DISINTEGRATION OF NONEVOLUTIONAL SHOCK WAVES

R. V. POLOVIN and K. P. CHERKASOVA

Physico-Technical Institute, Academy of Sciences, Ukrainian S.S.R.

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Disintegration of a magnetohydrodynamic shock wave with a small density discontinuity is investigated. The initial shock wave is a compression wave for which all boundary conditions are satisfied and the entropy increases. If the initial shock wave is evolutional, it cannot disintegrate. A non-evolutional shock wave can disintegrate into six magnetohydrodynamical waves (shock or self-similar waves, depending on the magnitude and direction of the initial magnetic field).

1. In magnetohydrodynamics, satisfaction of the boundary conditions on the discontinuity surface and an increase in the entropy are not sufficient to ensure existence of a shock wave. It is necessary also that the evolutionarity conditions be satisfied,[1,2] namely that the number of outgoing waves be equal to the number of independent boundary conditions on the discontinuity surface. In the opposite case, the problem of small perturbations of the shock wave has no solution, indicating that the initial shock wave disintegrates into several shock and self-similar waves.

The non-evolutionarity regions apparently coincide with the regions where the initial shock wave can disintegrate. A direct proof of this theorem was given only for the particular case when the velocity of the shock wave is close to the Alfven velocity, and the magnetic field on both sides of the shock wave makes a small angle with the normal to the discontinuity surface.

In the present paper we prove this theorem for another particular case, that of a shock wave with a small density jump. Such a shock wave will be evolutionary if the jumps of all the magnetohydrodynamic quantities are small, or non-evolutionary if it is close to a 180° Alfven discontinuity (these waves were investigated in detail by Bazer and Ericson[3]).

2. We consider first the possibility of disintegration of an evolutionary shock wave in which the jumps of all the magnetohydrodynamic quantities are small.

In this case we can use the results obtained by Lyubarskii and Polovin,[4] who determined the amplitudes of the waves produced as a result of disintegration of a small discontinuity:*

$$\Delta_{\pm}^{(\varepsilon)}\rho = \pm\frac{1}{2R}\left\{\varepsilon\frac{U_y^2c^2[\Delta\rho-(\partial\rho/\partial s)_p\Delta s]}{U_{\pm}^2-U^2}-\varepsilon\frac{\Delta H_y^2}{8\pi}\right.$$
$$\left.+\frac{\rho U_x^2}{U_{\pm}}\left[\frac{H_y\Delta v_y}{H_x}+\frac{U_y^2\Delta v_x}{U_{\pm}^2-U_x^2}\right]\right\}. \quad (1)$$

Here

$$\mathbf{U}\equiv\mathbf{H}/\sqrt{4\pi\rho}, \qquad U_{\pm}=\left[(U^2+c^2\pm R)/2\right]^{1/2},$$
$$R=\left[(U^2+c^2)^2-4c^2U_x^2\right]^{1/2};$$

$\Delta\rho$, Δs, Δv, and ΔH_y are the jumps in the density, entropy, velocity, and transverse magnetic field on the initial discontinuity; c is the velocity of sound; the upper symbol in the ± sign pertains to the rapid magnetoacoustic wave, while the lower one pertains to the slow wave; $\epsilon = +1$ for waves propagating in the direction of positive x relative to the medium, while $\epsilon = -1$ pertains to waves propagating in the opposite direction; the x axis is directed along the normal to the discontinuity. A shock wave corresponds to $\Delta_{\pm}^{(\epsilon)}\rho > 0$, while a self-similar wave corresponds to $\Delta_{\pm}^{(\epsilon)}\rho < 0$.

If the initial discontinuity is a shock wave of low intensity, then the jumps in the magnetohydrodynamic quantities are related by the following equations

$$\Delta v_x/\Delta\rho=\varepsilon U_{\pm}/\rho, \qquad \Delta v_y/\Delta\rho=-\varepsilon H_xH_yU_{\pm}/4\pi\rho^2(U_{\pm}^2-U_x^2),$$
$$\Delta H_y/\Delta\rho=U_{\pm}^2H_y/\rho(U_{\pm}^2-U_x^2), \qquad \Delta p/\Delta\rho=c^2. \quad (2)$$

Assuming, for the sake of being definite, that the initial discontinuity moved in the direction of negative x ($\epsilon = -1$) and substituting (2) into (1), we obtain

$$\Delta_{\pm}^{(-)}\rho=\Delta_{\pm}\rho, \qquad \Delta_{\mp}^{(-)}\rho=\Delta_{\pm}^{(+)}\rho=\Delta_{\mp}^{(+)}\rho=0.$$

This means that an evolutionary shock wave of low intensity cannot disintegrate.

3. We now proceed to investigate the disintegration of a non-evolutionary shock wave with small

*An error has crept into this formula in the papers of Polovin and Lyubarskii.[2,4]

density jump. The jumps in the magnetohydrodynamic quantities in such a wave are related by

$$\Delta v_x = U_{1x}\rho_1^{-1}\Delta\rho, \qquad \Delta p = \left[c_1^2 + (\gamma - 1)U_{1y}^2\right]\Delta\rho,$$

$$\Delta H_y = -2H_{1y}\left[1 - \frac{U_{1x}^2 - c_1^2 - (\gamma - 1)U_{1y}^2}{2U_{1y}^2}\frac{\Delta\rho}{\rho_1}\right],$$

$$\Delta v_y = -2U_{1y}\left[1 - \frac{U_{1x}^2 - c_1^2 - \left(\gamma - \frac{1}{2}\right)U_{1y}^2}{2U_{1y}^2}\frac{\Delta\rho}{\rho_1}\right]. \qquad (3)$$

The subscript 1 pertains to the region in front of the shock wave. We note that as $\Delta\rho \to 0$, this wave goes into an Alfven discontinuity, which rotates the magnetic field through 180°. We note also that the entropy jump on such a wave is a second-order quantity, $\Delta s = (U_{1y}^2/\rho_1 T_1)\Delta\rho$.

The initial shock wave (3) can break up into seven waves; three waves moves to the right (fast magnetoacoustic, Alfven discontinuity, and slow magnetoacoustic), three waves move to the left (fast, Alfven, and slow), and a contact discontinuity, at rest relative to the medium, in the middle.

Let us examine first the Alfven discontinuity. The jumps in the magnetohydrodynamic quantities on the Alfven discontinuity are related by the equations

$$\Delta_A^{(\varepsilon)}H_y = H_{1y}(\eta_\varepsilon - 1), \qquad \Delta_A^{(\varepsilon)}v_y = \varepsilon U_{1y}(1 - \eta_\varepsilon). \qquad (4)$$

The jumps in the other magnetohydrodynamic quantities are equal to zero; the subscript 1 pertains to the region in front of the discontinuity, ϵ has the same meaning as in (2), $\eta_\epsilon = 1$ if there is no Alfven discontinuity, and $\eta_\epsilon = -1$ if the Alfven discontinuity rotates the magnetic field through 180°.

Expressing ΔH_y and Δv_y in terms of $\Delta_\pm^{(\epsilon)}H_y$, $\Delta_\pm^{(\epsilon)}v_y$, $\Delta_A^{(\epsilon)}H_y$, and $\Delta_A^{(\epsilon)}v_y$, and noting that $\Delta_\pm^{(\epsilon)}H_y$ and $\Delta_\pm^{(\epsilon)}v_y$ tend to zero as $\Delta\rho \to 0$, we obtain $\eta_- = -1$ and $\eta_+ = +1$. This means that disintegration of the initial shock wave results in a 180° Alfven discontiniity moving in the same direction as the initial wave. There is no Alfven discontinuity moving in the opposite direction.

We now proceed to determine the amplitudes of the magnetoacoustic waves and of the contact discontinuity. Each of these five waves is characterized by a single parameter—the density jump. On the other hand, the sum of the jumps of each of the five magnetohydrodynamic quantities (p, ρ, v_x, v_y, and H_y) on the seven resultant waves is equal to the initial jump. Since the number of unknown amplitudes is equal to five ($\Delta_+^-\rho$, $\Delta_-^-\rho$, Δ_k, $\Delta_-^+\rho$, and $\Delta_+^+\rho$), we obtain a system of five equations with five unknowns, the solution of which yields

$$\Delta_k\rho = -(\gamma - 1)c_1^{-2}U_{1y}^2\Delta\rho,$$

$$\Delta_+^+\rho = \left[\frac{U_{1x}^2 + U_{1-}^2}{2(U_{1x}^2 - U_{1-}^2)} + \frac{U_1^2 - c_1^2 - \gamma U_{1y}^2}{U_{1y}^2} - b_+\left(\frac{U_{1x}U_{1+}}{U_{1x}^2 - U_{1-}^2} + \frac{U_{1+}^2 + U_{1x}^2 - U_{1+}U_{1x}}{U_{1-}^2 - U_{1x}^2}\right)\right]\frac{\Delta\rho}{a},$$

$$\Delta_+^-\rho = \left[\frac{U_{1x}^2 + U_{1-}^2}{2(U_{1x}^2 - U_{1-}^2)} + \frac{U_1^2 - c_1^2 - \gamma U_{1y}^2}{U_{1y}^2} + \frac{U_{1+}Rb_+}{U_{1x}U_{1y}^2}\right]\frac{\Delta\rho}{a},$$

$$\Delta_-^{(\varepsilon)}\rho = \left[\frac{U_{1+}^2}{2(U_{1+} - U_{1x})} - \frac{U_{1x}^2}{2(U_{1+} + U_{1x})} - \frac{U_{1+}(U_1^2 - c_1^2 - \gamma U_{1y}^2)}{U_{1y}^2} + \frac{1}{2}\frac{U_{1+} - U_{1x}}{U_{1+} + U_{1x}}U_{1+}b_+\right]\frac{\Delta\rho}{U_{1-}a} - \frac{\varepsilon b_-}{2}\Delta\rho, \qquad (5)$$

where

$$R = \sqrt{(U_1^2 + c_1^2)^2 - 4c_1^2U_{1x}^2},$$

$$x = (U_{1+}^2 + U_{1x}^2)/(U_{1+}^2 - U_{1x}^2) + 2U_{1x}U_{1+}/(U_{1x}^2 - U_{1-}^2),$$

$$b_\pm = \pm\frac{U_{1y}^2}{R}\left[\frac{U_1^2 - c_1^2 - \gamma U_{1y}^2}{U_{1y}^2} - \frac{c_1^2 + (\gamma - 1)U_{1y}^2}{U_{1\mp}^2 - U_1^2}\right].$$

The character of the wave (shock or self-similar) is determined by the sign of the quantity $\Delta_\pm^{(\epsilon)}\rho$.

If $U_1 \ll c_1$, then all the waves produced during the disintegration are shock waves.

On the other hand, if $U_1 \gg c_1$, $H_{1x} \ll H_{1y}$, or $H_{1y} \ll H_{1x}$, then the slow wave that moves in the same direction as the initial shock wave will be a shock wave. The remaining waves can be either shock or self-similar, depending on the values of U_1/c_1, H_{1x}/H_{1y}, and γ.

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