POLARIZATION CROSS SECTION FOR THE SCATTERING OF FAST NUCLEONS

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The contribution of pion poles to the cross section for the NN → NN process is investigated for the case where the polarization of one of the final nucleons is measured. It is shown that not only the contribution from the quadratic term but also that from the linear term vanishes at all angles. Some practical consequences of this fact are discussed.

In recent times measurements of polarization in the elastic scattering of fast nucleons at small angles have attracted great interest. In the present article we hope to direct attention to the fact that such experiments are also of considerable theoretical interest.

Let us write the NN → NN scattering amplitude in the form of the sum

$$M = P_1 + P_{-1} + X, \quad (1)$$

where P_1 and P_{-1} arise from single-meson poles lying respectively in the regions $\cos\theta > 1$ and $\cos\theta < -1$ of the plane of the cosine of the scattering angle (center-of-mass system). Their explicit form is well known; the principal experimental interest, therefore, lies in the term X, which is determined by more distant singularities (many-pion contributions[1,2]).

As is known, the individual terms of relation (1) have the form (μ is the mass of the π meson)

$$P_1 = \frac{g^2}{\mu^2 - t} S_5,$$

$$P_{-1} = \frac{g^2}{\mu^2 - \bar{t}}\left(\frac{1}{4}S_1 - S_2 - \frac{3}{2}S_3 - S_4 + \frac{1}{4}S_5\right); \quad (2)$$

$$X = \sum_{i=1}^{5} x_i S_i, \quad (3)$$

where

$$S_1 = 1, \quad S_2 = \sum_{\mu}\gamma_\mu^1\gamma_\mu^2, \quad S_3 = \sum_{\mu<\nu} i\gamma_\mu^1\gamma_\nu^1 i\gamma_\mu^2\gamma_\nu^2,$$

$$S_4 = \sum_{\mu} i\gamma_\mu^1\gamma_5^1 i\gamma_\mu^2\gamma_5^2, \quad S_5 = \gamma_5^1\gamma_5^2;$$

$$t = -2q^2(1-\cos\theta), \quad \bar{t} = -2q^2(1+\cos\theta)$$

(q is the length of the momentum vector in the center-of-mass system). The gamma-matrix superscripts distinguish between the spaces of the first and second fermions.

Let us begin by examining the first single-meson term P_1, and let us write the sum $P_{-1} + X$ as

$$Y = \sum_{i=1}^{5} y_i S_i.$$

The expression for the cross section contains, besides the quadratic terms $|P_1|^2$ and $|Y|^2$, the interference terms P_1^+Y and Y^+P_1. If it were now possible to assume that all coefficients y_i except y_5 were negligibly small, y_5 could be expressed in terms of the unpolarized cross section by solving a simple quadratic equation. However, since Y contains generally speaking all the structure terms, it is not possible to distinguish further between contributions to the cross section from terms of the type $|Y|^2$ and interference terms.

At high energies this "undesirable" contribution from the interference terms becomes, generally speaking, stronger and stronger, since the poles asymptotically approach the boundary of the physical region (for example, at 9 Bev the cosines of the poles $P_{\pm 1}$ equal ±1.002 in the center-of-mass system). At small angles (which are important from the experimental point of view) the quadratic term $|P_1|^2$ is of practically no significance for the cross section, since it vanishes together with its derivative at $\cos\theta = 1$.[1] On the other hand, the interference term $P_1^+X + X^+P_1$ has a larger derivative at $\cos\theta = 1$. Therefore at small angles the effect of the pole is almost exclusively determined by the sharp dependence of the interference term on the cosine.

In order to find all five coefficients x_i it is necessary to carry out a "complete experiment" consisting of five independent experiments. We wish to draw attention below to the fact that in experiments where the polarization of one of the nucleons is measured the effect of the interference term $P_{\pm 1}^+X + X^+P_{\pm 1}$ is completely eliminated throughout the region $-1 \leq \cos\theta \leq 1$ (the vanishing of the quadratic term $|P_{\pm 1}|^2$ in such experiments is generally known).

The polarization cross section is proportional to the expression

$$\mathrm{Sp}_1\mathrm{Sp}_2\left(M^+\frac{-i\hat{p}+m}{2m}\frac{-i\hat{k}+m}{2m}M\frac{-i\hat{k}'+m}{2m}\frac{-i\hat{p}'+m}{2m}\Sigma e\right), \tag{4}$$

where $\hat{p} = \gamma_\mu p_\mu$; $\mathbf{\Sigma e}$ is the Racah spin tensor of nucleon polarization; Σ_i here is determined by the relation (σ_i is the Pauli matrix)

$$\Sigma_i \equiv \begin{pmatrix} \sigma_i & 0 \\ 0 & \sigma_i \end{pmatrix} = i\gamma_5\gamma_i\beta \qquad (\beta = \gamma_4).$$

Multiplying this relation by the polarization vector $e \equiv (0, \mathbf{e})$, we obtain $\Sigma\mathbf{e} = i\gamma_5\hat{e}\beta$.

To calculate the interference between amplitude (1) and pole (2) it is necessary to replace one of the M's in (4) by a pseudoscalar term and the other by an expression for X. The computation can be carried out in the rest system of the polarized nucleon. The problem consists of calculating five sums of double traces which are combinations of sixteen expressions of the type

$$i\mathrm{Sp}\left(\frac{-i\hat{p}+m}{2m}\gamma_4\gamma_5\gamma_4\frac{1+\gamma_4}{2}\hat{e}\gamma_4\gamma_5\gamma_A\right)$$
$$\times\,\mathrm{Sp}\left(\frac{-i\hat{k}+m}{2m}\gamma_4\gamma_5\gamma_4\frac{-i\hat{k}'+m}{2m}\gamma_A\right), \tag{5}$$

where γ_A designates any of the sixteen terms of the appropriate γ-algebra. It is necessary to form five sums from these terms so as to obtain the structure terms $S_1, \ldots, S_5$. All these sums equal zero, because the expression $\epsilon^{\alpha\beta\gamma\delta}p_\alpha p'_\beta k'_\gamma k_\delta$ vanishes owing to conservation of energy and momentum ($\epsilon^{\alpha\beta\gamma\delta}$ is a unit antisymmetric tensor).

The second single-meson term P_{-1}, which in the chosen representation has a complicated structure, can be reduced to the form $\gamma_5^1\gamma_5^2$ by the Fierz transformation.

As has been shown above, the effect of the single-meson term on the cross section is completely eliminated in polarization experiments, and it becomes possible to measure directly the contributions of higher meson approximations. Of course, there are some nonsingle-meson terms whose spin structure resembles that of the single-meson terms, and which thus also make no contribution to the polarization cross section. But although it is impossible to determine the magnitude of all x_i ($i = 1, 2, \ldots, 5$) by such an experiment, we can still obtain a lower estimate of the total contribution of nonsingle-meson terms. At the present time very little is known about them theoretically, but it is generally considered that they are small.

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[1] G. F. Chew, Phys. Rev. **112**, 1380 (1958); Ann. Rev. Nucl. Sci. **9**, 29 (1959).

[2] Amati, Leader, and Vitale, Nuovo cimento **17**, 68 (1960).

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