THE PROPERTIES OF SOME STRONGLY DEFORMED NUCLEI

LIU YÜAN, N. I. PYATOV, V. G. SOLOV'EV, I. N. SILIN, and V. I. FURMAN

Joint Institute for Nuclear Research

Submitted to JETP editor December 26, 1960

J. Exptl. Theoret. Phys. (U.S.S.R.) **40**, 1503-1510 (May, 1961)

An improved scheme of single-particle levels of a self-consistent field is employed to investigate some properties of strongly deformed nuclei in the range $150 < A < 190$, on the basis of the superfluid model of the nucleus. The mean values of the pair coupling constants are evaluated by comparing the calculated pair energies with the experimental data. The density of the low-energy single-particle energy levels of odd nuclei is calculated and found to agree with the experimental data and to be approximately double the level density predicted by the Nilsson scheme. Some regularities in the behavior of weakly excited states in even-even nuclei are noted. On the average, the calculation error due to conservation of the number of particles does not exceed 6 percent.

THE superfluid model of the nucleus, proposed by one of the authors[1] and based on the unified or the shell model, takes account of the residual short-range interactions of the nucleons in the nucleus by the Bogolyubov variational principle.[2] To describe such interactions, we use a Hamiltonian in the form

$$H = \sum_{s\sigma} [E_s - \lambda]\, a^+_{s\sigma} a_{s\sigma} - G \sum_{ss'} a^+_{s+} a^+_{s-} a_{s'-} a_{s'+}, \qquad (1)$$

where E_s are the single-particle levels of the self-consistent field, λ is a parameter that plays the role of the chemical potential, $(s\sigma)$ are the quantum characteristics of the level $(\sigma = \pm 1)$, and G is the short-range pair-coupling constant. We note that the number of particles is conserved in the mean.

The properties of the transuranic elements were investigated earlier (see reference 3, where the principal equations of the problem are given). In the present paper we investigate, on the basis of the superfluid model, the properties of strongly deformed nuclei in the region $150 < A < 190$.

From a comparison of the calculated pair energies with the experimental data we shall determine and compare with experiment the pair-coupling constants G and calculate the spectra of single-particle levels for some odd and even-even nuclei.

MODIFICATION OF THE NILSSON SCHEME

We use the levels of the Nilsson scheme[4] as the single-particle levels E_s in our calculations. An analysis of the scheme, carried out on the basis of known spectroscopic data,[5-7] shows that the proton shell is in general satisfactorily described by the Griffin and Rich scheme.[8] We shall make significant changes only in the neutron level schemes for $82 < N < 126$:

a) All the eigenvalues of neutrons with $N = 6$ will be increased by $0.25\hbar\omega_0^0$ (corresponding to a parameter $\mu = 0.33$) (see Nilsson's paper[4] for notation), with the exception of the states $i\,{}^{13}\!/_2$, which will be increased by $0.06\hbar\omega_0^0$ ($\mu = 0.42$).

b) We drop the $h\,{}^{11}\!/_2$ subshell $0.3\hbar\omega_0^0$ ($\mu = 0.65$);

c) The $f\,{}^5\!/_2$ eigenvalues are raised $0.06\hbar\omega_0^0$ ($\mu = 0.42$).

d) The $p\,{}^3\!/_2$ eigenvalues are dropped $0.01\hbar\omega_0^0$ to agree with experimental data on nuclei with 109 neutrons.

e) The ${}^1\!/_2{}^-$[521] level is raised $0.04\hbar\omega_0^0$, as in reference 8.

The corrected Nilsson scheme, shown in Fig. 1, gives correct ground states for all nuclei with $93 < N < 109$ and gives for some nuclei the required sequence of first excited levels. It agrees essentially with the corrected scheme given by Nilsson and Prior,[9] with the exception of the ${}^{11}\!/_2{}^-$[505] level.

The ground states given in reference 3 were calculated on a "Strela" computer. The problem was reduced to the least-squares method and solved by linearization. The specified energy levels of the average field and of the coupling constant G were used to calculate the correlation function C, the chemical potential λ, the mean square fluctuation of the number of particles $(\overline{\Delta n^2})^{1/2}$, and the system energy ϵ, accurate to 4 – 6 significant decimal places.

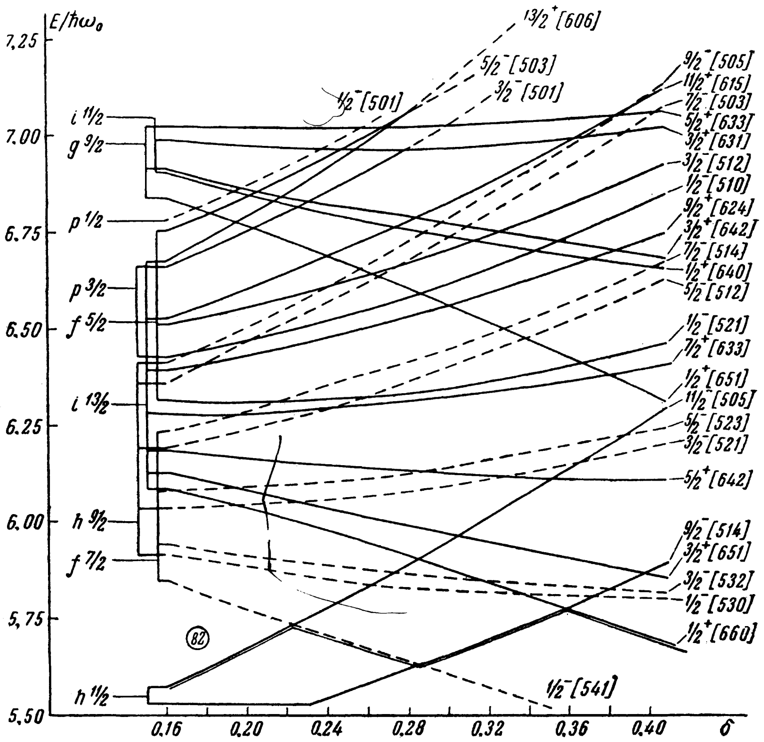

FIG. 1. Nilsson scheme for 82 < N < 126.

PAIR ENERGIES AND CORRELATION FUNCTIONS

To determine G we calculated the neutron and proton pair energies P_N and P_Z by the formula

$$P_N = 2\varepsilon(Z, N-1) - \varepsilon(Z, N) - \varepsilon(Z, N-2). \quad (2)$$

The calculated pair energies of the neutron system for $G_N = 0.020$ and $0.024\,\hbar\omega_0^0$ and of the proton system for $G_Z = 0.024$ and $0.028\,\hbar\omega_0^0$, along with the corresponding experimental data,[10,11] are shown in Figs. 2 and 3. From a comparison of the calculated pair energies with the experimental

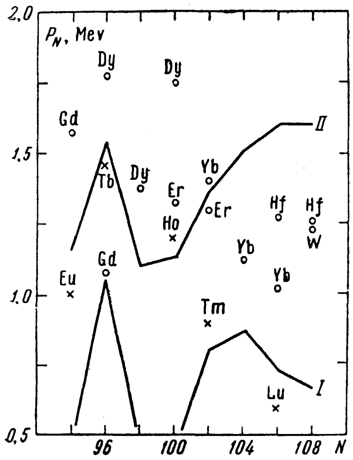

FIG. 2. Calculated (continuous curves) and experimental[10,11] values of the neutron pair energies (○ – even-even nuclei, × – nuclei with odd A); curve I – for $G = 0.020\hbar\omega_0^0$, curve II – for $G = 0.024\hbar\omega_0^0$.

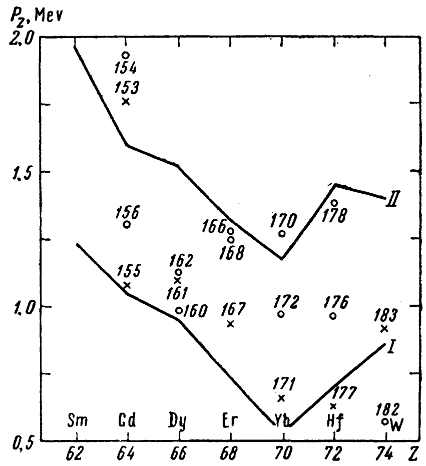

FIG. 3. Calculated (continuous curves) and experimental[10,11] values of proton pair energies (○ – even-even nuclei, × – nuclei with odd A); curve I – for $G = 0.024\hbar\omega_0^0$, curve II – for $G = 0.028\hbar\omega_0^0$.

data we have determined the average values of the pair coupling constants

$$G_N = 0.024\,\hbar\omega_0^0, \qquad G_Z = 0.026\,\hbar\omega_0^0. \quad (3)$$

We note that the pair energy decreases with the increasing deformation, owing to the diminished role of the residual interactions.

By fixing the scheme of single-particle levels on the basis of an analysis of the experimental spectra of odd nuclei, with allowance for superfluidity, and by determining the constants G_N and

G_Z from the pair energies, we eliminate ipso facto all arbitrariness from the subsequent calculations.

Let us investigate the behavior of the correlation function $C = G\Sigma_s u_s v_s$ and of the chemical potential λ corresponding to the ground states of even-even and odd nuclei. The variation of the chemical potential λ with the degree of filling of the shell, for a system with an even number of neutrons, is shown in Fig. 4. The value of λ fluctuates about the Fermi-surface energy, the deviations reaching an order of 1 Mev. The fluctuations in the difference $|\lambda - E_F|$ become even greater for the excited states. The deviation of λ from E_F is particularly great in the excited states of even systems.

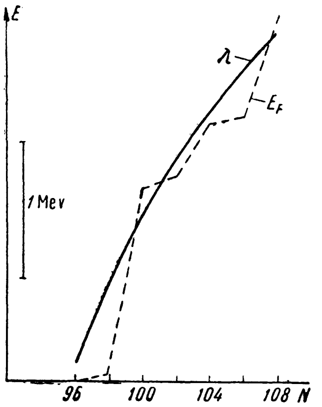

FIG. 4. Variation of Fermi-surface energy E_F and chemical potential λ of the ground states of an even neutron system; $G = 0.024\hbar\omega_0^0$.

The behavior of the functions C for the ground states of even and odd systems are demonstrated in Figs. 5 and 6. We note that the values of C for the ground states of odd systems depend strongly on the course of the single-particle levels that are closest to the Fermi-surface energy. It is seen from the scheme for single-particle neutron levels (Fig. 1) that in the region of deformations $\delta = 0.26 - 0.36$ the energy difference between the levels $N = 97$ and $N = 99$ reaches 1.4 Mev, which leads to a sharp reduction in C for the corresponding odd nuclei. The values of C for the ground states of even systems are less sensitive to the behavior of single-particle levels. The values of C for odd systems $(2n-1)$ are on the average 20 — 30 percent less than the values of C for even systems $(2n)$, in agreement with the estimates of Nilsson and Prior.[9]

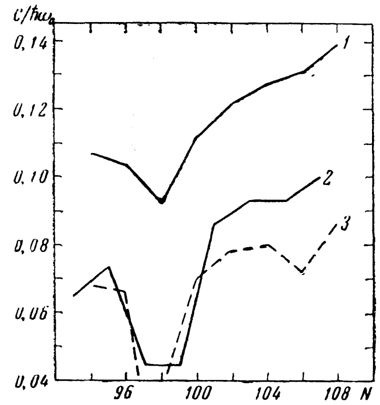

FIG. 5. Behavior of the correlation function C of the neutron system as a function of the number of neutrons N in a nucleus (ground state): 1 – even-even system with $G = 0.024\hbar\omega_0^0$, 2 – odd system with $G = 0.024\hbar\omega_0^0$, 3 – even-even system with $G = 0.020\hbar\omega_0^0$.

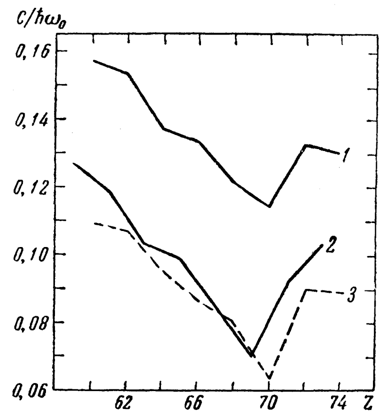

FIG. 6. Behavior of correlation function C of proton system as a function of the number of protons Z in the nucleus (ground state): 1 – even-even system with $G = 0.028\hbar\omega_0^0$, 2 – odd system with $G = 0.028\hbar\omega_0^0$, 3 – even-even system with $G = 0.024\hbar\omega_0^0$.

Thus, the appearance of a single quasi-particle leads to a considerable weakening of superfluidity. It should be noted that the values of C decrease as a rule with increasing deformation, thus demonstrating the diminished role of pair correlations with increasing δ. The function C for single-quasi particle states of odd systems has a minimum for the ground state and increases for the excited states with increasing excitation energy, approaching thereupon the value for the ground state of the corresponding even system, as can be readily seen from Table I. (The symbol $K+1$ denotes the level following the level K, etc.) In the case of an even system, the function C for two quasi-particle excited states diminishes by 30 percent and more, vanishing sometimes. With increasing excitation energy, the magnitude of this correlation function increases.

SINGLE-PARTICLE LEVELS OF ODD NUCLEI

On the basis of the superfluid model of the nucleus let us calculate now, in the region of interest to us, the spectrum of single-particle levels of nuclei with odd N as well as odd Z. In Fig. 7 we show by way of an example the calculated and experimental values of the levels of Dy^{161} and

Table I. Characteristics of ground and excited states of odd nuclei

State of system		$C/\hbar\omega_0^0$	$(\overline{\Delta n^2})^{1/2}$	$\frac{E_F-\lambda}{\hbar\omega_0^0}$
$N=95,\ \delta=0.30,\ G=0.024\,\hbar\omega_0^0$				
$K-3$;	$11/2^-$ [505]	0.091	2.01	−0.018
$K-2$;	$3/2^+$ [651]	0.090	2.00	−0.017
$K-1$;	$3/2^-$ [521]	0.077	1.77	−0.004
K;	$5/2^+$ [642]	0.074	1.71	+0.010
$K+1$;	$5/2^-$ [523]	0.076	1.75	+0.016
$K+2$;	$7/2^+$ [633]	0.094	2.08	+0.033
$K+3$;	$1/2^-$ [521]	0.095	2.09	+0.034
$Z=71,\ \delta=0.28,\ G=0.028\,\hbar\omega_0^0$				
$K+3$;	$1/2^-$ [541]	0.101	—	+0.123
$K+2$;	$9/2^-$ [514]	0.089	—	+0.118
$K+1$;	$5/2^+$ [402]	0.080	—	+0.113
K;	$7/2^+$ [404]	0.068	—	+0.106
$K-1$;	$1/2^+$ [411]	0.104	—	+0.067
$K-2$;	$7/2^-$ [523]	0.106	—	+0.065
$K-3$;	$3/2^+$ [411]	0.110	—	+0.065

Lu^{175}. We note that the excitation energies which we have calculated agree better with the experimental data than those given by the Nilsson scheme. However, in view of the strong dependence of the determined levels of the odd nuclei on the behavior of the levels of the average field, it is difficult to expect a detailed agreement with the experimental data. We shall therefore investigate the densities of the single-particle levels.

The average density of the calculated neutron levels was found to be 3.3 times the 1-Mev level for $99 \leq N \leq 109$, compared with 3.1 for the similarly averaged experimental levels.[12] The average density of the calculated proton levels for $63 \leq Z \leq 73$ is found to be 3.4 times the 1-Mev level, compared with 3.6 for the experimental levels. The average densities of the calculated (single-particle) proton and neutron levels are 1.7 times the level densities in the corresponding Nilsson scheme. The calculations are in agreement with Bakke's investigations[13] of the density of single-particle levels.

Thus, as in the transuranic region,[3] the density of low-energy levels agrees with experiment and is approximately twice the level density given by the Nilsson scheme. We note that the increase in the level density is connected with the superfluid properties of the ground and excited states. The necessary level density cannot be obtained by any modification of the single-particle levels in the independent-particle model.

SPECTRA OF EVEN-EVEN NUCLEI AND ESTIMATE OF THE CALCULATION ACCURACY

The most interesting and promising is the application of methods based on the superfluid nuclear model to the analysis of spectra of even-even nuclei. It was shown in reference 3 that in the excited state of an even system $|K, K+1\rangle$, where one quasi-particle is at the K level (K denotes the totality of quantum numbers corresponding to the Fermi-surface energy of the given nu-

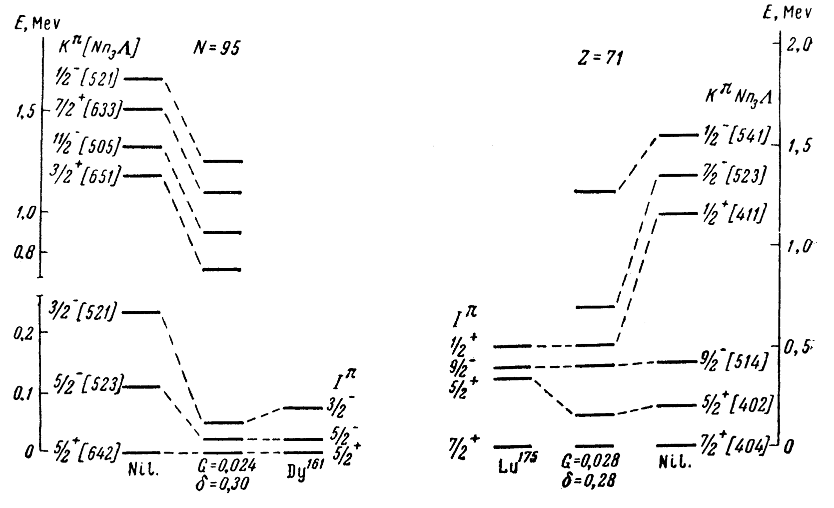

FIG. 7. Spectra of single-particle excitations of nuclei with odd number of particles. The letters "Nil" denote the spectrum after Nilsson, the spectrum in the center has been obtained for the superfluid model of the nucleus, and the spectra of Dy and Lu are experimental without rotational lines). G is given in units of $\hbar\omega_0^0$ ($\hbar\omega_0^0 = 41\ A^{-1/3}$ Mev).

Table II. Spectrum of $_{70}Yb^{172}_{102}$

State	$C/\hbar\omega_0^0$	$(\overline{\Delta n^2})^{1/2}$	$\frac{E_F-\lambda}{\hbar\omega_0^0}$	K^π	E, Mev
		Proton levels $\delta=0.29$, $G_Z=0.028\ \hbar\omega_0^0$			
$0\rangle$	0.109	2.00	−0.084	0^+	0
$K, K+1\rangle$	0	0	−0.130	3^+, (4^+)	1.54
$K, K+2\rangle$	0	0	−0.114	(2^+), 3^+	1.69
$K-1, K+1\rangle$	0	0	−0.124	(0^-), 7^-	1.76
$K-1, K+2\rangle$	0 ~	0	−0.102	1^-, (6^-)	1.91
$K, K+3\rangle$	0 ~	0	−0.101	(4^-), 5^-	1.91
$K, K\rangle$	0.084	1.82	−0.165	0^+	2.29
$K+1, K+1\rangle$	0.084	1.77	0	0^+	2.40
$K-2, K+1\rangle$	0.019	0.40	−0.099	(2^+), 5^+	2.50
$K-1, K\rangle$	0.086	1.84	−0.165	(3^-), 4^-	2.50
$K+1, K+2\rangle$	0.086	1.79	0	(1^+), 6^+	2.55
$K+2, K+2\rangle$	0.087	1.81	0	0^+	2.68
$K-1, K-1\rangle$	0.088	1.87	−0.166	0^+	2.70
		Neutron levels $\delta=0.29$, $G_N=0.024\ \hbar\omega_0^0$			
$0\rangle$	0.108	2.33	0	0^+	0
$K, K+1\rangle$	0.034	0.87	−0.023	(2^+), 3^+	0.98
$K+1, K+1\rangle$	0.053	1.45	+0.013	0^+	1.08
$K-1, K+1\rangle$	0.046	1.20	−0.032	1^-, (6^-)	1.11
$K, K\rangle$	0.057	1.49	−0.050	0^+	1.17
$K, K+2\rangle$	0.054	1.41	−0.002	3^+, (4^+)	1.25
$K-1, K\rangle$	0.061	1.59	−0.052	(3^-), 4^-	1.30
$K-1, K+2\rangle$	0.058	1.54	−0.011	(0^-), 7^-	1.33
$K+1, K+2\rangle$	0.061	1.62	+0.019	(1^+), 6^+	1.38
$K-1, K-1\rangle$	0.065	1.67	−0.054	0^+	1.42
$K+2, K+2\rangle$	0.067	1.74	+0.025	0^+	1.66
$K, K+3\rangle$	0.064	1.61	+0.005	(4^-), 5^-	2.03
$K+1, K+3\rangle$	0.068	1.73	+0.023	2^-, (7^-)	2.16

cleus with G = 0) and the other is at the level K+1, the superfluidity of the system is greatly reduced, and in many cases simply vanishes. This is connected with the fact that the levels K and K+1 are blocked for correlated pairs, and therefore a large gap appears in the states accessible to pairs. Inasmuch as the number of states below this gap is equal to the number of particles, and the pairs cannot favor, from the energy point of view, the levels K+2 and higher, owing to the large loss in kinetic energy, the superfluidity in the state $|K, K+1\rangle$ is very small. In this connection, the energy of the system in the state $|K, K+1\rangle$ is reduced and as a rule the energy difference in this state is essentially less than the gap width $2C_0$, where C_0 is the value of the correlation function in the ground state.

Let us illustrate these arguments with $_{70}Yb^{172}$ as an example, for which the calculated characteristics of the excited states are listed in Table II. There is no superfluidity in the excited states $|K, K+1\rangle$, $|K, K+2\rangle$ and others, i.e., C = 0, whereas in the ground state $C_0 = 0.12\hbar\omega_0^0 = 0.85$ Mev; in higher energy states the superfluidity increases, approaching the value of the ground state. In the case of a neutron system, the superfluid properties are much weaker in the state $|K, K+1\rangle$, inasmuch as $C = 0.034\hbar\omega_0^0$, whereas in the ground state $C_0 = 0.126\hbar\omega_0^0 \approx 0.93$ Mev.

Let us compare now the calculated spectrum of Yb^{172} with the experimental data[14] obtained in an investigation of the decay of $_{71}Lu^{172}$ with the configuration {neutrons $\frac{1}{2}^-[521]\downarrow$; protons $\frac{7}{2}^+[404]\uparrow$}, i.e., $I^\pi = 4^-$, k = 4. There is no doubt that the excited state 3^+3 with energy 1.17 Mev in the spectrum of Yb^{172} is a single-particle one, and can be either a proton state of the form $\{\frac{7}{2}^+[404]\downarrow\} - \{\frac{1}{2}^+[411]\downarrow\}$, or a neutron state $\{\frac{1}{2}^-[521]\downarrow + \frac{5}{2}^-[512]\uparrow\}$. The level 3^+3 is an example of the $|K, K+1\rangle$ state, and its energy is less than the energy of the gap, since the neutron gap is $2C_0 = 1.86$ Mev and the proton gap is $2C_0 = 1.7$ Mev. In view of the reduced superfluidity of the system in the $|K, K+1\rangle$ state, the moment of inertia J of the system in this state should increase, as confirmed by experiment, since $\hbar^2/2J = 13$ for the ground state and $\hbar^2/2J = 11$ for the state $|K, K+1\rangle$.

We note that the moment of inertia of a system in an excited state depends on the superfluid properties of both the given state and of the other states. Therefore the sharp reduction in the correlation function C for a given state does not lead to a considerable change in the moment of inertia of the system in the same excited state, compared with the moment of inertia of the system in the ground or other excited states.

We note that those states 0^+ in which both quasi-particles are at the same level have been calculated with lower accuracy than the other states. The conservation of the number of particles in the mean leads to several difficulties which are concentrated at these states. One of the two quasi-particle states is superfluous, and the ground state and the 0^+ state are not orthogonal to each other. Actually, in the case of Er^{166}, with $\delta = 0.31$, $G_N = 0.024\hbar\omega_0^0$ and $G_Z = 0.028\hbar\omega_0^0$, estimates of the non-orthogonality lead to the following results:

a) proton states

$$\begin{aligned} \langle K-1,\ K-1 \mid 0\rangle &= 0.30, \\ \langle K,\ K \mid 0\rangle &= -0.39, \\ \langle K+1,\ K+1 \mid 0\rangle &= 0.38, \\ \langle K+1,\ K+1 \mid K-1,\ K-1\rangle &= -0.10, \\ \langle K,\ K \mid K-1,\ K-1\rangle &= -0.02, \\ \langle K+1,\ K+1 \mid K,\ K\rangle &= -0.13; \end{aligned}$$

b) neutron states

$$\begin{aligned} \langle K-1,\ K-1 \mid 0\rangle &= 0.08, \\ \langle K,\ K \mid 0\rangle &= 0.12, \\ \langle K+1,\ K+1 \mid 0\rangle &= -0.10, \\ \langle K+1,\ K+1 \mid K-1,\ K-1\rangle &= -0.15, \\ \langle K,\ K \mid K-1,\ K-1\rangle &= 0. \\ \langle K+1,\ K-1 \mid K,\ K\rangle &= -0.23; \end{aligned}$$

here $|\,0\rangle$ is the ground state.

Using the formulas of reference 15, let us estimate the error due to conservation of the number of particles in the mean. The calculated rms values $(\overline{\Delta n^2})^{1/2}$ of the fluctuation of the number of particles are given in Tables I and II. The relative magnitude of the fluctuation $(\overline{\Delta n^2})^{1/2}/2\Omega$ (Ω is the number of summed levels) changes appreciably on going from the ground state to the excited ones, but is nowhere more than 6 percent. Thus, the accuracy of our calculations is restricted not by the conservation of number of particles in the mean, but principally by the accuracy to which the single-particle levels of the self-consistent field are known.

In conclusion we consider it our pleasant duty to thank N. N. Bogolyubov, K. L. Gromov, B. S. Dzhelepov, and L. K. Peker for very fruitful discussions of the work.

[1] V. G. Solov'ev, Doklady Akad. Nauk SSSR **133**, 325 (1960), Soviet Phys.-Doklady **5**, 778 (1961).

[2] N. N. Bogolyubov, Doklady Akad. Nauk SSSR **119**, 244 (1958), Soviet Phys.-Doklady **3**, 292 (1958); JETP **34**, 73 (1958), Soviet Phys. JETP **7**, 51 (1958).

[3] V. G. Solov'ev, Preprint, Joint Institute for Nuclear Research, D-545, 1960; JETP **40**, 654 (1961), Soviet Phys. JETP **13**, 456 (1961).

[4] S. G. Nilsson, Kgl. Dan. Vid. Selsk. Mat.-Fys. Medd. **29**, No. 16 (1955).

[5] B. R. Mottelson and S. G. Nilsson, Mat.-Fys. Skr. Dan. Vid. Selsk. **1**, No. 8 (1959).

[6] B. S. Dzhelepov and L. K. Peker, Схемы распада радиоактивных ядер (Decay Schemes of Radioactive Nuclei), Academy of Sciences U.S.S.R. (1958).

[7] Материалы совещания по нейтронно-дефицитным изотопам (Materials of Conference on Neutron-Deficient Isotopes), Dubna, (1959).

[8] J. J. Griffin and M. Rich, Phys. Rev. Lett. **3**, 342 (1959).

[9] S. G. Nilsson and O. Prior, Preprint.

[10] W. H. Johnson, Jr. and V. B. Bhanot, Phys. Rev. **107**, 1669 (1957).

[11] V. A. Kravtsov, JETP **36**, 1224 (1959), Soviet Phys. JETP **9**, 871 (1959).

[12] Harmatz, Handley, and Mihelich, Phys. Rev. **119**, 1345 (1960).

[13] F. N. Bakke, Nucl. Phys. **9**, 670 (1958/59).

[14] R. G. Wilson and M. L. Pool, Phys. Rev. **118**, 1067 (1960).

[15] S. T. Belyaev, Mat.-Fys. Medd. Dan. Vid. Selsk. **31**, No. 11 (1959). V. G. Solov'ev, JETP **35**, 823 (1958) and **36**, 1869 (1959), Soviet Phys. JETP **8**, 572 (1959) and **9**, 1331 (1959).

Translated by J. G. Adashko
255