HIGH-FREQUENCY MAGNETIC SUSCEPTIBILITY OF A UNIAXIAL ANTIFERROMAGNET IN A LONGITUDINAL MAGNETIC FIELD

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The high-frequency magnetic susceptibility tensor of an antiferromagnet is calculated for various values of a constant magnetic field applied along the axis of the specimen. The calculation is based on the Landau-Lifshitz equation of motion for the sublattice moments.

THE dispersion of the magnetic susceptibility of an antiferromagnet is fundamentally related to rotation of the magnetic moments of the sublattices in the effective magnetic field (including the external magnetic field). The imaginary part of the magnetic susceptibility has resonance characteristics; the resonance frequencies coincide with the natural frequencies of rotation of the system of moments. The resonance frequencies of an antiferromagnet were calculated for various equilibrium configurations by Kittel[1] and by Turov.[2] The present authors[3] calculated the high-frequency magnetic susceptibility of a uniaxial antiferromagnet in the absence of a constant magnetic field.

We know[4] that a sufficiently strong constant magnetic field can change the equilibrium configuration of the sublattice moments; this naturally produces a change in the nature of the dispersion. The subject of this article is the calculation of the high-frequency magnetic susceptibility of an antiferromagnet for various equilibrium configurations of the moments. The results obtained make it possible to determine the equilibrium structures and the type of transition between them from high-frequency measurements.

We consider a uniaxial antiferromagnet with two sublattices, located in a constant and uniform magnetic field directed along the chosen axis, and also in a weak alternating magnetic field of frequency ω. For calculation of the magnetic susceptibility of the antiferromagnet, it is necessary to consider the forced motion of the magnetic moments of the sublattices under the influence of the alternating field. This motion is known to be describable by the Landau-Lifshitz equation

$$\partial \mathbf{M}_s / \partial t = g\,[\mathbf{M}_s \mathbf{H}_e^{(s)}] - (\gamma / M^2)\,[\mathbf{M}_s\,[\mathbf{M}_s \mathbf{H}_e]], \qquad (1)^*$$

where $\mathbf{M}_s$ is the vector magnetization of the s-th sublattice (s = 1, 2); M is the magnitude of the magnetization of each of the sublattices and is assumed to be constant; g is the gyromagnetic ratio, γ a relaxation constant, and $\mathbf{H}_e^{(s)}$ the effective field acting on the s-th sublattice:

$$\mathbf{H}_e^{(s)} = -\partial \mathscr{H} / \partial \mathbf{M}_s. \qquad (2)$$

κ is the energy density of the antiferromagnet,

$$\mathscr{H} = \alpha \mathbf{M}_1 \mathbf{M}_2 - \tfrac{1}{2} \lambda\,[(\mathbf{M}_1 \mathbf{n})^2 + (\mathbf{M}_2 \mathbf{n})^2] + \eta\,(\mathbf{M}_1 \mathbf{n})\,(\mathbf{M}_2 \mathbf{n}) - \mathbf{H}\,(\mathbf{M}_1 + \mathbf{M}_2). \qquad (3)$$

Here α is the exchange interaction constant ($\alpha > 0$), and λ and η are anisotropy constants, which will be assumed to be positive.* Positiveness of the constants λ and η insures that in the absence of an external field, the magnetic moments will be directed antiparallel and along the axis of the antiferromagnet (**n** is a unit vector along this axis).

It is known[1] that if a constant magnetic field of magnitude greater than $H_1 = \sqrt{(\lambda + \eta)(2\alpha - \lambda + \eta)}\,M$ is applied along the axis, then the energetically preferred magnetic state is one in which the vectors $\mathbf{M}_1$ and $\mathbf{M}_2$ are oriented symmetrically with respect to the axis **n**, at an angle θ such that

$$\cos\theta = H / M\,(2\alpha - \lambda + \eta). \qquad (4)$$

The transition to the new ground state involves the surmounting of a potential barrier and is accompanied by evolution of heat (a transition of the first kind). Under these circumstances, of course, there is a range of fields in which the antiparallel orientation of moments is metastable. The upper limit of the metastable states is the field

$$H_2 = \sqrt{(\lambda + \eta)(2\alpha + \lambda + \eta)}\; M$$

*[MH] = M × H; (MH) = M · H.

*Since the anisotropy energy is related to relativistic interactions, λ and $\eta \ll \alpha$.

(the lability field). According to Eq. (4) the angle between the magnetic moments depends on the applied field when $H > H_2$ and becomes zero when

$$H = H_3 = (2\alpha - \lambda + \eta) M.$$

Subsequent increase of field does not change the structure of the magnetic state. At $H = H_3$ there occurs a phase transition of the second kind: the longitudinal component χ_{zz} of the magnetic susceptibility (the z axis is chosen along **n**) changes discontinuously from the value $2/(2\alpha - \lambda + \eta)$ for $H < H_3$ to zero for $H \geq H_3$.* We remark that the components χ_{xx} and χ_{yy} of the magnetic susceptibility are continuous at $H = H_3$.

On decrease of the field from values below H_3, it is possible to carry over into the low-field region the configuration with a symmetrical orientation of the moments with respect to the axis. The lability field in this case is

$$H_4 = [\lambda (2\alpha - \lambda + \eta)^2/(2\alpha + \lambda + \eta)]^{1/2} M.$$

We note that $H_4 < H_1$. The difference $H_2 - H_4$ determines the width of the hysteresis loop in the magnetization of the antiferromagnet.

We shall derive an expression for the high-frequency magnetic susceptibility tensor $\chi_{ik}(\omega)$ for various values of the constant field H. The tensor $\chi_{ik}(\omega)$ is calculated by means of Eq. (1), linearized with respect to the high-frequency field, at the equilibrium configurations considered above.

1. $H < H_1$. In this case, as is usual in gyrotropic media, it is convenient to describe the magnetic susceptibility by giving the values of $\chi_\pm$ in

$$m_\pm = \chi_\pm h_\pm,$$

where $h_\pm = h_x \pm ih_y$, $m_\pm = m_x \pm im_y$; **h** is the high-frequency magnetic field and **m** the alternating part of the resultant magnetic moment. With the abbreviations

$$\Omega^2 = (g^2M^2 + \gamma^2)(\lambda + \eta)(2\alpha + \lambda + \eta) - \gamma^2 H^2/M^2,$$
$$\Omega_1^2 = 2(g^2M^2 + \gamma^2)(\lambda + \eta), \quad (5)$$

the expressions for the components of the tensor χ_{ik} can be written

$$\chi_\pm = \frac{\Omega_1^2 - 2i\omega\gamma}{\Omega^2 - (\omega \mp gH)^2 - 2i\omega\gamma(\alpha + \lambda + \eta)}, \quad \chi_{zz} = 0. \quad (6)$$

From (6) it is evident that the role of antiferromagnetic resonance line width is played by the quantity $2\gamma(\alpha + \lambda + \eta)$.[3] At $H = 0$ the values of χ_+ and χ_- are equal:

$$\chi_+ = \chi_- = \chi_{xx} = \chi_{yy}.$$

The off-diagonal components χ_{xy} and χ_{yx} are then zero. This means that at $H = 0$ there is no gyrotropy.[3] It should be mentioned that in reference 3 a mistake was made, in consequence of which an incorrect frequency dependence of the imaginary part of χ_{xx} was obtained.*

2. $H_1 < H < H_3$. Because of the dependence of the angle between the magnetic moments upon the magnetic field, in this case $\chi_{zz} \neq 0$. The calculations lead to the following result:

$$\chi_{zz}(\omega) = \chi_{zz}(0)\frac{\nu^2 + i\nu\omega}{\nu^2 + \omega^2}; \quad (7)$$
$$\chi_{zz}(0) = 2/(2\alpha - \lambda + \eta),$$
$$\nu = (2\alpha - \lambda + \eta)\gamma \sin^2\theta = \gamma(1 - H^2/H_3^2) H_3/M. \quad (8)$$

Here the angle θ is determined by formula (4). We notice that for $\omega \neq 0$ the value of χ_{zz} approaches zero as $H \to H_3$, i.e., as $\theta \to 0$.

Formula (7) describes a behavior with relaxation time $\tau = 1/\nu$ in the neighborhood of the phase transition of the second kind (where $H \approx H_3$):

$$\tau = \frac{1}{\gamma}\frac{MH_3}{H_3^2 - H^2}. \quad (8')$$

The approach of the relaxation time to infinity at $H = H_3$ is in accordance with a result of the general theory of phase transitions of the second kind.[5]

We have assumed that the temperature of the body was fixed. If we take into account that all the parameters on which the tensor χ_{ik} depends are functions of temperature, then the equation $H_3 = H$ at fixed H must be regarded as an equation for determination of the temperature T_c of the phase transition of the second kind. Then formula (8′) determines the relaxation time τ in the neighborhood of T_c:

$$\tau = \frac{G}{|T_c - T|}, \quad G = M/2\gamma \left|\frac{dH_3}{dT}\right|_{T=T_c} \quad (8'')$$

Formulas (7), (8), (8′), and (8″) show that at fixed frequency there is a field or a temperature at which the absorption in a high-frequency magnetic field along the z axis reaches a maximum. The height and position of the maximum depend on the frequency ω. The maximum always occurs in that phase in which the angle between the sublattice moments is different from zero (this angle

*Strictly, $\chi_{zz} \neq 0$ for $H \geq H_3$. The value of χ_{zz} is determined by the dependence of the energy of spin waves upon the magnetic field (the "paraprocess"). At $T = 0$, χ_{zz} becomes zero. At any temperature below the Curie-Néel temperature, the value of χ_{zz} takes a finite jump at $H = H_3$.

*Our attention was directed to this fact by E. A. Turov, to whom we are very grateful.

plays the role of the parameter η in the general theory[5]). The transverse components of the magneitc susceptibility tensor in this case are

$$\chi_{xx} = \frac{\omega_1^2 - 2i\omega\gamma}{\omega_0^2 - \omega^2 - 2i\alpha\gamma'\omega}\cos^2\theta,$$

$$\chi_{yy} = \frac{1}{\alpha}\frac{\omega_0^2 - 2i\omega\gamma}{\omega_0^2 - \omega^2 - 2i\alpha\gamma'\omega},$$

$$\chi_{xy} = -\chi_{yx} = \frac{2igM\omega\cos\theta}{\omega_0^2 - \omega^2 - 2i\alpha\gamma'\omega}. \quad (9)$$

Here

$$\omega_0^2 = (g^2M^2 + \gamma^2)(4\alpha^2\cos^2\theta - 2(\lambda + \eta)\alpha\sin^2\theta),$$
$$\omega_1^2 = 4\alpha(g^2M^2 + \gamma^2), \quad (10)$$
$$\gamma' = \gamma\left[1 + \cos^2\theta - \frac{1}{2\alpha}(\lambda + \eta)\sin^2\theta\right].$$

The x axis lies in the plane of the magnetic moments, the y axis perpendicular to this plane.

In contrast to the preceding case ($H < H_1$), here there is a single resonance frequency $\omega = \omega_0$. This is connected with the fact that we have not taken account of anisotropy in the basal plane (the xy plane). Calculation of the spin-wave spectrum in this case leads, it is known,[2] to the result that one of the frequencies of the spectrum goes to zero as the wave vector goes to zero.

3. $H > H_3$. As was pointed out above, for $H > H_3$ the two magnetic moments in the equilibrium state are parallel to each other and are directed along the magnetic field ($\mathbf{H} \parallel \mathbf{n}$). The high-frequency magnetic susceptibility tensor in this case coincides with that of a uniaxial ferromagnet:

$$\chi_{xx}(\omega) = \chi_{yy}(\omega) = \chi_\perp(0)\frac{\omega_f^2 - i\omega\gamma_f}{\omega_f^2 - \omega^2 - 2i\omega\gamma_f},$$

$$\chi_{xy}(\omega) = -\chi_{yx}(\omega) = \frac{2igM\omega}{\omega_f^2 - \omega^2 - 2i\omega\gamma_f},$$

$$\chi_{xz} = \chi_{yz} = \chi_{zz} = 0, \quad (11)$$

where

$$\chi_\perp(0) = 2M/(H + (\lambda - \eta)M),$$
$$\omega_f^2 = g^2[H + (\lambda - \eta)M]^2(1 + \gamma^2/g^2M^2),$$
$$\gamma_f = \gamma(H/M + \lambda - \eta). \quad (12)$$

Comparison of formulas (11) and (12) with formulas (9) and (10) shows that at $H = H_3 = (2\alpha - \lambda + \eta)M$, all components of the tensor $\chi_{ik}(\omega)$ are continuous.

Knowledge of the frequency dependence of the magnetic susceptibility tensor enables us to solve the problem of the dependence of the resonance frequency on the form of an ellipsoidal specimen,[6] and also to calculate the frequencies of nonuniform resonance.[7] Both problems reduce to the finding of the characteristic solutions of the system of equations*

$$\operatorname{rot}\mathbf{h} = 0,$$
$$\operatorname{div}\mathbf{h} = \begin{cases} -4\pi\operatorname{div}\hat{\chi}\mathbf{h} & \text{(inside the body)} \\ 0 & \text{(outside the body)} \end{cases}$$

with zero boundary conditions at infinity and with the usual boundary conditions at the surface of the body (here the value $\hat{\chi}$ of the tensor at $\gamma = 0$ is used).

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*rot = curl.