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ON THE NATURE OF COLLECTIVE LEVELS IN NONSPHERICAL NUCLEI

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A microscopic interpretation of the so-called β- and γ-vibrational levels in nonspherical nuclei is presented. By means of the method of approximate second quantization an equation is derived for determining the frequencies of these levels. Electromagnetic transition probabilities for these levels are computed. The theory is compared with experiment.

1. INTRODUCTION

At present in addition to rotational levels other levels of a collective nature have also been observed experimentally within the domain of strongly deformed nuclei. In particular, among such levels are included the so-called β- and γ-vibrational levels (excitations of the quadrupole type). The β-vibration levels correspond to internal excitations with a component of the angular momentum along the nuclear axis $K = 0$, while the γ-vibrational levels have this component given by $K = 2$. The energies of such levels lie in the range from 0.5 to 1.2 Mev, while the reduced probabilities of electric quadrupole transitions between these levels and the ground state are several times greater than the values obtained from a single particle estimate.

It is generally assumed[1] that these levels correspond to the excitation of oscillations of the nuclear surface about its equilibrium shape. An estimate of the frequencies of such oscillations made on the basis of the hydrodynamical model yields values $\hbar\omega \sim 1.5 - 2$ Mev which are considerably greater than the observed values. Moreover, it is not possible within the framework of the hydrodynamical model to take into account effects associated with the shell structure of the nucleus. On the other hand, a calculation within the framework of the independent particle model[2] leads to frequencies of the corresponding oscillations of the order of magnitude of the energy gap between the shells ($\hbar\omega_0 \sim 5 - 6$ Mev).

Within recent years a theory of collective excitations in spherical nuclei has been developed.[3-5] This theory is based on introducing the quadrupole-quadrupole interaction in addition to the residual interaction between nucleons which leads to pair correlation. The existence of a residual quadrupole-quadrupole interaction follows from experiment. This interaction manifests itself, for example, by the fact that the quadrupole moments of odd nearly-magic nuclei turn out to be greater than those predicted by the single-particle model. For the same reason the probabilities of E2 transitions in these nuclei are enhanced in comparison with the single-particle values.

The residual quadrupole-quadrupole interaction leads to the formation of a bound state of the quasiparticles. It should be expected that a similar effect must also exist in deformed nuclei. The energy required for the creation of a pair of quasiparticles in a deformed nucleus is $\geq 2\Delta$, where Δ is a constant characterizing the energy of the pair correlation. The quadrupole-quadrupole interaction leads to the formation of a bound state of the quasiparticles the energy of which is $< 2\Delta$.

An investigation of collective excitations in a Fermi system can be carried out in two ways: 1) by the method of the two-particle Green's function[6] and 2) by the method of approximate second quantization.[7] The collective nature of the lowest 2^+ levels in spherical nuclei has been investigated in Belyaev's paper[4] by the first method and by Marumori[3] and Baranger[5] by the second method.

In this paper we investigate collective excitations of the quadrupole type in deformed nuclei. The investigation is carried out with the aid of the method of approximate second quantization. An equation is obtained from which the frequencies of β- and γ-excitations can be determined. Wavefunctions are obtained for excited states of the type mentioned. Reduced probabilities for E2 and E0 transitions from these states to the ground state are found.

2. THE METHOD OF INVESTIGATION. EXCITATION SPECTRUM, WAVE FUNCTIONS.

We assume that the component of the angular momentum of the nucleons along the nuclear axis is a good integral of the motion. We shall write the Hamiltonian in the system of coordinates fixed with respect to the nuclear axes in the form

$$H = H_0 + H_Q, \tag{1}$$

where H_0 is the reduced Bardeen Hamiltonian for nucleons moving in an axially-symmetric deformed potential, while

$$H_Q = -\frac{1}{2}\varkappa \sum_{\mu}\sum_{(\lambda)} (q_{2\mu})_{\lambda\lambda'} (q^*_{2\mu})_{\lambda_1\lambda'_1} a^+_{\lambda} a_{\lambda'} a^+_{\lambda_1} a_{\lambda'_1}; \tag{2}$$

$q_{2\mu}$ are quadrupole moment operators; $\kappa > 0$ is the quadrupole-quadrupole interaction constant; a^+_λ, a_λ are the creation and annihilation operators for a particle in the state λ; λ is a set of quantum numbers for single nucleon states in the given potential. We restrict ourselves to consideration of nucleons of one kind.

The Hamiltonian H_0 can be diagonalized by the method of canonical transformation due to Bogolyubov.[7] By neglecting the interaction between the quasiparticles we can represent H_0 in the form

$$H_0 = \sum_{\lambda} E_\lambda (\alpha^+_\lambda \alpha_\lambda + \beta^+_\lambda \beta_\lambda), \qquad E_\lambda = \sqrt{\Delta^2 + \varepsilon^2_\lambda}, \tag{3}$$

where α_λ, β_λ; α^+_λ, β^+_λ are the annihilation and creation operators for quasiparticles of energy E_λ; ϵ_λ are the single particle levels referred to the Fermi surface ϵ_0. The summation in (3) is carried out over the positive components of the angular momentum. It can be shown[16] that taking into account the interaction between the quasiparticles in (3) leads to small corrections to the energies of the β-levels.

In future we shall be interested in collective excitations of the boson type characterized by the component of the total angular momentum along the nuclear axis K = 0; 2. We write $H_Q = H'_Q + H''_Q$ in terms of the operators α and β with

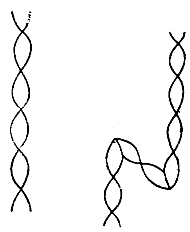
$$H'_Q = -\frac{1}{2}\varkappa \sum_{\mu}\sum_{(\lambda)} (q_{2\mu})_{\lambda\lambda'} (q^*_{2\mu})_{\lambda_1\lambda'_1} (u_\lambda v_{\lambda'} + u_{\lambda'} v_\lambda)(u_{\lambda_1} v_{\lambda'_1} + u_{\lambda'_1} v_{\lambda_1})(A^+_{\lambda\lambda'} + A_{\lambda'\lambda})(A^+_{\lambda_1\lambda'_1} + A_{\lambda'_1\lambda_1}). \tag{4}$$

Here u_λ, v_λ are the coefficients of the canonical transformation from the particle operators to the quasiparticle operators:

$$u^2_\lambda = \tfrac{1}{2}(1 + \varepsilon_\lambda / E_\lambda), \qquad v^2_\lambda = \tfrac{1}{2}(1 - \varepsilon_\lambda / E_\lambda),$$

and $A^+_{\lambda\lambda'} = \alpha^+_\lambda \beta^+_{\lambda'}$, $A_{\lambda'\lambda} = \beta_\lambda \alpha_{\lambda'}$ are the creation and annihilation operators for pairs of quasiparticles.

In investigating collective excitations we must in our case restrict ourselves to the part of the interaction Hamiltonian H'_Q, and this corresponds to summing graphs of the form shown in the figure.



The neglect of H''_Q in the investigation of collective effects introduces an error $\sim A^{-1/3}$. In the first order approximation with respect to the interaction the operators A, A^+ can be treated as Bose operators, i.e.,

$$[A_{\lambda\lambda_1}, A^+_{\lambda'\lambda'_1}] \approx \delta_{\lambda\lambda'}\delta_{\lambda_1\lambda'_1}. \tag{5}$$

We let

$$H^{red}|K\rangle = E_K|K\rangle, \qquad H^{red}|0\rangle = E_0|0\rangle,$$
$$H^{red} = H_0 + H'_Q, \tag{6}$$

where $|0\rangle$ is the wave function of the ground state, $|K\rangle$ are the wave functions of the excited states of the boson type, characterized by the energy E_K and by the component of angular momentum along the nuclear axis K.

In order to obtain the excitation energy ω_K we write the equations of motion for the operators A, A^+. In matrix form these equations appear as follows ($\hbar = 1$)

$$\omega_K \langle K | A^+_{\lambda\lambda'} | 0\rangle + \langle K | [A^+_{\lambda\lambda'}, H^{red}] | 0\rangle = 0,$$
$$\omega_K \langle K | A_{\lambda'\lambda} | 0\rangle + \langle K | [A_{\lambda'\lambda}, H^{red}] | 0\rangle = 0. \tag{7}$$

By utilizing (3) — (6) we obtain the system of integral equations for the amplitudes $\langle K | A^+_{\lambda\lambda'} | 0\rangle$ and $\langle K | A_{\lambda'\lambda} | 0\rangle$:

$$(\omega_K - E_\lambda - E_{\lambda'})\langle K | A^+_{\lambda\lambda'} | 0\rangle + \varkappa (q^*_{2K})_{\lambda'\lambda} f_{\lambda\lambda'} \sum_{\lambda\lambda'} (q_{2K})_{\lambda\lambda'} f_{\lambda\lambda'} \{\langle K | A^+_{\lambda\lambda'} | 0\rangle + \langle K | A_{\lambda'\lambda} | 0\rangle\} = 0,$$
$$(\omega_K + E_\lambda + E_{\lambda'})\langle K | A_{\lambda'\lambda} | 0\rangle - \varkappa (q^*_{2K})_{\lambda'\lambda} f_{\lambda\lambda'} \sum_{\lambda\lambda'} (q_{2K})_{\lambda\lambda'} f_{\lambda\lambda'} \{\langle K | A^+_{\lambda\lambda'} | 0\rangle + \langle K | A_{\lambda'\lambda} | 0\rangle\} = 0, \tag{8}$$

where we have introduced the notation $f_{\lambda\lambda'} = u_\lambda v_{\lambda'} + u_{\lambda'} v_\lambda$. The condition that the system (8) should have a solution gives an equation for the determination of ω_K:

$$1 = 2\varkappa \sum_{\lambda\lambda'} |(q_{2K})_{\lambda\lambda'}|^2 \frac{E_\lambda E_{\lambda'} - \varepsilon_\lambda \varepsilon_{\lambda'} + \Delta^2}{2E_\lambda E_{\lambda'}} \frac{E_\lambda + E_{\lambda'}}{(E_\lambda + E_{\lambda'})^2 - \omega^2_K}. \tag{9}$$

Equation (9) has been obtained previously[8] by the method of two-particle Green's function. As can be seen from (9) the frequencies of collective excitations depend on the specific form of the self-consistent potential and on the magnitude of the constant $\varkappa$.

We now proceed to determine the wave functions of the excited states. We introduce the operators $\mathfrak{A}_K$, $\mathfrak{A}_K^+$, which satisfy the operator equations

$$\omega_K \mathfrak{A}_K^+ + [\mathfrak{A}_K^+, H^{red}] = 0, \qquad \omega_K \mathfrak{A}_K - [\mathfrak{A}_K, H^{red}] = 0. \quad (10)$$

On the basis of (6) it follows from (10) that

$$\mathfrak{A}_K |0\rangle = 0, \qquad \mathfrak{A}_K^+ |0\rangle = |K\rangle. \quad (11)$$

The normalization condition $\langle K | K' \rangle = \delta_{KK'}$ yields

$$[\mathfrak{A}_K, \mathfrak{A}_{K'}^+] = \delta_{KK'}. \quad (12)$$

The operators $\mathfrak{A}$ and $\mathfrak{A}^+$ can be expressed in the form of a linear superposition of the operators A and A^+:

$$\mathfrak{A}_K = \sum_{\lambda\lambda'} (X_{\lambda\lambda'}^K A_{\lambda\lambda'} - Y_{\lambda'\lambda}^K A_{\lambda'\lambda}^+),$$
$$\mathfrak{A}_K^+ = \sum_{\lambda\lambda'} (X_{\lambda\lambda'}^{K*} A_{\lambda\lambda'}^+ - Y_{\lambda'\lambda}^{K*} A_{\lambda'\lambda}), \quad (13)$$

where the coefficients X and Y satisfy the normalization conditions

$$\sum_{\lambda\lambda'} (X_{\lambda\lambda'}^{K'} X_{\lambda\lambda'}^{K*} - Y_{\lambda'\lambda}^{K'} Y_{\lambda'\lambda}^{K*}) = \delta_{KK'}. \quad (14)$$

With the aid of (10) we obtain the equations for the determination of X and Y:

$$(\omega_K - E_\lambda - E_{\lambda'}) X_{\lambda\lambda'}^K + \varkappa (q_{2K}^*)_{\lambda'\lambda} f_{\lambda\lambda'} \sum_{\lambda\lambda'} (q_{2K})_{\lambda\lambda'} f_{\lambda\lambda'} (X_{\lambda\lambda'}^K + Y_{\lambda'\lambda}^K) = 0,$$
$$(\omega_K + E_\lambda + E_{\lambda'}) Y_{\lambda'\lambda}^K - \varkappa (q_{2K}^*)_{\lambda'\lambda} f_{\lambda\lambda'} \sum_{\lambda\lambda'} (q_{2K})_{\lambda\lambda'} f_{\lambda\lambda'} (X_{\lambda\lambda'}^K + Y_{\lambda'\lambda}^K) = 0. \quad (15)$$

It can be seen from (15) that in the representation in which the matrix elements $(q_{2K})_{\lambda\lambda'}$ are real, the coefficients X and Y are also real.

3. PROBABILITIES OF ELECTROMAGNETIC TRANSITIONS

The reduced probability of transitions of EL type for deformed nuclei has the form [9]

$$B(EL; I_i K_i \to I_f K_f) = e_{eff}^2 \langle K_f | Q(EL, \mu) | K_i \rangle^2 \langle I_i L K_i K_i - K_f | I_i L I_f K_f \rangle^2, \quad (16)$$

where I_i, K_i, I_f, K_f are respectively the spins and the components of angular momentum along the nuclear axis for the initial and the final states; $Q(EL, \mu)$ is the operator for the multipole moment in the system of coordinates fixed with respect to the nuclear axes; e_{eff} is the effective charge. The second factor in (16) is a Clebsch-Gordan coefficient. In the case $K_i = 0$ and $K_f \neq 0$ the magnitude of B(EL) is greater by a factor of two than would follow from (16).

We first obtain the expressions for B(E2). In terms of α and β, the quadrupole moment operator has the form

$$Q_{2\mu} = 2\sum_\lambda (q_{20})_{\lambda\lambda} v_\lambda^2 \delta_{\mu 0} + \sum_{\lambda\lambda'} (q_{2\mu})_{\lambda\lambda'} (u_\lambda v_{\lambda'} + u_{\lambda'} v_\lambda)(\alpha_\lambda^+ \beta_{\lambda'}^+ + \beta_\lambda \alpha_{\lambda'}) + \sum_{\lambda\lambda'} (q_{2\mu})_{\lambda\lambda'} (u_\lambda u_{\lambda'} - v_\lambda v_{\lambda'})(\alpha_\lambda^+ \alpha_{\lambda'} + \beta_\lambda^+ \beta_{\lambda'})$$
$$= Q_0 \delta_{\mu 0} + Q'_{2\mu} + Q''_{2\mu}. \quad (17)$$

The operator $Q''_{2\mu}$ makes no contribution to the collective effects, and therefore

$$\langle K | Q_{2K} | 0 \rangle \approx \langle K | Q'_{2K} | 0 \rangle = \sum_{\lambda\lambda'} (q_{2K})_{\lambda\lambda'} f_{\lambda\lambda'} \langle K | (A_{\lambda\lambda'}^+ + A_{\lambda'\lambda}) | 0 \rangle. \quad (18)$$

It can be easily seen from (13) and (15), that

$$X_{\lambda\lambda'}^K = \varkappa (q_{2K}^*)_{\lambda'\lambda} f_{\lambda\lambda'} (E_\lambda + E_{\lambda'} - \omega_K)^{-1} \langle K | Q_{2K} | 0 \rangle,$$
$$Y_{\lambda'\lambda}^K = \varkappa (q_{2K}^*)_{\lambda'\lambda} f_{\lambda\lambda'} (E_\lambda + E_{\lambda'} + \omega_K)^{-1} \langle K | Q_{2K} | 0 \rangle. \quad (19)$$

By utilizing the normalization condition (14) we obtain

$$\langle K | Q_{2K} | 0 \rangle^2 = \frac{1}{4\varkappa^2 \omega_K} \left\{ \sum_{\lambda\lambda'} (q_{2K})_{\lambda\lambda'}^2 f_{\lambda\lambda'}^2 \frac{E_\lambda + E_{\lambda'}}{[(E_\lambda + E_{\lambda'})^2 - \omega_K^2]^2} \right\}^{-1}. \quad (20)$$

We proceed to evaluate B(E0). As can be seen from (16), E0 transitions are possible only between the states $|0\rangle$ and $|K = 0\rangle$ (β-excitation). The matrix element of this transition has the form

$$B^{1/2}(E0; 0^+ \to 0^+) = \sum_{\lambda\lambda'} (r^2)_{\lambda\lambda'} (X_{\lambda\lambda'}^{K=0} + Y_{\lambda'\lambda}^{K=0}). \quad (21)$$

With the aid of (19) we obtain

$$\frac{B(E0; 0^+ \to 0^+)}{\langle K = 0 | Q_{20} | 0 \rangle^2} = \left[2\varkappa \sum_{\lambda\lambda'} (r^2)_{\lambda\lambda'} (q_{20})_{\lambda\lambda'} f_{\lambda\lambda'}^2 \frac{E_\lambda + E_{\lambda'}}{(E_\lambda + E_{\lambda'})^2 - \omega_{K=0}^2} \right]^2. \quad (22)$$

It follows from (20) and (22) that the probabilities of E2 and E0 transitions can be evaluated if we know the set of single particle levels, the corresponding frequencies of collective excitations and the value of the constant $\varkappa$.

4. ANALYSIS OF RESULTS

The formulas derived in the proceeding sections become considerably simplified in the case when the collective levels lie sufficiently low: $\omega_K \ll 2\Delta$ (adiabatic approximation). If the interval $\sim \Delta$ contains a sufficiently large number of single particle levels ($\rho_0 \Delta \gg 1$), then we can use the quasiclassical approximation. Particularly simple and clear results can be obtained in the case of the deformed oscillator potential. This case has been discussed previously.[8] For example, in the adiabatic approximation the following relation was obtained for the frequency of the β-excitation

$$\omega_{K=0} / 2\Delta = \left\{\frac{3}{2}\left[1 - \varkappa' \sum_{\lambda} (q_{20})^2_{\lambda\lambda} \delta(\varepsilon_\lambda)\right]\right\}^{1/2}, \tag{23}$$

where

$$\varkappa' = \varkappa \left[1 - \varkappa \sum_{\lambda \neq \lambda'} (q_{20})^2_{\lambda\lambda'} \frac{E_\lambda E_{\lambda'} - \varepsilon_\lambda \varepsilon_{\lambda'} + \Delta^2}{2E_\lambda E_{\lambda'} (E_\lambda + E_{\lambda'})}\right]^{-1}.$$

In (23) and in subsequent relations the summation over λ is taken over both signs of the component of angular momentum of the nucleons along the nuclear axis. An estimate of the value of the constant κ, and consequently also of κ' is available[10]:

$$\varkappa' \sim \varepsilon_0 / AR^4, \tag{24}$$

where A is the number of nucleons, R is the nuclear radius. By utilizing the obvious estimate

$$\sum_{\lambda} (q_{20})^2_{\lambda\lambda} \delta(\varepsilon_\lambda) \sim \rho_0 R^4, \tag{25}$$

we obtain

$$\varkappa' \sum_{\lambda} (q_{20})^2_{\lambda\lambda} \delta(\varepsilon_\lambda) \sim 1.$$

From this it follows that $\omega_{K=0}$ can be appreciably smaller than the quantity 2Δ.

From (23) it follows, in particular, that the method utilized in this paper is applicable if

$$\varkappa' \sum_{\lambda} (q_{20})^2_{\lambda\lambda} \delta(\varepsilon_\lambda) < 1.$$

Equation (9) can be rewritten in the form

$$1 = \varkappa'' \omega_K^2 \sum_{\lambda\lambda'} |(q_{2K})_{\lambda\lambda'}|^2 \frac{E_\lambda E_{\lambda'} - \varepsilon_\lambda \varepsilon_{\lambda'} + \Delta^2}{E_\lambda E_{\lambda'} (E_\lambda + E_{\lambda'})} \frac{1}{(E_\lambda + E_{\lambda'})^2 - \omega_K^2}, \tag{26}$$

where

$$\varkappa'' = \varkappa \left[1 - \varkappa \sum_{\lambda\lambda'} |(q_{2K})_{\lambda\lambda'}|^2 \frac{E_\lambda E_{\lambda'} - \varepsilon_\lambda \varepsilon_{\lambda'} + \Delta^2}{2E_\lambda E_{\lambda'} (E_\lambda + E_{\lambda'})}\right]^{-1}.$$

It can be easily shown that in the case of a deformation such that $\beta \ll 1$ the sum appearing in the definition of κ'' is independent of K with quasiclassical accuracy. Therefore, in the approximation $\omega_K \ll 2\Delta$ we obtain the ratio of the frequencies of the β- and γ-excitations in the form

$$\frac{\omega_\beta^2}{\omega_\gamma^2} = \sum_{\lambda\lambda'} |(q_{22})_{\lambda\lambda'}|^2 \mathscr{K}(\varepsilon_\lambda, \varepsilon_{\lambda'}) / \sum_{\lambda\lambda'} |(q_{20})_{\lambda\lambda'}|^2 \mathscr{K}(\varepsilon_\lambda, \varepsilon_{\lambda'}), \tag{27}$$

where

$$\mathscr{K}(\varepsilon_\lambda, \varepsilon_{\lambda'}) = \frac{E_\lambda E_{\lambda'} - \varepsilon_\lambda \varepsilon_{\lambda'} + \Delta^2}{2E_\lambda E_{\lambda'} (E_\lambda + E_{\lambda'})^3}. \tag{27'}$$

Thus, in the adiabatic approximation the ratio (27) is independent of the constant κ. In the quasiclassical approximation we obtain (cf. Appendix)

$$\omega_\beta^2 / \omega_\gamma^2 = \sum_{\lambda\lambda'} |(q_{22})_{\lambda\lambda'}|^2 F_{\lambda\lambda'} / \sum_{\lambda\lambda'} |(q_{20})_{\lambda\lambda'}|^2 F_{\lambda\lambda'}, \tag{28}$$

where we have used the notation

$$F_{\lambda\lambda'} = \frac{\delta(\varepsilon_\lambda)}{(\varepsilon_\lambda - \varepsilon_{\lambda'})^2}\left[1 - g\left(\frac{\varepsilon_\lambda - \varepsilon_{\lambda'}}{2\Delta}\right)\right],$$

$$g(x) = \frac{\ln(x + \sqrt{1 + x^2})}{x\sqrt{1 + x^2}}.$$

In the simplified Nilsson model without spin-orbit coupling[11] we obtain

$$\omega_\beta^2 / \omega_\gamma^2 = 3\int_0^1 [1 - g(tx)] \frac{(1 - 3x^2 + 2x^3)}{(tx)^2} dx, \qquad t = \frac{2D\varepsilon_0}{\omega_0 \Delta}. \tag{29}$$

where D is the Nilsson parameter, ω_0 is the gap between the shells in the oscillator potential. As $D \to 0$ the ratio $\omega_\beta/\omega_\gamma \to 1$. It can be shown by using (29) that for $D \neq 0$ the ratio $\omega_\beta/\omega_\gamma < 1$. Thus, in the adiabatic approximation the energies of the β-excitations are lower than the energies of the γ-excitations.

Systematic data on the position of the levels of β- and γ-excitations are available in the region of Th, U, Pu nuclei. For all these nuclei levels corresponding to β-excitations lie below levels corresponding to γ-excitations.[12]

A numerical solution of (9) was carried out for the case of β-excitations in the range of nuclei mentioned above. The Nilsson model was used as the single particle model. In expression (9) the sum over $\lambda \neq \lambda'$ was included in the renormalization of the constant κ. The calculation was carried out in accordance with the formula

$$1 = \widetilde{\varkappa} \sum_{\lambda} (q_{20})^2_{\lambda\lambda} \frac{\Delta^2}{\sqrt{\Delta^2 + \varepsilon_\lambda^2}} \frac{1}{4(\varepsilon_\lambda^2 + \Delta^2) - \omega_\beta^2}. \tag{30}$$

The value of the constant $\widetilde{\kappa}$ was determined from the position of the β-level in the Th^{232} nucleus. The values of the deformation β were taken from [13]. The quantity Δ was determined from the binding energy of appropriate nuclei. The energy of the Fermi boundary was identified with the position of the single particle level of the odd neutron in the ground state of the preceding odd nucleus.

The results of the calculation and their comparison with experiment are given in the table. The results of the calculation show that in agreement with experiment ω_β varies smoothly from one nucleus to another. It seems to us that this fact supports the hypothesis of the collective nature of this level.

Nucleus	β	Δ, Mev	ε_0	ω_β, Mev	
				experiment	theory
Th^{230}	0.23	0.85	633, 5/2	0.635	0.47
Th^{232}	0.24	0.85	633, 5/2	0.740	—
U^{232}	0.26	0.65	633, 5/2	0.700	0.79
U^{234}	0.25	0.80	633, 5/2	0.830	0.85
U^{236}	0.27	0.70	743, 7/2	0.925	0.96
U^{238}	0.27	0.70	631, 1/2	0.990	0.78
Pu^{238}	0.27	0.70	743, 7/2	0.940	0.96
Pu^{240}	0.28	0.60	631, 1/2	0.860	0.81

We now proceed to analyze the formulas for the probabilities of electromagnetic transitions. In the approximation $\omega_K \ll 2\Delta$

$$\langle K \mid Q_{2K} \mid 0\rangle^2 = \frac{1}{2\varkappa^2\omega_K}\left[\sum_{\lambda\lambda'} (q_{2K})^2_{\lambda\lambda'}\mathscr{K}(\varepsilon_\lambda, \varepsilon_{\lambda'})\right]^{-1}. \quad (31)$$

In (31) transitions over the shell can be neglected with an accuracy $\sim A^{-1/3}$. In the quasiclassical approximation we obtain (cf. Appendix)

$$\langle K \mid Q_{2K} \mid 0\rangle^2 = \frac{1}{2\varkappa^2\omega_K}\left[\sum_{\lambda\lambda'} (q_{2K})^2_{\lambda\lambda'}F_{\lambda\lambda'}\right]^{-1}. \quad (32)$$

We now discuss the excitation of β-levels. If the single particle diagram is such that in the sum (30) only diagonal matrix elements are important (for example, the axially-symmetric oscillator potential), then

$$\langle K=0 \mid Q_{20} \mid 0\rangle^2 = \frac{3\Delta^2}{\varkappa^2\omega_\beta}\left[\sum_{\lambda} (q_{20})^2_{\lambda\lambda}\delta(\varepsilon_\lambda)\right]^{-1}. \quad (33)$$

By utilizing the estimate (25) we obtain

$$B_\beta(E2) \sim e^2_{eff}\Delta^2/\varkappa\omega_\beta \sim B_{sp}(E2)\cdot\rho_0\Delta\cdot 2\Delta/\omega_\beta, \quad (34)$$

where $B_{sp}(E2) \sim e^2_{eff}R^4$ is the reduced probability for the single particle transition. Thus, it follows from (34) that in the adiabatic case there is an increase in the probability of the E2 transition in comparison with the single particle probability by approximately a factor of $A^{1/3}$.

By utilizing (27) and (31) we obtain in the adiabatic approximation

$$B_\gamma(E2;\ 0^+ \to 2^+)/B_\beta(E2;\ 0^+ \to 2^+) = 2\omega_\gamma/\omega_\beta. \quad (35)$$

From this it follows that in this approximation

$$B_\gamma(E2;\ 0^+ \to 2^+)/B_\beta(E2;\ 0^+ \to 2^+) > 2. \quad (36)$$

We go on to the discussion of E0 transitions. For $\omega_\beta \ll 2\Delta$ we obtain from (22) in the quasiclassical approximation

$$\frac{B(E0;\ 0^+ \to 0^+)}{B_\beta(E2;\ 0^+ \to 2^+)} \approx \varkappa^2\left[\sum_{\lambda\lambda'} (r^2)_{\lambda\lambda'}(q_{20})_{\lambda\lambda'}\delta(\varepsilon_\lambda)\right]^2. \quad (37)$$

On introducing the particle density near the Fermi surface

$$\rho(\varepsilon_0, \mathbf{r}) = \sum_\lambda \varphi^*_\lambda(\mathbf{r})\varphi_\lambda(\mathbf{r})\delta(\varepsilon_\lambda), \quad (38)$$

we obtain

$$\sum_{\lambda\lambda'}{}' (r^2)_{\lambda\lambda'}(q_{20})_{\lambda\lambda'}\delta(\varepsilon_\lambda) \approx \int \rho(\varepsilon_0, \mathbf{r})\, r^4 Y_{20}\, d\mathbf{r}. \quad (39)$$

The integral can be easily evaluated by assuming that the particle density is constant over the volume of the nucleus. We take the surface of the nucleus to be of the form

$$R(\theta) = R(1 + \beta Y_{20}(\theta)). \quad (40)$$

For the ratio (37) we have

$$B(E0;\ 0^+ \to 0^+)/B_\beta(E2;\ 0^+ \to 2^+) \approx (\varkappa\rho_0 R^7/V)^2\beta^2, \quad (41)$$

where V is the volume of the nucleus. By utilizing the estimate (24), and on taking into account that $\rho_0 \sim A/\epsilon_0$, we obtain $\kappa\rho_0 R^7/V \sim 1$, and consequently

$$B(E0;\ 0^+ \to 0^+)/B_\beta(E2;\ 0^+ \to 2^+) \sim \beta^2. \quad (42)$$

The estimate (42) is in qualitative agreement with the available experimental data.[14]

5. CONCLUSION

From the foregoing discussion it follows that the so-called β- and γ-vibrational levels can be interpreted from the microscopic point of view as collective excitations of boson type in a Fermi system of finite dimensions. Relations have been obtained for finding the frequencies of β- and γ-excitations, and also the probabilities of electromagnetic transitions. If the self-consistent potential, and, consequently, the single-particle energy level scheme, are sufficiently well known, then all the aforementioned quantities can be evaluated by means of automatic computers. The magnitude of the quadrupole-quadrupole interaction constant κ for a given domain of deformed nuclei (rare earths or actinides) can be regarded as constant. Therefore, having obtained the value of κ from the known β- and γ-level for one nucleus we can calculate the positions of β- and γ-levels for other nuclei. Knowing the value of the constant κ we can also evaluate the probabilities of electromagnetic transitions for a given group of nuclei.

Unfortunately, at the present time we do not yet have sufficient experimental data pertaining to the aforementioned type of levels. As has been pointed out already, systematic data on the positions of β- and γ-levels are available only for Th, U, Pu nuclei. In the rare earth region there are only a few such data. The low lying 0^+ level has been identified only in a few cases. A search for low-lying 0^+ levels in the region of the rare earths would be of interest. In accordance with the theory, if there exists a low lying 2^+ level corresponding to a γ-excitation, then nearby there should also exist a 0^+ level corresponding to a β-excitation. Measurement of the probabilities of electromagnetic transitions is also of interest since so far there are very few data on this subject and they are not accurate.

In conclusion the authors express their deep gratitude to Yu. T. Grin' who participated in the initial stages of this work.

APPENDIX

We evaluate the sum

$$S \equiv \sum_{\lambda\lambda'} |(q_{2K})_{\lambda\lambda'}|^2 \mathscr{K}(\varepsilon_\lambda, \varepsilon_{\lambda'}) \qquad (A.1)$$

in the quasiclassical approximation. For this we use the method proposed by Migdal.[15] It can be easily seen that the quantity $\mathscr{K}$ regarded as a function of ϵ_λ has a sharp maximum when the difference $d = \epsilon_\lambda - \epsilon_{\lambda'}$ is held fixed. This maximum has a width of $\sim \Delta$ near the Fermi surface. We denote

$$\int \mathscr{K}(\varepsilon_\lambda, \varepsilon_\lambda - d)\, d\varepsilon_\lambda = L\left(\frac{d}{2\Delta}\right).$$

Since the width Δ encompasses many levels and $\mathscr{K}$ has the required property near the Fermi surface, we have

$$S \approx \sum_{\lambda\lambda'} |(q_{2K})_{\lambda\lambda'}|^2 L\left(\frac{\varepsilon_\lambda - \varepsilon_{\lambda'}}{2\Delta}\right) \delta(\varepsilon_\lambda). \qquad (A.2)$$

It can be easily seen that

$$L\left(\frac{d}{2\Delta}\right) = \frac{1}{4}\int_{-\infty}^{\infty} \frac{d\varepsilon}{\sqrt{\Delta^2+\varepsilon^2}\sqrt{\Delta^2+(\varepsilon-d)^2}\,(\sqrt{\varepsilon^2+\Delta^2}-\sqrt{\Delta^2+(\varepsilon-d)^2})},$$

$$-\frac{1}{4d^2}\int_{-\infty}^{\infty} \frac{(\sqrt{\varepsilon^2+\Delta^2}-\sqrt{\Delta^2+(\varepsilon-d)^2})^2\, d\varepsilon}{\sqrt{\Delta^2+\varepsilon^2}\sqrt{(\varepsilon-d)^2+\Delta^2}\,(\sqrt{\varepsilon^2+\Delta^2}-\sqrt{\Delta^2+(\varepsilon-d)^2})}$$

$$= I_1 + I_2. \qquad (A.3)$$

The first integral in (A.3) is equal to

$$I_1 = \frac{1}{(2\Delta)^2} g\left(\frac{d}{2\Delta}\right), \qquad (A.4)$$

where

$$g(x) = [x\sqrt{1+x^2}]^{-1} \ln (x + \sqrt{1+x^2}).$$

The integral I_2 can also be easily evaluated and reduces to the expression

$$I_2 = \frac{1}{d^2}\left\{1 - \left[1 + \left(\frac{d}{2\Delta}\right)^2\right] g\left(\frac{d}{2\Delta}\right)\right\}. \qquad (A.5)$$

As a result of this we obtain for S with quasiclassical accuracy the expression utilized in (28) and (32).

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