E - 140)^2, \quad 140 < E < 164.8 \text{ Mev}$$

$$(\sigma - \bar{\sigma})/4\pi = (Aa + Bb) + (Ca + Db)E,$$

$$164.8 < E < 172.5 \text{ Mev}, \quad (\sigma - \bar{\sigma})/4\pi = b(E - 190)^2,$$

$$172.5 < E < 190 \text{ Mev},$$

where the coefficients  $A = 1.38 \times 10^4$ ,  $B = -6.56 \times 10^3$ ,  $C = -79.9$ , and  $D = 39.8$  were determined from the condition that the curves match at the thresholds.

By minimizing  $\chi^2$ , the weighted sum of square deviations of the points from the line, we obtained  $a = -7.1 \times 10^{-4}$  and  $b = 2.14 \times 10^{-3}$ . The resulting broken line is shown in Fig. 1 together with its error corridor as determined by the procedure described previously.<sup>[3]</sup> Now  $f = 20$  and  $\chi^2/f = 1.37$ ; the probability of random scatter of the points about the broken line has increased 100 times. It is evident from Fig. 1 that the error corridors of the two interpolations do not overlap at the thresholds. This gives some basis for recommending that the cross sections in this region be remeasured with accuracy sufficient to show the presence and nature of the threshold pion-scattering singularities which are able to give additional information concerning the pion-pion interaction.

The foregoing possibility that threshold singularities are present in the cross section must be regarded with caution. An analogous calculation, with nonnormalized data, on the scattering of negative pions by protons<sup>[4]</sup> gave a negative result. It

was not possible to fit the cross section near the  $\pi^-p \rightarrow \pi^0\pi^0n$ ,  $\pi^-p \rightarrow \pi^-\pi^0p$ , and  $\pi^-p \rightarrow \pi^+\pi^-n$  reaction thresholds at 160.7, 164.9, and 172.5 Mev, respectively, any better with a broken line with straight segments than with a parabola. The result was  $\chi^2/f = 1.08$  with  $f = 21$  for the best line with four segments, while the parabola gave  $\chi^2/f = 1.06$  with  $f = 23$ . Therefore, only further experiments will allow us to ascertain the existence of threshold singularities in the cross section for scattering of positive pions on protons.

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<sup>1</sup>A. I. Baz', JETP **33**, 923 (1957) and **36**, 1762 (1959), Soviet Phys. JETP **6**, 709 (1958) and **9**, 1256 (1959).

<sup>2</sup>Klepikov, Meshcheryakov, and Sokolov, preprint D-584, Joint Institute for Nuclear Research, 1960.

<sup>3</sup>N. P. Klepikov and S. N. Sokolov, preprint R-235, Joint Institute for Nuclear Research, 1958.

<sup>4</sup>Zinov, Konin, Korenchenko, and Pontecorvo, JETP **38**, 1708 (1960), Soviet Phys. JETP **11**, 1233 (1960).