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ON THE EXCITATION OF NUCLEI BY MUONS IN HEAVY MESIC ATOMS

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The ratio of the width for nonradiative nuclear excitation to the radiative width is calculated for levels of mu-mesonic atoms and shown to be weakly dependent on the muon matrix elements.

1. After a mesonic atom is formed in a highly excited state ($n \sim 14$), the muon cascades down to the ground state, the final transition of this cascade being the 2p-1s transition.[1] Zaretskii[2] showed that the full 2p-1s transition energy can be transferred to the nucleus (nonradiative excitation process). The ratio of the probability for nonradiative nuclear excitation with subsequent decay of the nucleus through various nuclear channels, W_{nuc}, to the probability for γ emission by the muon, W_γ, is of interest.

If the nuclear levels at an excitation energy equal to the 2p-1s muon transition energy are wide enough so that they can be considered to be overlapping, then this ratio is

$$W_{\text{nuc}}/W_\gamma = \Gamma_{\text{nr}}/\Gamma_\gamma, \tag{1}$$

where Γ_{nr} and Γ_γ are the nonradiative and radiative widths, respectively, of the 2p muon level.

If the nuclear levels are non-overlapping, the ratio of W_{nuc} to W_γ is determined[3,4] by the quantities $\Gamma_{\text{nr}}/\Gamma_\gamma$ and $\rho\Gamma_{\text{nuc}}$, where Γ_{nuc} is the average nuclear level width and ρ is the nuclear density at the energy of excitation of the nucleus.

Nonradiative nuclear excitation can also be important in transitions between other muon levels, for example, 3p-1s or the transition from any highly excited s or d state to 2p.

We calculate below the ratio $\Gamma_{\text{nr}}/\Gamma_\gamma$ for a dipole transition between arbitrary muon levels. The calculation is performed in the nonrelativistic approximation.

2. The muon-nucleus dipole interaction operator which leads to nonradiative muon transitions with nuclear excitation is

$$V = -\frac{4\pi}{3}e^2 \sum_m \sum_{i=1}^{Z} Y_{1m}(i) Y_{1m}^*(\mu) \times \begin{cases} r_i/r_\mu^2 \text{ for } r_\mu \geqslant r_i \\ r_\mu/r_i^2 \text{ for } r_\mu \leqslant r_i \end{cases}, \tag{2}$$

where $\mathbf{r}_i$ and $\mathbf{r}_\mu$ are the position vectors of the protons and the muon, respectively; the summation is over the protons in the nucleus with charge Ze.

In the initial state of the muon-nucleus system, let the muon be in the level with quantum numbers n_1, j_1, and l_1 and let the nucleus be in its ground state with spin I_0. As basis functions for the muon-nucleus system, we choose the eigenfunctions of the total angular momentum of the system, J_{in}, and its projection $M_{J_{\text{in}}}$:

$$\Psi_{J_{\text{in}} M_{J_{\text{in}}}} = \sum_{m_{j_1} m_{I_0}} (I_0 j_1 m_{I_0} m_{j_1} \mid J_{\text{in}} M_{J_{\text{in}}}) \psi_{I_0 m_{I_0}} R_{n_1 l_1} \Phi_{j_1 m_{j_1}}, \tag{3}$$

where $\psi_{I_0 m_{I_0}}$ is the wave function of the nucleus in its ground state, $R_{n_1 l_1} \Phi_{j_1 m_{j_1}}$ is the muon wave function, and $(\ldots|\ldots)$ is a Clebsch-Gordan coefficient.

Similarly, we write the wave function of the final state with total angular momentum J_f:

$$\Psi_{J_f M_f} = \sum_{m_{j_2} m_{I_f}} (I_f j_2 m_{I_f} m_{j_2} \mid J_f M_{J_f}) \psi_{I_f m_{I_f}} R_{n_2 l_2} \Phi_{j_2 m_{j_2}}, \tag{4}$$

where I_f is the spin of the final nuclear state and n_2, j_2, and l_2 are the quantum numbers of the muon final state.

In this representation, the matrix element of the operator (2) is (for brevity, we omit the dependence of the sign of the matrix element on the quantum numbers)

$$\begin{aligned} \langle J_f M_{J_f} | V | J_{\text{in}} M_{J_{\text{in}}} \rangle &= e^2 \left[\tfrac{4}{3}\pi (2l_1+1)(2j_2+1)\right]^{1/2} (1 l_1 0 0 \mid l_2 0) \\ &\times W(l_1 l_2 j_1 j_2;\ 1\,{}^1/_2) \sum (I_0 j_1 m_{I_0} m_{j_1} \mid J_{\text{in}} M_{J_{\text{in}}}) (J_f j_2 m_{I_f} m_{j_2} \mid J_f M_{J_f}) \\ &\times (1 j_2 m m_{j_2} \mid j_1 m_{j_1}) \left\langle \psi_{I_f m_{I_f}} \left| \sum_{i=1}^{Z} r_i f(r_i) Y_{1m}(i) \right| \psi_{I_0 m_{I_0}} \right\rangle, \end{aligned} \tag{5}$$

$$f(r_i) = r_i^{-3} \int_0^{r_i} r^3 R_{n_1 l_1} R_{n_2 l_2}\, dr + \int_{r_i}^{\infty} R_{n_1 l_1} R_{n_2 l_2}\, dr, \tag{6}$$

where W is the Racah coefficient and the summation in (5) is over m, m_{j_1}, m_{j_2}, m_{I_0}, and m_{I_f}.

The nuclear matrix element in (5) can be written:

$$\left\langle \psi_{I_f m_{I_f}} \left| \sum_i r_i f(r_i) Y_{1m} \right| \psi_{I_0 m_{I_0}} \right\rangle = (1 I_0 m m_{I_0} \mid I_f m_{I_f}) \left\langle I_f \left\| \sum_i r_i f(r_i) Y_1 \right\| I_0 \right\rangle. \tag{7}$$

where the dependence on the magnetic quantum numbers is separated out.

Since we are considering a nonradiative transition of a closed system, the total angular momentum and its projection are conserved. Thus, using (7) and carrying out the summation over magnetic quantum numbers in (5), we obtain

$$\langle J_f M_{J_f} | V | J_{in} M_{J_{in}} \rangle = e^2 [\tfrac{4}{3}\pi (2l_1+1)(2j_1+1)(2j_2+1) \times (2I_f+1)]^{1/2} (1 l_1 0 0 | l_2 0) W(I_0 j_1 I_f j_2; J_{in} 1) \left\langle I_f \left| \sum_i q_i f(r_i) \right| I_0 \right\rangle \delta_{J_f J_{in}} \delta_{M_{J_f} M_{J_{in}}} \quad (8)$$

where we have set $q_i = r_i Y_1(i)$, for brevity.

The nonradiative width is

$$\Gamma_{nr} = 2\pi \overline{|\langle |V| \rangle|^2} \rho. \quad (9)$$

The bar in (9) denotes summation over the final state quantum numbers J_f, M_{J_f}, j_2, I_f, averaging over the possible values of the total angular momentum of the system J_{in} and $M_{J_{in}}$, and also averaging the absolute square of the nuclear matrix element over those nuclear levels which lie within an energy interval of about Γ_{nr} around the transition energy. Substituting (8) into (9) and using the fact that the probability that the total muon-nucleus angular momentum be J_0 is $(2J_0+1)/(2I_0+1)(2j_1+1)$, we obtain

$$\Gamma_{nr} = \frac{8\pi^2}{9} e^4 (1 l_1 0 0 | l_2 0)^2 \sum_{I_f} \frac{2I_f+1}{2I_0+1} |\langle I_f | q_i f(r_i) | I_0 \rangle|^2_{av} \rho_{I_f}, \quad (10)$$

where ρ_{I_f} is the density of nuclear levels with spin I_f. The radiative width for the muon transition between the same states is

$$\Gamma_\gamma = \frac{4}{3} \frac{e^2 \omega^3}{c^3} (1 l_1 0 0 | l_2 0)^2 \left| \int_0^\infty r^3 R_{n_1 l_1} R_{n_2 l_2} dr \right|^2. \quad (11)$$

It is convenient to transform the integral in (11) in the following way. We write the equation of motion of the muon in matrix form:

$$m_\mu \langle \ddot{r} \rangle = -\langle \nabla U \rangle, \quad (12)$$

where m_μ is the muon mass and U is the potential energy of the muon in the field of the nucleus,

$$U = -Ze^2 \times \begin{cases} 1/r_\mu & \text{for } r_\mu \geq R \\ \tfrac{1}{2} R^{-1}[3-(r_\mu/R)^2] & \text{for } r_\mu \leq R \end{cases}, \quad (13)$$

where R is the nuclear radius.

With (12) and (13), we obtain

$$\int_0^\infty r^3 R_{n_1 l_1} R_{n_2 l_2} dr = \frac{Ze^2}{m_\mu \omega^2} \left[\frac{1}{R^3} \int_0^R r^3 R_{n_1 l_1} R_{n_2 l_2} dr + \int_R^\infty R_{n_1 l_1} R_{n_2 l_2} dr \right] = \frac{Ze^2}{m_\mu \omega^2} f(R). \quad (14)$$

f(x) in (14) is the same as that in (6). Thus, the ratio (1) in which we are interested is

$$\frac{\Gamma_{nr}}{\Gamma_\gamma} = \frac{2\pi^2}{3} \frac{m_\mu^2 \omega c^3}{Z^2 e^2} \sum_{I_f} \frac{2I_f+1}{2I_0+1} \left| \left\langle I_f \left| q_i \frac{f(r_i)}{f(R)} \right| I_0 \right\rangle \right|^2_{av} \rho_{I_f}. \quad (15)$$

The cross section for dipole photoexcitation is[5]

$$\sigma = (8\pi^3/3)(\omega/c)[|\langle Q_{1,1}\rangle|^2_{av} + |\langle Q_{1,-1}\rangle|^2_{av}]\rho. \quad (16)$$

We will consider the photoexcitation of the nucleus in the transition $\psi_{I_0} \to \psi_{I_f}$. We separate out the dependence of the matrix elements in (16) on the magnetic quantum numbers, as we did in (7); then, in the notation introduced previously, we obtain

$$|\langle Q_{1,1}\rangle|^2_{av} = \frac{e^2}{2I_0+1} \sum_{m_{I_0} m_{I_f}} (1 I_0 1 m_{I_0} | I_f m_{I_f})^2 |\langle I_f | q_i | I_0 \rangle|^2_{av} = \frac{e^2}{3} \frac{2I_f+1}{2I_0+1} |\langle I_f | q_i | I_0 \rangle|^2_{av}. \quad (17)$$

Thus, the photoexcitation cross section for the nuclear transition considered above, summed over the final values of the nuclear spin, is

$$\sigma = \frac{16\pi^3}{9} \frac{\omega}{c} e^2 \sum_{I_f} \frac{2I_f+1}{2I_0+1} \left| \left\langle I_f \left| \sum_i{}' q_i \right| I_0 \right\rangle \right|^2_{av} \rho_{I_f}. \quad (18)$$

Substituting (18) into (15) and letting

$$B = \frac{\sum_{I_f} (2I_f+1) \left| \left\langle I_f \left| \sum_i q_i f(r_i) \right| I_0 \right\rangle \right|^2_{av} \rho_{I_f}}{\sum_{I_f} (2I_f+1) \left| \left\langle I_f \left| \sum_i q_i f(R) \right| I_0 \right\rangle \right|^2_{av} \rho_{I_f}}, \quad (19)$$

we obtain, finally,

$$\Gamma_{nr} / \Gamma_\gamma = (3/8\pi)(m_\mu c^2 / Ze^2)^2 \sigma B. \quad (20)$$

3. The form factor B takes into account the effect of the finite nuclear size on the ratio (1). For heavy mesonic atoms like uranium, B is of the order of 2. For a point nucleus, B = 1 and the ratio (1) does not depend at all on the muon matrix elements. This is easily understood if one notes that the nonradiative nuclear excitation process can be interpreted as the inverse of internal conversion; then (20) is the inverse internal conversion coefficient. Since the internal conversion coefficient becomes independent of the nuclear matrix elements in the case of a point nucleus, it is natural that the dependence on the muon matrix elements disappears from (20). We note also that (20) depends on the transition energy mainly through the photoexcitation cross section, which is to be taken at the transition energy; B depends rather weakly on the energy.

The magnitude of the form factor B can be estimated as follows. The matrix element in (19) can be written

$$\langle k | \sum_i q_i f(r_i) | 0 \rangle = \sum_{k'} \langle k' | \sum_i q_i | 0 \rangle \langle k | f(r_i) | k' \rangle; \quad (21)$$

where the summation in (21) is over all intermediate states. In squaring (21) we neglect the cross terms. This is permissible if the phases of the terms in the summation in (21), each of which is the product of essentially different dipole and monopole transition matrix elements, are random. Then

$$\left|\left\langle k \left| \sum_i q_i f(r_i) \right| 0 \right\rangle\right|^2 = \sum_{k'} \left|\left\langle k' \left| \sum_i q_i \right| 0 \right\rangle\right|^2 |\langle k | f(r) | k' \rangle|^2, \quad (22)$$

where $f(r)$ is a very smooth function. For example, for the 2p-1s transition in mesonic uranium, using the numerical muon wave functions given by Pustovalov,[6] we obtain

$$f(r) = 1 - 0.71\,(r/R)^2 + 0.3\,(r/R)^{7/2} - 0.06\,(r/R)^5. \quad (23)$$

The state k is a highly excited nuclear state (in the mesonic uranium 2p-1s transition, $E_K \sim 6$ Mev) and therefore $|\langle k\ f(r)\ k' \rangle|^2$ has a maximum at $k' = k$ and for $k' \neq k$ it is quasiclassically small. On the other hand, the quantity $|\langle k' | \Sigma_i q_i | 0 \rangle|^2$, proportional to the photoexcitation cross section, is a smooth function of k' and can therefore be taken outside the summation over k':

$$\left|\left\langle k \left| \sum_i q_i f(r_i) \right| 0 \right\rangle\right|^2 \approx \left|\left\langle k \left| \sum_i q_i \right| 0 \right\rangle\right|^2 \sum_{k'} |\langle k | f(r) | k' \rangle|^2$$
$$= \left|\left\langle k \left| \sum_i q_i \right| 0 \right\rangle\right|^2 \langle k | f^2(r) | k \rangle. \quad (24)$$

Substituting (24) into (19), and assuming a uniform charge distribution in the excited state of the nucleus, we obtain

$$B \approx \frac{3}{f^2(1)} \int_0^1 x^2 f^2(x)\,dx, \quad x = \frac{r}{R}. \quad (25)$$

For the 2p-1s transition in U^{238}, $B \approx 1.8$. The photofission cross section for the above-mentioned energy in U^{238} is about 12 mb,[7] and the ratio of the fission width to the sum of fission and neutron widths is about 4.[8] Thus, assuming that the photoexcitation cross section is due to the absorption of electric dipole quanta, we obtain $\sigma \approx 50$ mb. From this

$$(\Gamma_{nr} / \Gamma_\gamma)_{2p-1s} \approx 0.7. \quad (26)$$

The same ratio holds also for Th, since the photoexcitation cross sections for U^{238} and Th are the same.[8]

For the 3p-1s transition in the same elements we obtain $E_{3p-1s} = 9.5$ Mev, $B \approx 1.4$, $\sigma \approx 320$ mb, and

$$(\Gamma_{nr} / \Gamma_\gamma)_{3p-1s} \approx 3.5. \quad (27)$$

The results, (26) and (27), can be qualitatively understood by the following argument. The radiation of a γ quantum by the muon is a first order process with a probability of order $e^2/\hbar c$; nonradiative nuclear excitation is a second order process with a probability of order $(e^2/\hbar c)(Ze^2/\hbar c)$ Thus, the ratio of the widths for these two processes is of order $Ze^2/\hbar c$. For the elements mentioned above, $Ze^2/\hbar c \sim 1$. However, for other nuclei and other transitions, the ratio (1) can be much larger or much smaller than $Ze^2/\hbar c$ because of the nuclear matrix elements.

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