NUCLEON CORRELATIONS AND PHOTONUCLEAR REACTIONS. I
THE PHOTODISINTEGRATION OF He^4

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We consider a model in which pair correlations are taken into account and discuss the photodisintegration of the He^4 nucleus. The available experimental data allow us to estimate the range of the pair correlations r_K^S (S: spin of the correlated pair). In triplet states $r_K^1 \approx (1.3 \text{ to } 1.4)\ 10^{-13}$ cm. We obtain an upper limit for the correlation range in singlet states: $r_K^0 \lesssim r_K^1/3$.

1. One of the directions in which the theory of the nucleus has been developed is via an account of nucleon pair correlations. There are a number of papers[1-3] devoted to this problem. On the other hand, Levinger[4] has some time ago indicated the importance of nucleon correlations for the theory of photonuclear reactions in the high energy region.

Gorbunov[5] has obtained experimental data on the reactions

$$He^4(\gamma, np)\,D, \tag{A}$$

$$He^4(\gamma, 2n)\,2p, \tag{B}$$

and in particular the cross section for the reaction A and the ratio of the number of cases in which reaction B is observed to the number of cases of reaction A (which is approximately equal to $1/8$). We shall show in the present paper that these experimental data enable us to reach some conclusions about the character of the nucleon correlations in the He^4 nucleus. As the correlation is essentially determined by nucleon-nucleon interactions at small distances, these conclusions can apparently also be applied to other nuclei. We can here note beforehand that the small difference between the cross sections for the reactions B and A indicates that the correlations of nucleon pairs in singlet states is appreciably weaker than in triplet states: our calculations corroborate this assumption.

2. We write the He^4 nucleus wave function in the form

$$\Psi = N \prod_S \prod_{ij} (1 + \chi_{ij}^S \hat{Q}^S)\, \Psi_{IPM}, \tag{1}$$

where Ψ_{IPM} is the independent particle model wave function, $\hat{Q}^S$ the projection operator into a nucleon pair state with total spin S and vanishing total relative orbital angular momentum; χ_{ij}^S are correlators.

We shall assume that the correlators have the following properties: 1) they are functions of $|\mathbf{r}_i - \mathbf{r}_j|$ only and $\chi_{ij}^S(0) = -1$ corresponding to a strong nucleon-nucleon repulsion at small distances while $\chi_{ij}^S \to 0$ as $r \to \infty$; the distance over which the correlation differs appreciably from zero is called the correlation range; 2) the correlation range may depend on the spin of the correlated nucleon pair; 3) the correlators are independent of the third component of the isotopic spin of the pair in virtue of the charge-independence of the nuclear forces. We shall also assume that the correlators are small, and shall neglect their products with one another. The criterion that they are small is expressed by the inequality

$$\sum_{mn}{}' |c_{\alpha\beta}^{mn}|^2 \ll 1,$$

where the $c_{\alpha\beta}^{mn}$ are the expansion coefficients

$$\chi_{ij}^S \varphi_i^\alpha \varphi_j^\beta = \sum_{mn} c_{\alpha\beta}^{mn} \varphi_i^m \varphi_j^n.$$

The φ_i^α are here the single-particle functions out of which Ψ_{IPM} is constructed; i and j are particle numbers; α and β are their individual quantum numbers.

The function Ψ must be normalized using the condition $|\langle \Psi | \Psi_{IPM} \rangle|^2 = 1$. To evaluate the normalizing factor N we write the two-particle function $\psi_\beta^S(ij) = (1 + \chi_{ij}^S \hat{Q}^S)\varphi_i^\alpha \varphi_j^\beta$ in the following form

$$\psi_{\alpha\beta}^S(ij) = (z_{\alpha\beta}^S + \hat{q}\chi_{ij}^S \hat{Q}^S)\varphi_i^\alpha \varphi_j^\beta,$$
$$z_{\alpha\beta}^S = 1 + \langle \varphi_i^\alpha \varphi_j^\beta | \chi_{ij}^S \hat{Q}^S | \varphi_i^\alpha \varphi_j^\beta \rangle, \tag{2}$$

where the operator $\hat{q}$ makes the components of

$\chi_{ij}^{S}\hat{Q}^{S}\varphi_{i}^{\alpha}\varphi_{j}^{\beta}$ which belong to Ψ_{IPM} vanish. When writing it in this form the condition*

$$\langle\varphi_i^{\alpha}\varphi_j^{\beta}\mid\hat{q}\chi_{ij}^{S}\hat{Q}^{S}\mid\varphi_i^{\alpha}\varphi_j^{\beta}\rangle=0$$

is satisfied.

Using the fact that the correlators are small we have

$$N=\Big\{\prod_{S\alpha\beta} z_{\alpha\beta}^{S}\Big\}^{-1}\equiv Z^{-1}$$

and

$$\Psi\approx\Psi_{IPM}+\sum_{ij}\{z_{ij}^{S}\}^{-1}\hat{q}\chi_{ij}^{S}\hat{Q}^{S}\Psi_{IPM}. \qquad (3)$$

We note the resemblance of Eq. (3) with the expression for the wave function in the first order of usual perturbation theory.

3. It is convenient to take as the wave function $\Psi^{(-)}$ of the final state an eigenfunction of the Hamiltonian

$$H_f=\sum_{i=1}^{4}T_i+V_{12}+V_{34},$$

where to fix our ideas the numbers 1 and 2 indicate respectively a proton and a neutron with momenta $\hbar\mathbf{k}_1$ and $\hbar\mathbf{k}_2$, while the numbers 3 and 4 indicate the proton and the neutron which in reaction A form a deuteron and in reaction B move with some energy of the relative motion $E_{rel}=\hbar^2\kappa^2/2\mu$ ($\mu=m/2$ is the reduced mass). Such a choice of H_f is connected with the fact that the interaction V_{34} which leads to the formation of a deuteron must be taken into account exactly. The remaining part of the interaction

$$V'=\sum_{i>j}V_{ij}-V_{12}-V_{34}$$

can be taken into account approximately but we shall not do that as the correction will only be important near the threshold of the reaction.

We only evaluate the total cross section for the reactions, and restrict ourselves therefore in the operator for the electromagnetic transition H_ζ to the electrical dipole term, i.e., we put

$$H_\zeta\sim\frac{e}{2}\sum_i\nabla_i\zeta\tau_3^i,$$

where ζ is the polarization vector of the γ quantum and τ_3^i the third component of the isotopic spin of the i-th nucleon.

If we separate the motion of the center of mass of the nucleon pairs 1-2 and 3-4 from their relative motion, which is possible when we choose oscillator wave functions,† we get the cross section for the $He^4(\gamma, np)D$ reaction in the following form:*

$$d\sigma^A\sim\frac{3}{4}\left|G(\mathbf{K})g_0\tilde{g}_1(\mathbf{k})\,\mathbf{k}\zeta\right|^2 d\mathbf{K}\,d\mathbf{k},\quad g_0=\int\varphi_d(\mathbf{r})\varphi_{\mathbf{rel}}(\mathbf{r})\,d\mathbf{r},$$

$$\tilde{g}_S(\mathbf{k})=(2\pi)^{-3/2}\int e^{-i\mathbf{kr}}\hat{q}\chi^S\hat{Q}^S\varphi_{\mathbf{rel}}(\mathbf{r})\,d\mathbf{r},\quad S=0,1,$$

$$G(\mathbf{K})=(2\pi)^{-3/2}\int e^{-i\mathbf{KR}}\Phi(\mathbf{R})\,d\mathbf{R}. \qquad (4)$$

Here φ_d is the deuteron wave function, φ_{rel} the wave function of the relative motion of the nucleon pair, $\mathbf{k}=\frac{1}{2}(\mathbf{k}_1-\mathbf{k}_2)$, $\mathbf{K}=\mathbf{k}_1+\mathbf{k}_2$, and $\Phi(\mathbf{R})$ is the wave function of the center of mass of one pair of nucleons in the He^4 nucleus with respect to the other pair. In deriving Eq. (4) we have assumed the phase shift in the p state to be equal to zero.

For a given energy E^A of the incident γ quantum the magnitudes of the momenta $\hbar\mathbf{K}$ and $\hbar\mathbf{k}$ are connected through the energy conservation law. Integrating (4) over the angles of the vectors $\mathbf{K}$ and $\mathbf{k}$ and over all values of k possible for a given E^A we obtain the cross section for the reaction A as function of the energy E^A. It is, finally, necessary to give the explicit form of the correlators. The most convenient form for calculations is

$$\chi_{ij}^{S}=-\exp\{-\beta_S r_{ij}^2\}, \qquad (5)$$

and then the correlation range $r_K^S\approx\beta_S^{-1/2}$. A comparison of the calculated and the experimental cross sections for the $He^4(\gamma, np)D$ reaction (see Fig. 1) enables us to choose $\beta_1\approx(0.5\text{ to }0.6)\times10^{26}\text{ cm}^{-2}$ corresponding to a correlation range in the triplet state of $r_K^1\approx(1.4\text{ to }1.3)\times10^{-13}$ cm. This result corroborates the validity of the assumptions made in Sec. 2 involving the smallness of the correlators.

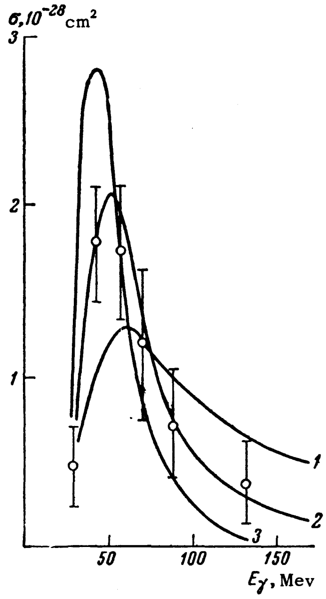

FIG. 1. Total cross section for the $He^4(\gamma, np)D$ reaction. The points are Gorbunov's experimental data[5] and the curves 1, 2, and 3 the theoretical total cross sections for $\beta_1=0.7$, 0.5, and 0.3×10^{26} cm^{-2}, respectively.

*One usually puts $\langle\varphi_i^{\alpha}\varphi_j^{\beta}\mid\chi_{ij}\mid\varphi_i^{\alpha}\varphi_j^{\beta}\rangle=0$. It is, however, practically impossible to find a function χ_{ij} with arbitrary parameters satisfying this condition.

†The elimination of the center-of-mass motion was performed by the method used by Lipkin.[6]

*For the sake of simplicity we have omitted from Eqs. (4) and (6) the factor $4\{z^S\}^{-2}(e^2/\hbar c)(2\pi)^2\hbar^4m^{-2}E_\gamma^{-1}$, where E_γ is the energy of the γ quantum.

Indeed, for the above-mentioned values of β we have $\sum_{mn}' |c_{\alpha\beta}^{mn}|^2 \approx 0.048$ and 0.040.

4. We can obtain information about the triplet correlation range only from the data on the $He^4(\gamma, np)D$ reaction. It turns out, that one can determine an upper limit for r_K^0 by comparing the cross sections of the reactions A and B (in practice one compares the number of cases where the reactions A and B are observed under identical circumstances[5]).

We evaluate the cross section for the $He^4(\gamma, 2n)2p$ reaction for the case where $\mathbf{k}_1$ and $\mathbf{k}_2$ are the same as the corresponding momenta in the reaction A and the nucleons 3 and 4 move with a relative momentum κ. The energy necessary for this case is $E^B = E^A + \epsilon_d + \hbar^2\kappa^2/2\mu$, where ϵ_d is the deuteron binding energy. We get

$$d\sigma^B \sim (4\pi)^{-1} \sum_S \frac{3^S}{4} \Big| G(\mathbf{K})\, g_{0S}(\varkappa)\, \tilde{g}_S(\mathbf{k})\, \mathbf{k}\zeta$$

$$+ G(\mathbf{K})\, g_{0S}(k)\, \tilde{g}_S(\varkappa)\, \varkappa\zeta \Big|^2 d\mathbf{K}\, d\mathbf{k}\, d\varkappa,$$

$$\times \quad g_{0S}(u) = \sqrt{\frac{2}{\pi}} \int \frac{\sin(ur + \delta^S)}{ur} \varphi_{\mathrm{rel}}(r)\, r^2\, dr. \qquad (6)$$

Here, δ^S is the phase of the s wave in the corresponding spin state.

Integrating (4) over the angles of the vector $\mathbf{k}$ and multiplying it by the number of γ quanta in the beam with energy E^A (denoted by n^A) we get for the reaction A the number of reactions for a given momentum $\hbar\mathbf{K}$. On the other hand, integrating (6) over the angles of the vectors $\mathbf{k}$ and κ, multiplying it then by n^B and integrating over κ from zero to its maximum value determined by the end-point energy of the γ spectrum and the energy E^A we obtain the number of reactions B with energies from $E^A + \epsilon_d$ to E_{max}^B (for the same fixed values of $\mathbf{K}$ and $\mathbf{k}$). We denote this quantity by $p^B(E^A, k)$. The ratio p^B/p^A can then be expressed in the form

$$p^B/p^A \equiv C(E^A, k)$$

$$= [n^A g_0^2 \tilde{g}_1^2(k)]^{-1} \sum_S \left(\tfrac{1}{3}\right)^{1-S} \Big\{ \tilde{g}_S^2(k) \int n^B g_{0S}^2(\varkappa) \varkappa^2\, d\varkappa$$

$$+ k^{-2} g_{0S}^2(k) \int n^B \tilde{g}_S^2(\varkappa)\, \varkappa^4\, d\varkappa \Big\}. \qquad (7)$$

We note that the momentum $\hbar\mathbf{K}$ has disappeared from this ratio.

The quantity

$$x = \int p^A(E, k)\, C(E^A, k)\; dE\, dk \Big/ \int p^A(E, k)\, dE\, dk$$

$$(E \equiv E^A)$$

in which we are interested can be evaluated for different assumed values of the parameter β_0. The results are given in Fig. 2.

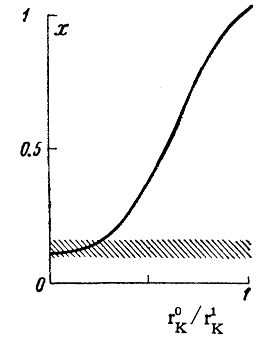

FIG. 2. The results of the calculations of x. The shaded band corresponds to the experimental value $x \approx 1/8$ (see reference 5).

If we take into consideration a certain lack of precision both in the experimental data and in the calculations, we can state that the correlation range in the singlet state of a nucleon pair is not larger than about $1/3$ that of the triplet correlation range. This result depends only little on the precise choice of the form of the correlators.

There are at the present no calculations which enable us to find the form of the correlators of the correlation range from a general theory and it is thus difficult to interpret the result obtained here. It is, apparently, connected with the large difference between the nucleon-nucleon forces in the ^3S and the ^1S states at small distances.

I consider it a pleasant duty to express my deep gratitude to M. Ya. Amus'ya for valuable discussions connected with this work.

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