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ELECTROMAGNETIC CORRECTIONS TO WEAK INTERACTIONS

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The procedure for calculating electromagnetic corrections to weak interactions is studied. It is shown that at present there is no basis for asserting that the coupling constants of various weak interaction processes are equal or unequal with an accuracy better than electromagnetic.

A number of papers appeared in recent years containing calculations of electromagnetic corrections to the simpler weak interaction processes (μ and β decays[1-5]). The total weak interaction Lagrangian is usually taken in the form[6]

$$L_G = G : j^+ j :, \qquad (0.1)$$

where j is a parity-nonconserving charged current:

$$j = (\bar{e}O\nu) + (\bar{\mu}O\nu) + (\bar{n}Op) + \ldots,$$
$$O^\mu = \gamma^\mu a_+, \qquad a_\pm = \tfrac{1}{2}(1 \pm i\gamma^5). \qquad (0.2)$$

Afterwards one proceeds in the standard manner used in field theory to calculate corrections to the probabilities and cross sections obtained from Eq. (0.1) due to electromagnetic

$$L_e = e\sum_\alpha : \bar{c}_\alpha \hat{A} c_\alpha : + \ldots \qquad (0.3)$$

and strong (n stands for nucleon)

$$L_g = g : \bar{n}\gamma^5 n\pi : + \ldots \qquad (0.4)$$

interactions.

When this is done one studies, in effect, two different problems. The first problem consists of finding more accurate expressions for angular and energetic distributions in μ and β decays with electromagnetic corrections taken into account. To the study of this problem are devoted the papers of Behrends, Finkelstein, Kinoshita, and Sirlin,[1] Durand, Landovitz, and Marr,[2] Berman,[4] and Kuznetsov,[3] in which, however, bremsstrahlung terms connected with recoil are not fully taken into account. (The corresponding correction terms for μ decay were obtained, in effect, in Kuznetsov's second paper.[7])

The second problem consists of the determination of the renormalization of the coupling constants of the various weak interaction processes as a consequence of the interactions (0.3) and (0.4). Gell-Mann and Feynman have proposed the conserved vector current hypothesis,[6] according to which the vector part of the weak interaction does not undergo renormalization due to strong interactions. In the papers of Goldberger and Treiman and Chou Kuang Chao[8] the magnitude of the renormalization due to strong interactions is discussed for the axial vector current in β decay and other processes (its value is taken from experiments on the asymmetry in β decay; see, e.g., Alikhanov[9]). (The observed deviations from the V-A scheme (V-A → V-λA) in β decay may also be explained within this framework by taking into account only electromagnetic corrections with appropriate form factors. This means that experiment could be consistent with the absence of renormalization of the axial vector current due to strong interactions. In this connection one should also note Nambu's work.[10])

It is also known that the β decay of mirror nuclei ($0^+ \to 0^+$ transitions) is due to only the vector part of the weak interaction, which, as indicated, is not renormalized by the strong interactions. In these decays, as in μ decay, the basic weak interaction is deformed only by electromagnetic corrections. The coupling constants in these cases turn out to be very close in magnitude. This raises the question of comparing the electromagnetically renormalized coupling constants in these processes. The near equality of these constants has been discussed in a number of papers.[1,2,5]

The present work is devoted to an analysis of the starting assumptions that form the basis of such a comparison. It turns out that in the framework of contemporary field theory it is not possible to determine uniquely the renormalized coupling constants. In different processes these constants may be determined only simultaneously with a certain (generally speaking arbitrary) normalization constant. At the moment it does not seem possible to establish a relation between the normalization constants in different processes. The gauge invariance property, contrary to the expectations of a number of authors,[1,2,5] does not save the situation. It follows therefore that a comparison of coupling constants for the above mentioned processes to an accuracy better than electromagnetic makes no sense.

Below we formulate and prove the concept of renormalization for the class of problems under consideration. We then prove the gauge invariance of the method of calculation used. In the end we consider the implications of the indicated facts for μ and β decay.

1. RENORMALIZABILITY

It is known that the four-fermion interaction (0.1) is not renormalizable, although for observable processes the inclusion of terms of higher order in G does not, apparently, change the V-A form.[11] Therefore a consistent discussion of terms of higher order in G is at this time simply impossible.* As a result we are forced to consider a semi-phenomenological theory in which only first order terms in G are kept, but all terms of the perturbation theory expansion in e and g corresponding to Eqs. (0.3) and (0.4) are taken into account. The justification of this approach is usually found in the extraordinary smallness of G.

We shall show that such a theory is renormalizable in the conventional sense. This means that the divergent expressions that appear in the course of calculations using perturbation theory can be eliminated by introducing into the Lagrangian counter terms of the type (0.3), (0.4), (0.1) and of the type of the free field Lagrangian or, which is the same, by renormalizing the masses and wave functions of the particles and the coupling constants of the interactions.

As is known, by an appropriate redefinition of the T products at the equal argument points, it is possible to reduce these infinite renormalizations to finite arbitrary multiplicative factors in the indicated quantities and in the simpler Green's functions related to them. Below, when referring to operations with divergent quantities, we shall understand operations with finite (defined up to a constant) quantities that result after the T products are redefined. One may also consider these divergent quantities to be regularized by the introduction of an appropriate cut-off. Then the above-noted finite arbitrariness in the determination of the coupling constants and particle masses corresponds, in part, to the circumstance that there is no need to choose the cut-off momenta to be the same in different, unrelated to each other, Feynman diagrams.

*It should be noted in addition that so far no processes of higher order in G have been observed.

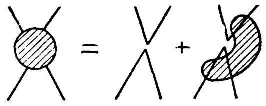

FIG. 1

We shall carry out our considerations on the simplest examples of μ and β decay, following the presentation in the book by Bogolyubov and Shirkov (Chap. 4).[12] We shall restrict ourselves in the matrix elements to only first order terms in G arising from the Lagrangians

$$L_\mu = G:(\bar{\mu}O\nu)(\bar{\nu}Oe):=G:(\bar{\mu}Oe)(\bar{\nu}O\nu):, \tag{1.1}$$

$$L_\beta = G:(\bar{n}Op)(\bar{\nu}Oe):=\frac{1}{2}:[\bar{n}(G_V\gamma^\mu + G_A\gamma^\mu i\gamma^5)p](\bar{\nu}\gamma^\mu a_+ e): \quad (G_V = G_A = G). \tag{1.1'}$$

As indicated above we will, however, take into account all terms of the perturbation-theory series corresponding to Eqs. (0.3) and (0.4). After the usual renormalizations of the coupling constants e and g, and the masses and wave functions of the particles, there remains in such a theory only one (logarithmic) divergence, corresponding to the diagram of Fig. 1. This is easily established by the conventional counting of the powers involved in the diagram.

For such G-vertices we can write an equation which, in terms of Feynman diagrams, corresponds to Fig. 1:

$$-iS_\mu = G:(\bar{\mu}Fe)(\bar{\nu}O\nu): = G:(\bar{\mu}Oe)(\bar{\nu}O\nu): + G:(\bar{\mu}Te)(\bar{\nu}O\nu):, \tag{1.2}$$

$$-iS_\beta = \frac{1}{2}:\bar{n}_\alpha p_\beta \bar{\nu}_\delta e_\gamma : F_{\alpha\beta;\gamma\delta} = \frac{1}{2}:[\bar{n}(G_V\gamma^\mu + G_A\gamma^\mu i\gamma^5)p](\bar{\nu}\gamma^\mu a_+ e): + \frac{1}{2}:\bar{n}_\alpha p_\beta \bar{\nu}_\delta e_\gamma : R_{\alpha\beta;\gamma\delta}. \tag{1.2'}$$

In other words

$$F_{\alpha\beta} = O_{\alpha\beta} + T_{\alpha\beta}, \tag{1.3}$$

$$F_{\alpha\beta;\gamma\delta} = GO_{\alpha\beta}O_{\gamma\delta} + R_{\alpha\beta;\gamma\delta}. \tag{1.3'}$$

Here T and R are contributions due to the strong and electromagnetic interactions. They are logarithmically divergent.

After some transformations, in which use is made of the operator structure of the strong and electromagnetic interactions,* of the properties of direct products of operators† (see, e.g., Slavnov and Sukhanov[13]), and of the two-component nature of the neutrino, the divergent parts of R and T are isolated. In this way one obtains, as usual,

$$T_{\alpha\beta} = (Z - 1)\, O_{\alpha\beta} + T^{\mathrm{reg}}_{\alpha\beta}, \tag{1.4}$$

$$R_{\alpha\beta;\,\gamma\delta} = [G_V\,(Z_1 - 1)\,\gamma^\mu + G_A\gamma^\mu i\gamma^5 (Z_2 - 1)]_{\alpha\beta} O_{\gamma\delta} + R^{\mathrm{reg}}_{\alpha\beta;\,\gamma\delta}. \tag{1.4'}$$

Here T^{reg} and R^{reg} are finite functions of the momenta and the renormalized constants e, g, m, M, μ, etc.

After introducing into the Lagrangian appropriate counter terms we find that the divergence under consideration leads to a multiplicative renormalization of the coupling constant G (the constants G_V and G_A in β decay):‡

$$G \to ZG; \tag{1.5}$$

$$G_V \to Z_1 G_V, \quad G_A \to Z_2 G_A, \quad Z_1 \neq Z_2. \tag{1.5'}$$

It should be noted once more, that after the separation (1.4) of T(R) into a regularized and divergent part there remains in the definition of $T^{\mathrm{reg}}(R^{\mathrm{reg}})$ a finite arbitrariness, corresponding to different possible choices of the subtraction constants. This arbitrariness corresponds to the finite multiplicative renormalization of the type (1.5). At that the constants Z and $Z_1(Z_2)$ are in no way related to each other or to the analogous constants of other renormalizations (electromagnetic and strong). At first sight it seems that the situation is opposite in electrodynamics where the subtraction constant for the vertex diagram is related to the subtraction constant for the fermion self energy diagram as a consequence of gauge invariance. This contradiction with electrodynamics is apparent only, the G-vertex diagram being only superficially similar to the vertex diagram in electrodynamics. This distinction will be discussed in more detail in the next Section.

*For the here relevant most strongly divergent parts these interactions give direct products of the type $(\gamma^5\gamma^\mu) \times (\gamma^\mu\gamma^5)$ and $(\gamma^\mu\gamma^\nu) \times (\gamma^\nu\gamma^\mu)$ respectively.

†The following of them are of importance to us: $C_\pm^{-1}(\gamma^\alpha\gamma^\beta\gamma^\mu a_\pm) \times (\gamma^\mu a_+ \gamma^\alpha\gamma^\beta) = D_\pm^{-1}(\gamma^\mu a_\pm \gamma^\alpha\gamma^\beta)(\gamma^\mu a_+ \gamma^\alpha\gamma^\beta) = (\gamma^\mu a_\pm)(\gamma^\mu a_+)$ ($C_\pm$ and $D_\pm$ are certain numbers irrelevant for the calculations).

‡This result may also be derived by writing the S matrix as

$$S = T\left\{\left[1 + i\int L_G(x)\,dx\right]\exp\left[i\int dx\,(L_e(x) + L_g(x))\right]\right\}$$

and then computing its matrix elements as variational derivatives with respect to appropriate fields. The electromagnetic and strong divergences are in that case separated from the divergence in the G-vertex.

Thus, after the renormalization of the coupling constants e, g, and G, the particle masses and their wave functions, the theory under consideration contains no divergences.

2. GAUGE INVARIANCE

We have indicated at the end of the preceding section that the renormalization constants Z and $Z_1(Z_2)$ are unrelated to each other or to the analogous constants for other electromagnetic or strong Green's functions (diagrams). In contrast to this in a number of papers[1,3-5] it has been in essence assumed that the constant Z is related to the renormalization constant for the fermion mass operator by a relation of the type of Ward's identity in electrodynamics. We shall now show that the requirement of gauge invariance in the processes under consideration does not impose any restrictions on the constant Z and that therefore the above indicated arbitrariness in the determination of the constant Z persists.

To prove this it is necessary to show that the procedure for calculating electromagnetic corrections to weak interactions is gauge invariant. We shall show that the matrix element of the weak interaction, including electromagnetic and strong-interaction corrections, is after renormalization independent of the field intensity of longitudinal and scalar photons d_l. A full proof of this assertion is somewhat clumsy in perturbation theory; we shall therefore not present it but will limit ourselves to the consideration of only the terms of lowest order in e. The proof can also be carried out with the help of contour integration. In that case one need only to repeat almost verbatim the corresponding discussion in the book by Bogolyubov and Shirkov[12] (Sec. 41).

It is more convenient to give the proof for the μ-decay process. The generalization to processes in which the contribution of the strong interactions is relevant presents no difficulty; we leave it out here only to avoid a greater, as compared to μ decay, complexity. We shall write the weak interaction matrix element in the form

$$-iS_\mu = G{:}(\bar{\mu}Fe)\,(\bar{\nu}O\nu){:}.$$

FIG. 2

At that
$$GF = G\,(O + \sum_{n\geqslant 1} e^{2n}F_n). \qquad (2.1)$$

Since we are concerned with a theory in which the electromagnetic renormalizations of the charge e, and the particle masses and wave functions, have already been carried out, it is sufficient in second order to consider only the diagram of Fig. 2, in fourth order the six diagrams of Fig. 3,* etc.

In the series (2.1) the term of the 2n-th order in e is a polynomial of n-th order in d_l:

$$F_n = \sum_{k=0}^{n} d_l^k F_n^{(k)}. \qquad (2.2)$$

To second order in e we have

$$F_1^{(1)} = \frac{1}{(2\pi)^4}\int \frac{\hat{k}}{k^2}\,\frac{\hat{p}-\hat{k}+\mu}{k^2-2pk}\,O\,\frac{\hat{q}-\hat{k}+m}{k^2-2qk}\,\frac{\hat{k}}{k^2}\,dk.$$

Here p and q are the momenta of the electron and muon respectively. Since in Eq. (2.1) F stands between the operators $\overline{\psi}_\mu$ and ψ_e, which satisfy their respective Dirac equations, it follows that

$$F_1^{(1)} = C_1 O. \qquad (2.3)$$

Here C_1 is a "divergent constant":

$$C_1 = \frac{1}{(2\pi)^4}\int \frac{dk}{k^4}.$$

An analogous discussion of the fourth order terms in e, when the Dirac equations for the electron and muon and Lorentz invariance are taken into account, leads to the result

$$F_2^{(2)} = F_{2a}^{(2)} + F_{2b}^{(2)} = C_2^{(2)} O, \qquad (2.4)$$
$$F_2^{(1)} = C_2^{(1)} O + C_1 F_1^{(0)}. \qquad (2.5)$$

Here $C_2^{(2)}$ and $C_2^{(1)}$ are "divergent constants."

Substituting Eqs. (2.3) — (2.5) into Eq. (2.1) we find

$$GF = G\,(O + e^2 C_1 d_l O + e^2 F_1^{(0)} + e^4 d_l^2 C_2^{(2)} O + e^4 d_l C_1 F_1^{(0)} + e^4 d_l C_2^{(1)} O + e^4 F_2^{(0)} + \ldots).$$

Accurate to within terms of higher order this means that

$$GF = G''\,(O + e^2 F_1^{(0)} + e^4 F_2^{(0)} + \ldots) = G'' F^{(0)}. \qquad (2.6)$$

Here G'' is the weak-interaction coupling constant, renormalized by longitudinal and scalar photons:

$$G'' = G\,(1 + e^2 C_1 d_l + e^4 C_2^{(2)} d_l^2 + e^4 C_2^{(1)} d_l + \ldots). \qquad (2.7)$$

The function F^0 requires further renormalization, as stated in Sec. 1, however it no longer depends on d_l. Consequently the dependence on d_l is contained entirely in the renormalization of the coupling constant G. An analogous situation occurs in mesodynamics, where the renormalization constant of the vertex operator including electrodynamic corrections depends, as is easily seen, on d_l.

*Throughout we make use of the renormalized photon distribution function

$$D^{mn}(k) = k^{-2} d(k^2)\,[g^{mn} - k^{-2}k^m k^n] + d_l k^{-4} k^m k^n.$$

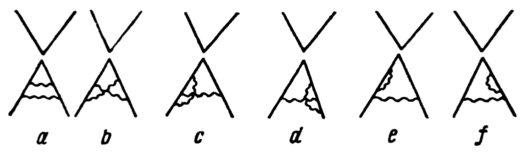

FIG. 3

Consequently, in order that the observable quantities be independent of d_l it is not necessary to impose any restrictions whatsoever on Z (Z_1 or Z_2). This is independent of what gauge invariant method is used to calculate other processes.

In contrast to this, in electrodynamics the condition of gauge invariance leads through the Ward identity to a relation between the normalization constants (or, which is the same, the cut-off momenta) for different processes. A relation is also established between the vertex diagram and the self-energy diagram. In this Section we have shown that the weak interaction matrix element is gauge invariant by itself, without imposing any conditions on the normalization constant. Therefore the normalization constant for the matrix element of each of the weak interaction processes is determined independently and represents an independent constant in the theory.

3. DISCUSSION

An elementary (but tedious) calculation of the matrix elements for μ and β decay, including bremsstrahlung, results in formulas for the probabilities of these processes of the type given in references 1 — 4 (including corrections due to recoil). These formulas, however, should contain additive terms corresponding to the finite arbitrariness noted above. At that it turns out, as was to be expected, that the parameters that characterize the spectrum (such as the Michel parameter and the asymmetry parameter) are independent of the normalization constant and are determined uniquely. At the same time the quantity G is determined from the lifetimes of the corresponding particles only in combination with the normalization constant C. Thus for μ decay it is only possible to determine the quantity $G[1-(\alpha/2\pi)C] = G(1-0.001162\,C)$. No convincing arguments whatever are known at this

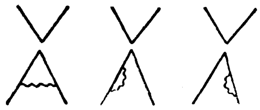

FIG. 4

time in favor of one or another choice of the normalization point.

In references 1, 3, and 4 a certain normalization for μ decay results essentially from the fact that the sum of the three divergent expressions, corresponding to the diagrams of Fig. 4, are assumed to be a definite number, in view of the generally accidental fact that the divergences cancel out. The authors of these papers forget, however, the arbitrariness that arises when divergent expressions are summed, and tacitly assume that the normalization constants (or, which is the same, the cut-off momenta) for the G-vertex and for the self energies of the electron and μ meson should necessarily coincide. The falsity of this point of view was discussed in Secs. 1 and 2. We can also note that the procedure used by these authors in substance prevents them from calculating corrections to the β decay even with a very rough account of the role of the strong interactions[8] through the constant form factor $\lambda = |G_A/G_V| = 1.2$.

With the help of dispersion-relations techniques Durand et al.[2] deduce unique expressions for the probabilities of μ and β decay, apparently from the condition that in the final expression the nonphysical photon mass λ_0 must vanish (i.e., in essence from the gauge invariance condition). It should, however, be noted that in applying the dispersion techniques to electromagnetic processes they automatically introduce into this field the determination of the renormalized coupling constant taken from the strong interactions. In reality the renormalized coupling constant can be determined in electrodynamics only when the emission of soft photons is taken into account. Therefore only the coupling constant determined in this way should be finite and independent of λ_0.* When this circumstance is taken into account the choice of a definite normalization point in the work of Durand et al.[2] is no longer unique.

In this way we are faced with a general situation in field theory of deducing experimental results from a theory in which the Lagrangians (0.1) — (0.4) are not fully known, and in which the constants (G, e, g, M, m, μ) entering these Lagrangians are not fully known. It is necessary, in addition, to specify a certain number C — the normalization constant. Only afterwards will the results be fully determined.

An analogous situation also arises in other versions of renormalizable theories. In electrodynamics, however, one usually introduces the additional boundary condition, which requires that the Coulomb law be satisfied at large distances. This means that the vertex operator is normalized at the point $k = 0$. In mesodynamics the determination of the g-charge as a subtraction at the point M corresponds to the normalization of the vertex operator as $\Lambda(M^2, M^2; m_\pi^2) = 1$. In the theory of μ and β decay there are no conditions of this type. For this reason we cannot have in this case a unique determination of the coupling constants G, G_A and G_V, related to each other. These constants can be determined only simultaneously with the normalization constant C, as discussed above.* Therefore, from our point of view, one should not talk about the existence of a discrepancy between theory and experiment, as is done by Feynman.[5]

If an intermediate boson, responsible for weak interactions, exists, one might expect that in that case it would be possible to relate the normalization constants for various weak interaction processes. However, until convincing arguments for the existence of such a boson are produced, the normalization constants of various processes (and, consequently, coupling constants) will remain unrelated to each other. It makes no sense therefore (even under the conserved vector current hypothesis) to talk at this time of the equality of coupling constants for various weak interaction processes to an accuracy better than the electromagnetic corrections.

In conclusion the authors express their gratitude to D. V. Shirkov and Ya. A. Smorodinskii for fruitful discussions, and also to A. I. Larkin, V. G. Vaks, and B. N. Valuev for valuable remarks.

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*In the language of Durand et al.[2] the finite quantity should be the sum $G^2 + G^2C \ln(\lambda_0^2/m^2)$, and not G^2 itself.

*One can require the equality of renormalized coupling constants for various weak interaction processes. In that case the experimental lifetimes and cross sections will allow one to relate the normalization constants.

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