SOVIET PHYSICS JETP VOLUME 13, NUMBER 4 OCTOBER, 1961

SCATTERING OF LOW-ENERGY PHOTONS ON A SYSTEM WITH SPIN ½

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Submitted to JETP editor November 1, 1960

J. Exptl. Theoret. Phys. (U.S.S.R.) **40**, 1148-1154 (April, 1961)

An expression for the scattering cross section of low-energy photons on a system with spin ½ is obtained within the framework of the local theory, with an accuracy up to terms quadratic in frequency. In addition to the constants e, M, and λ (which represent the charge, mass, and anomalous moment of the system respectively), three other parameters, α, β, and $\langle r_e^2 \rangle$ (representing, respectively, the electric and magnetic polarizabilities and the mean-square radius of the charge distribution of the system) also appear in the cross-section formula.

1. INTRODUCTION

Low[1] and Gell-Mann and Goldberger[2] have investigated the scattering amplitude of light on a system with spin ½ within the framework of the local theory. It has been shown that, from the requirement of the relativistic and gauge invariance of the theory, one can draw definite conclusions concerning the scattering amplitude, with an accuracy up to terms linear in frequency. For a comparison of the theory with experiment, it would be interesting to obtain not the amplitude but the differential cross section. It is the purpose of the present paper to obtain a formula for the cross section with an accuracy up to terms quadratic in the frequency. To this end, it is necessary to take into account in the expression for the amplitude the quadratic terms which interfere with the Thompson scattering amplitude and thus contribute to the quadratic terms in the expression for the cross section, in addition to the linear terms. No complete derivation of a general formula for the light-scattering cross section on a system with spin ½ has been given in the literature, although the importance of the quadratic terms was indicated, and the calculation of some of them carried out, by Klein[3] and Baldin.[4] (A preliminary report was presented by Baldin at the Elementary Particle Conference in Padua in 1957.)

It is also interesting to obtain the general formula for the cross section in order to find such characteristics of the nucleons as their electrical (α) and magnetic (β) polarizability. In the conclusion, we shall dwell briefly on the possibility of obtaining numerical estimates of α and β from the presently-available experimental data on the Compton effect on protons.[5,6]

2. CALCULATION OF THE SCATTERING AMPLITUDE

Within the assumptions made by Low,[1] we can write the following expression, quadratic in e, for the matrix element for the scattering of a photon with four-momentum k and polarization **e** on a nucleon* with momentum p. The scattered photon then has a momentum k′ and a polarization **e**′:

$$\langle \mathbf{e}'k'p's' \,|\, S \,|\, \mathbf{e}kps\rangle = -\frac{2i}{\sqrt{4kk'}} \Big\langle p's' \,\Big|\, e^2\, \mathbf{e}\mathbf{e}' \int \Phi^{\cdot}(x)\Phi(x) \times \exp[i(k-k')x]\,dx \,\Big|\, ps \Big\rangle - \frac{1}{\sqrt{4kk'}} \Big\langle p's' \,\Big|\, \int P[j(x)\mathbf{e}', j(y)\mathbf{e}] \times \exp[i(ky-k'x)]\,dxdy \,\Big|\, ps \Big\rangle \quad (1)$$

where p′ is the momentum of the final nucleon, s′ and s are the projections of the spin of the initial and final nucleons, (kx) is the scalar product of the four-vectors, and $\Phi(x)$ and $j_\nu(x)$ are time-dependent operators of the meson field and of the current in the electromagnetic representation of the interaction.

We shall change Eq. (1) over to time-independent operators, using the transformation

$$A(x) = e^{iH_0 t} A(\mathbf{x}) e^{-iH_0 t},$$

where H_0 is the total Hamiltonian of the meson and nucleon fields.

Writing the P-product explicitly and letting the operator H_0 operate on the indices of the matrix element, we obtain, after an integration with respect to t,

*Henceforth, we use the word nucleon as an abreviation for a "system with spin ½."

$$\langle|S|\rangle = -\frac{2\pi}{\sqrt{4kk'}}\delta(E(p')+k'-E(p)-k)\Big[2i\Big\langle\Big|e^2\mathbf{e}\mathbf{e}'\int\Phi^*(\mathbf{x})\Phi(\mathbf{x})\exp[i(\mathbf{k}-\mathbf{k}')\mathbf{x}]\,dx\Big|\Big\rangle$$
$$+\Big\langle\Big|\int\mathbf{j}(\mathbf{x})\mathbf{e}'\exp[-i\mathbf{k}'\mathbf{x}]\,dx\,\frac{1}{i(H_0-E(p)-k)}\int\mathbf{j}(\mathbf{y})\mathbf{e}\exp[i\mathbf{k}\mathbf{y}]\,dy\Big|\Big\rangle$$
$$+\Big\langle\int\mathbf{j}(\mathbf{y})\mathbf{e}\exp[i\mathbf{k}\mathbf{y}]\,dy\,\frac{1}{i(H_0-E(p)+k')}\int\mathbf{j}(\mathbf{x})\mathbf{e}'\exp[-i\mathbf{k}'\mathbf{x}]\,dx\Big|\Big\rangle\Big]. \tag{2}$$

We shall expand the last two terms in Eq. (2) in terms of the total system of physical states of the meson-nucleon field. We then have

$$\langle|S|\rangle = -\frac{2\pi}{\sqrt{4kk'}}\delta(E(p')+k'-E(p)-k)\Big[2i\Big\langle\Big|e^2\mathbf{e}\mathbf{e}'\int\Phi^*(\mathbf{x})\Phi(\mathbf{x})\exp[i(\mathbf{k}-\mathbf{k}')\mathbf{x}]\,dx\Big|\Big\rangle$$
$$+\sum_N\frac{\Big\langle\Big|\int\mathbf{j}(\mathbf{x})\mathbf{e}'\exp[-i\mathbf{k}'\mathbf{x}]\,dx\Big|N\Big\rangle\Big\langle N\Big|\int\mathbf{j}(\mathbf{y})\mathbf{e}\exp[i\mathbf{k}\mathbf{y}]\,dy\Big|\Big\rangle}{i[E_N-E(p)-k]}+\sum_N\frac{\Big\langle\Big|\int\mathbf{j}(\mathbf{y})\mathbf{e}\exp[i\mathbf{k}\mathbf{y}]\,dy\Big|N\Big\rangle\Big\langle N\Big|\int\mathbf{j}(\mathbf{x})\mathbf{e}'\exp[-i\mathbf{k}'\mathbf{x}]\,dx\Big|\Big\rangle}{i[E_N-E(p)+k']}\Big]. \tag{3}$$

In order to obtain the momentum-conservation law explicitly, we shall use the translational invariants of the matrix element. We show the detailed calculations only for the first term in Eq. (3):

$$\Big\langle p's'\Big|e^2\mathbf{e}\mathbf{e}'\int\Phi^*(\mathbf{x})\Phi(\mathbf{x})\exp[i(\mathbf{k}-\mathbf{k}')\mathbf{x}]\,dx\Big|ps\Big\rangle$$
$$=\exp[-i(\mathbf{p}'-\mathbf{p})\mathbf{a}]\Big\langle p's'\Big|e^2\mathbf{e}\mathbf{e}'\int\Phi^*(\mathbf{x}-\mathbf{a})$$
$$\times\Phi(\mathbf{x}-\mathbf{a})\exp[i(\mathbf{k}-\mathbf{k}')\mathbf{x}]\,dx\Big|ps\Big\rangle$$
$$=\exp[-i(\mathbf{p}'+\mathbf{k}'-\mathbf{p}-\mathbf{k})\mathbf{a}]$$
$$\times\Big\langle p's'\Big|e^2\mathbf{e}\mathbf{e}'\int\Phi^*(\mathbf{x}')\Phi(\mathbf{x}')$$
$$\times\exp[i(\mathbf{k}-\mathbf{k}')\mathbf{x}']\,dx'\Big|ps\Big\rangle. \tag{4}$$

Integrating the right- and left-hand sides of Eq. (4) with respect to **a**, we finally obtain

$$\Big\langle\Big|e^2\mathbf{e}\mathbf{e}'\int\Phi^*(\mathbf{x})\Phi(\mathbf{x})\exp[i(\mathbf{k}-\mathbf{k}')\mathbf{x}]\,dx\Big|\Big\rangle$$
$$=\frac{(2\pi)^3\delta(\mathbf{p}'+\mathbf{k}'-\mathbf{p}-\mathbf{k})}{V}\Big\langle\Big|e^2\mathbf{e}\mathbf{e}'$$
$$\times\int\Phi^*(\mathbf{x})\Phi(\mathbf{x})\exp[i(\mathbf{k}-\mathbf{k}')\mathbf{x}]\,dx\Big|\Big\rangle.$$

It can easily be seen that the coefficient ahead of the matrix element, appearing as a result of the transformation carried out, is equal to 1.* Henceforth, we shall write the matrix elements of operators containing integrals as

$$\Big\langle\Big|e^2\mathbf{e}\mathbf{e}'\int\Phi^*(\mathbf{x})\Phi(\mathbf{x})\exp[i(\mathbf{k}-\mathbf{k}')\mathbf{x}]\,dx\Big|\Big\rangle$$
$$=(2\pi)^3\delta(\mathbf{p}'+\mathbf{k}'-\mathbf{p}-\mathbf{k})$$
$$\times\Big[\frac{1}{V}\Big\langle\Big|e^2\mathbf{e}\mathbf{e}'\int\Phi^*(\mathbf{x})\Phi(\mathbf{x})\exp[i(\mathbf{k}-\mathbf{k}')\mathbf{x}]\,dx\Big|\Big\rangle\Big].$$

We assume the quantity in the brackets to be finite.

From the translational invariance of the two terms of Eq. (3), we can easily obtain the expressions

$$\langle|S|\rangle = -\frac{(2\pi)^4}{\sqrt{4kk'}}\delta(p'+k'-p-k)\Big\{\frac{2i}{V}\Big\langle\Big|e^2\mathbf{e}\mathbf{e}'\int\Phi^*(\mathbf{x})\Phi(\mathbf{x})\exp[i(\mathbf{k}-\mathbf{k}')\mathbf{x}]\,dx\Big|\Big\rangle$$
$$+\sum_{\nu\sigma}\Big[\frac{\langle|\mathbf{j}(0)\mathbf{e}'|\nu\sigma\mathbf{k}\rangle\langle\nu\sigma\mathbf{k}|\mathbf{j}(0)\mathbf{e}|\rangle}{i[E_\nu(k)-M-k]}+\frac{\langle|\mathbf{j}(0)\mathbf{e}|\nu\sigma-\mathbf{k}'\rangle\langle\nu\sigma-\mathbf{k}'|\mathbf{j}(0)\mathbf{e}'|\rangle}{i[E_\nu(k')-M+k']}\Big]$$
$$+\sum_{N'}\Big[\frac{\Big\langle\Big|\int\mathbf{j}(\mathbf{x})\mathbf{e}'\exp[-i\mathbf{k}'\mathbf{x}]\,dx\Big|N'\mathbf{k}\Big\rangle\Big\langle N'\mathbf{k}\Big|\int\mathbf{j}(\mathbf{y})\mathbf{e}\exp[i\mathbf{k}\mathbf{y}]\,dy\Big|\Big\rangle}{i[E_{N'}-M-k]V^2}$$
$$+\frac{\Big\langle\Big|\int\mathbf{j}(\mathbf{y})\mathbf{e}\exp[i\mathbf{k}\mathbf{y}]\,dy\Big|N-\mathbf{k}'\Big\rangle\Big\langle N-\mathbf{k}'\Big|\int\mathbf{j}(\mathbf{x})\mathbf{e}\exp[-i\mathbf{k}'\mathbf{x}]\,dx\Big|\Big\rangle}{i[E_{N'}-M+k']V^2}\Big]\Big\}. \tag{5}$$

We have separated the single-nucleon term explicitly, and set the momentum of the initial nucleon $\mathbf{p}=0$ (in the laboratory system). The indices ν and σ indicate the summation over positive and negative energies and over the projections of the nucleon spin in the intermediate state. It would be more consistent to work with positive energies only, adding to the single-nucleon expression those terms of the sum over all excited states which contain a pair in the intermediate state in addition to the nucleon. However, it is simpler for us to carry out the summation over negative energies.†

The amplitude component due to the contribution of single-nucleon terms can be calculated directly if we use the total formula for the matrix element of the current operator connecting single-nucleon states:‡

*This is true for the limiting case $V\to\infty$.

†See Heitler[7] pp. 213-214 concerning the equivalence of the results obtained.

‡See reference 1, Eq. (3.1).

$$\langle p_2|j_\mu(0)|p_1\rangle = i\bar{u}(p_2)\,[e\gamma_\mu f(\Delta p^2) - \sigma_{\mu\nu}\Delta p_\nu \lambda' g(\Delta p^2)]\,u(p_1),$$
$$\Delta p = p_2 - p_1, \qquad \lambda' = \lambda e/2M, \qquad \sigma_{\mu\nu} = i(\gamma_\nu\gamma_\mu - \gamma_\mu\gamma_\nu)/2. \tag{6}$$

The calculation of the single-nucleon part of Eq. (5), using Eq. (6), leads to the expression

$$\sum_{\nu\sigma}[\ldots] = iN(p')\,N(p)$$
$$\times \Big\langle \Big| \frac{e^2f^2}{M}\mathbf{ee}' - i\frac{e^2f}{2M}\frac{k}{M}(f+2\lambda g)\,\sigma\,[\mathbf{e}'\mathbf{e}]$$
$$- i\frac{e^2}{2M}\frac{k}{M}(f+\lambda g)^2\,\sigma\,[[\mathbf{ne}]\,[\mathbf{n}'\mathbf{e}']]$$
$$- i\frac{e^2f}{2M}\frac{k}{M}(f+\lambda g)\,((\mathbf{en}')\,(\sigma[\mathbf{n}'\mathbf{e}']) - (\mathbf{e}'\mathbf{n})\,(\sigma[\mathbf{ne}]))$$
$$- \frac{1}{2}\frac{e^2}{2M}\Big(\frac{k}{M}\Big)^2(f+\lambda g)^2\,(\mathbf{ee}'(1-\cos^2\theta) - (\mathbf{en}')\,(\mathbf{e}'\mathbf{n})$$
$$\times(1-\cos\theta))$$
$$- \frac{1}{2}\frac{e^2}{2M}\Big(\frac{k}{M}\Big)^2(2\lambda gf + \lambda^2g^2)\,(\mathbf{en}')\,(\mathbf{e}'\mathbf{n})\Big|\Big\rangle,$$
$$N(p') = \sqrt{1/2\,(1+M/E(p'))}, \qquad N(p) = 1, \quad \mathbf{p} = 0. \tag{7*}$$

In deriving Eq. (7), we have changed over to two-row matrices, and have limited ourselves to terms quadratic in frequency. Terms quadratic in k and containing the operator $\boldsymbol{\sigma}$ have been dropped, since their contribution to the cross section is of the order of k^3 and less. The functions f and g characterizing the spread of the charge and of the magnetic moment of a nucleon in the non-relativistic approximation can be written as

$$f(\Delta p^2) = 1 - \tfrac{1}{6}\langle r_e^2\rangle\,k^2, \quad g(\Delta p^2) = 1 - \tfrac{1}{10}\langle r_\mu^2\rangle\,k^2.$$

where $\langle r_e^2\rangle$ and $\langle r_\mu^2\rangle$ are the mean-square radii of the charge and of the magnetic moment. In our approximation, we must, in the expression for the amplitude, set the functions f and g equal to one in all terms except the term $e^2f^2M^{-1}\mathbf{e}\cdot\mathbf{e}'$.

As was shown by Low,[1] the sum of the first and third terms in Eq. (5) does not contribute to the single-nucleon term in the approximation linear in k. For the problem on hand, let us first construct the general form of the terms quadratic in k that are due to this sum and that do not contain the operator $\boldsymbol{\sigma}$. From $\mathbf{e}$, $\mathbf{e}'$, $\mathbf{k}$, and $\mathbf{k}'$ one can write the general expression

$$\gamma_1\mathbf{ee}'k^2 + \gamma_2(\mathbf{ee}')\,(\mathbf{kk}') + \gamma_3(\mathbf{ek}')\,(\mathbf{e}'\mathbf{k}). \tag{8}$$

Expressions of the type $\mathbf{e}\times\mathbf{k}$, $\mathbf{e}'\times\mathbf{k}'$, $\mathbf{e}\times\mathbf{k}'$, $\mathbf{e}'\times\mathbf{k}$ can be reduced, as can easily be ascertained, to the form (8). γ_1, γ_2, and γ_3 denote still unknown coefficients, whose meaning must be explained. To this end, we shall use the formula proved in reference 1

$$k_i'k_jg_{ij} = k'k\Big\langle\Big|\int P\,[\rho(x)\,\rho(y)]\exp[i(ky - k'x)]\,dx\,dy\Big|\Big\rangle, \tag{9}$$

where g_{ij} is defined in the following way

$$\langle|S|\rangle = -e_i'e_jg_{ij}/\sqrt{4kk'}.$$

Equation (9) is obtained as a result of the gauge invariance of the theory. Carrying out a transformation of the right-hand side of Eq. (9), analogous to those carried out earlier, we easily find that

$$k_i'k_jg_{ij} = i(2\pi)^4\delta(p'+k'-p-k)\Big\{\text{single-nucleon term}$$
$$- kk'k_i'k_j\sum_{N'}\Big[\frac{\langle 0|\int\rho(\mathbf{x})\,x_i dx\,|N'0\rangle\langle N'0|\int\rho(y)\,y_j dy\,|0\rangle}{(E_{N'}-M)\,V^2}$$
$$+ \frac{\langle 0|\int\rho(y)\,y_j\,dy\,|N'0\rangle\langle N'0|\int\rho(x)\,x_i\,dx\,|0\rangle}{(E_{N'}-M)\,V^2}\Big]\Big\}.$$

Under the summation sign we have the equivalent of a symmetrical tensor α_{ij}. For a particle with spin $1/2$, this tensor should be of the form $\alpha\delta_{ij}$. Thus, transitions to other states make an additional contribution of the form $\alpha k^2\mathbf{k}\cdot\mathbf{k}'$, in addition to the single-nucleon term. It is natural to name the constant

$$\alpha = \sum_{N'}\Big[\frac{\langle 0|\int\rho(\mathbf{x})\,x_i dx\,|0N'\rangle\langle 0N'|\int\rho(y)\,y_i dy\,|0\rangle}{(E_{N'}-M)\,V^2}$$
$$+ \frac{\langle 0|\int\rho(y)\,y_i dy\,|0N'\rangle\langle 0N'|\int\rho(x)\,x_i dx|0\rangle}{(E_{N'}-M)\,V^2}\Big] \tag{10}$$

the electrical polarizability of the nucleon.[4] Comparing the left-hand side of Eq. (9) with the right-hand one we see that, to have an equality, it is necessary that $\gamma_2 + \gamma_3 = 0$ and $\gamma_1 = -\alpha$. The equality of the single-nucleon terms on the left- and right-hand sides follows automatically from the requirement that the single nucleon terms should vanish for an exchange of $e \to k$ and $e' \to k'$ which, as can easily be checked, is satisfied in our case. As a result, we obtain the following form of the matrix element for the process under consideration

$$\langle|S|\rangle = -\frac{i(2\pi)^4}{\sqrt{4kk'}}\delta(p'+k'-p-k)\,\{\langle|(\ldots)$$
$$-\alpha k^2\mathbf{ee}' - \beta\,[\mathbf{ke}]\,[\mathbf{k}'\mathbf{e}']|\rangle\}. \tag{11}$$

The symbol . . . denotes the single-nucleon term in the amplitude, and $\beta = -\gamma_2 = +\gamma_3$.

In order to explain the physical meaning of the constant β, we shall carry out a Foldy transformation[8] of the expressions that lead to the term $\beta\mathbf{k}\times\mathbf{e}\cdot\mathbf{k}'\times\mathbf{e}'$. This transformation means that $\exp(i\mathbf{q}\cdot\mathbf{r})$ is transformed in the corresponding matrix elements according to the identity

*$[\mathbf{n}\,\mathbf{e}] = \mathbf{n}\times\mathbf{e}$.

$$\mathbf{e}e^{i\mathbf{qr}} = \int_0^1 \{\text{grad}\,(\mathbf{er}e^{is\mathbf{qr}}) - i\,[\mathbf{r}\,[\mathbf{qe}]]\,e^{is\mathbf{qr}}\}\,ds. \quad (12)$$

After substituting, we see that β is given by the following approximation:

$$\beta \sim \sum_{N'} \Big[\frac{\langle 0 | \int [\mathbf{j}(\mathbf{x})\,\mathbf{x}]\,dx \,|\, N'0\rangle \langle N'0 | \int [\mathbf{j}(\mathbf{y})\,\mathbf{y}]\,dy \,|\, 0\rangle}{(E_{N'} - M)\,V^2} + \frac{\langle 0 | \int [\mathbf{j}(\mathbf{y})\,\mathbf{y}]\,dy \,|\, N'0\rangle \langle N'0 | \int [\mathbf{j}(\mathbf{x})\,\mathbf{x}]\,dx \,|\, 0\rangle}{(E_{N'} - M)\,V^2} \Big]. \quad (13)$$

The "approximately-equal" sign, rather than the equality sign, is used above, since, in general, other terms not written here may also contribute to β. This problem requires additional investigation.

The first term in Eq. (13) has no simple physical meaning, while the second term, like α, can be called the magnetic polarizability. Henceforth, we shall define the magnetic polarizability as the sum of all contributions to β, i.e., as β itself.

3. DIFFERENTIAL CROSS SECTION

The differential cross section is given by the equation*

$$d\sigma = \frac{1}{4kk'}\,|Q|^2\,\frac{d\mathbf{p}'d\mathbf{k}'}{J\,(2\pi)^2}\,\delta(\mathbf{p}' + \mathbf{k}' - \mathbf{p} - \mathbf{k})\,\delta(E(p') + k' - E(p) - k),$$

where Q is the expression inside the braces in Eq. (11), and J is the current density of the colliding particles.

Averaging and summing up over the spin projections of the nucleons and of the photon polarization, we obtain a formula for the differential cross section for the scattering of an unpolarized photon beam on unpolarized nucleons:

$$\frac{d\sigma}{d\Omega} = \frac{1}{2} r_0^2 \Big\{ \Big[1 - 2\frac{k}{M}(1 - \cos\theta) + 3\Big(\frac{k}{M}\Big)^2 (1 - \cos)^2 \Big](1 + \cos^2\theta) + \Big(\frac{k}{M}\Big)^2 [(1 - \cos\theta)^2 + f(\theta)] - \Big[\frac{2}{3}\langle r_e^2\rangle M^2 + 2\frac{\alpha M^3}{e^2}\Big]\Big(\frac{k}{M}\Big)^2 (1 + \cos^2\theta) - 4\frac{\beta M^3}{e^2}\Big(\frac{k}{M}\Big)^2 \cos\theta \Big\},$$

$$r_0 = e^2/M, \qquad f(\theta) = a_0 + a_1\cos\theta + a_2\cos^2\theta,$$

$$a_0 = 2\lambda + \tfrac{9}{2}\lambda^2 + 3\lambda^3 + \tfrac{3}{4}\lambda^4, \qquad a_1 = -4\lambda - 5\lambda^2 - 2\lambda^3,$$

$$a_2 = 2\lambda + \tfrac{1}{2}\lambda^2 - \lambda^3 - \tfrac{1}{4}\lambda^4. \quad (14)$$

In deriving Eq. (14), the terms containing powers of frequency higher than k^2 were neglected. Equation (14) contains the three additional constants α, β, and $\langle r_e^2\rangle$, as compared with the formula of Powell[10] for the scattering of light on a charged particle with anomalous momentum λ. The value of $\langle r_e^2\rangle$ can be obtained from experiments on the structure of the nucleon. We can attempt to determine the constants α and β from protons by comparing Eq. (14) with the experimental data. The reader may refer to such a comparison carried out by Gol'danskii et al.[6]

*See reference 9, p. 292.

4. CONCLUSION

We shall make a few comments concerning the role of the separate terms in Eq. (14), and the region of its applicability. The role of the separate terms is characterized by the following data:

	10–20 Mev	50–60 Mev	90–100 Mev
Term containing λ:	1%	10%	50%
Term containing $\langle r_e^2\rangle$:	0.3%	4%	12%

Henceforth, we put $\frac{1}{2}(e^2/M)^2 = 1$. In the estimates, we have put $\lambda = 1.7$ and $\sqrt{\langle r_e^2\rangle} = 0.8 \times 10^{-13}$ cm. We cannot give a good estimate of the terms containing α and β, since there is no rigorous theory concerning these parameters. The estimate of the quantity

$$\alpha' = \frac{1}{3}\frac{e^2}{\hbar c}\langle r_e^2\rangle \frac{\hbar}{Mc} + \alpha,$$

carried out by Baldin[4] has yielded the following results:

$$0{,}4\cdot 10^{-42}\,\text{cm}^3 \leqslant \alpha' \leqslant 1{,}5\cdot 10^{-42}\ \text{cm}^3.$$

The lower limit of this estimate has been obtained from the relation between the polarizability and the amplitude of the electrical-dipole photoproduction of π mesons. The upper limit of the estimate has been obtained by an analysis of the experimental data on the Compton effect. Gol'danskii, et al.[6] have obtained the value of $\alpha = (0.9 \pm 0.2) \times 10^{-42}$ cm^3. It is easily concluded from this that the contribution of the term containing α may amount to 5 to 12%. No reliable theoretical estimates of β are at present available. From the fact that the measured cross section is not greatly different from the cross section given by the formula of Powell, we can conclude that the contribution of the term containing β is not greater than the contribution of that containing α.

Thus, each of the terms containing α and β makes a contribution of about 10%, in its order of magnitude. This indicates that we have to be very careful in determining α and β directly from experiments by comparing Eq. (14) with experimental data. This is because we do not have an exact estimate of the terms neglected in Eq. (14). We can expect that the contribution of the consecutive terms of the expansion will amount to $\sim k/\mu$ (where μ is the meson mass) of the contribution of terms containing α and β. If we want to fit the experimental data on the Compton effect on a proton with an accuracy of 5%, then Eq. (14) is satisfactory in the 0 — 70 Mev energy range. If we want to determine the parameters α and β to within 10 to 20% by comparing Eq. (14) with experimental data, then experiments on γ-ray scattering at 10 — 30 Mev energy are necessary, which are at present already on the border-line of experimental feasibility. A more exact evaluation of this limit depends on the estimate of the neglected terms in the amplitude, which has so far not been carried out.

In conclusion, the author wishes to thank A. M. Baldin for proposing the subject, and for his constant interest in the work.

Note added in proof (April 7, 1961): A more exact analysis shows that the summation over negative energies in the single-nucleon term (see footnote 3) does not take into account all the intermediate states involving pairs, but accounts only for those which correspond to usual diagrams of the perturbation theory. The remaining states should be included in the sum over N′.

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Translated by H. Kasha
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