RECONSTRUCTION OF THE ENERGY GAP IN A SUPERCONDUCTOR BY MEASUREMENT OF SOUND ATTENUATION

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A method is indicated for the reconstruction of the energy gap $\Delta(\mathbf{n})$ as a function of direction in an anisotropic superconductor on the basis of measurements of sound absorption $\alpha(\mathbf{n})$ in the low-temperature region. The method is based on the simple relation between the level lines of $\Delta(\mathbf{n})$ and $\alpha(\mathbf{n})$ on a stereographic projection of the Fermi surface.

IT was shown earlier[1] that ultrasonic damping $\alpha_s(\mathbf{q})$ in an anisotropic superconductor in the direction $\mathbf{n} = \mathbf{q}/q$ is determined by the asymptotic formula

$$\ln[(\alpha_s(\mathbf{q})/\omega_s(\mathbf{q})] = -\Delta_{min}^{(\mathbf{n})}/T, \quad (1)$$

where $\Delta_{min}^{(\mathbf{n})}$ is the minimum value of the energy gap $\Delta(\mathbf{n})$ in the vicinity of a stereographic projection of the Fermi surface perpendicular to $\mathbf{n}$. Equation (1) is applicable in the temperature region determined by the inequality

$$1 < T_k/T < (2/\alpha)\ln(v_F/c), \quad (2)$$

where v_F is the Fermi velocity, c = sound velocity, α = coefficient of anisotropy, equal to the ratio of $\Delta(\mathbf{n})$ on the Fermi surface to the minimum value of Δ_0.

The purpose of the present work is a detailed analysis of the problem of the possibility of reconstruction of the function $\Delta(\mathbf{n})$ according to the given measurements of $\alpha_s(\mathbf{q})$, and the demonstration of a simple experimental procedure for reconstructing the gap. We shall assume that the type of Fermi surface is known from other experiments. In what follows, the Fermi surface is assumed to be singly connected. We shall also assume that the lattice possesses radial symmetry. For simplicity, we introduce the notation $\Delta_{min}^{(\mathbf{n})} = f(\mathbf{n})$.

We consider a great circle on the sphere $C(\mathbf{n})$, perpendicular to the direction $\mathbf{n}$. Let $P(\mathbf{n})$ be the point on $C(\mathbf{n})$ at which $\Delta(\mathbf{n})$ takes its minimum value $f(\mathbf{n})$. Then the level line $\Gamma_{f(\mathbf{n})}$ of the function $\Delta(\mathbf{n})$ passing through $P(\mathbf{n})$ is tangent to the circle $C(\mathbf{n})$. From this it is evident that Γ_a is the envelope of a family of circles $C(\mathbf{n})$ for which $f(\mathbf{n}) = a$. In this way a method is given for constructing the function $\Delta(\mathbf{n})$ in terms of $f(\mathbf{n})$.

However, in this case certain peculiarities can arise which we shall consider in examples.

1. Let $\Delta(\mathbf{n})$ have a total of two minima in diametrically opposite points which we shall take to be the poles of a spherical system of coordinates. We shall assume that $\Delta(\vartheta, \varphi)$ is a monotonic function of ϑ $(0 < \vartheta < \pi/2)$ for fixed φ. We shall also assume that a certain level line Γ_a of the function $\Delta(\mathbf{n})$ is not convex. Then there exists a circle $C(\mathbf{n}_a)$ which is tangent to Γ_a at two points P_1, P_2 (Fig. 1). On an arbitrary circle which is tangent to Γ_a on the non-convex cut P_1P_2, the minimum of $\Delta(\mathbf{n})$ is less than a. It is then evident that one cannot define the function $\Delta(\mathbf{n})$ on the segment P_1P_2 of the curve Γ_a. The set of such segments for all Γ_a forms a region in which $\Delta(\mathbf{n})$ cannot be defined ("white spot").

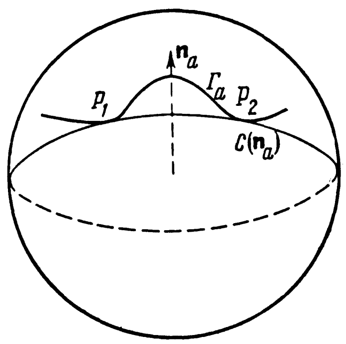

FIG. 1

We consider two families of circles tangent to Γ_a "on the left" (P_1) and "on the right" (P_2). It is obvious that these families are tangent to the circle $C(\mathbf{n}_a)$. For the function $f(\mathbf{n})$ this corresponds to the fact that its level line Γ_a has an angle point at $\mathbf{n}_a$.

2. Let there exist four absolute minima (in

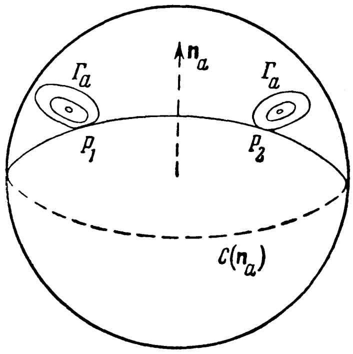

FIG. 2

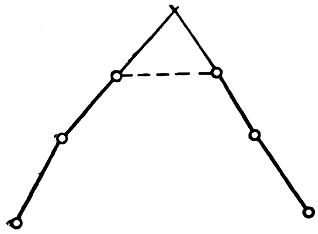

FIG. 3

two pairs of diametrically opposite points). Then, for certain values of a, the level lines Γ_a are split into four unconnected closed curves (Fig. 2). There exists a circle $C(\mathbf{n}_a)$ tangent to both these curves. In this case, just as in the previous, there are two families of circles tangent to Γ_a which are joined by $C(\mathbf{n}_a)$. Just as in case 2, this leads to a "white spot" for the function $\Delta(\mathbf{n})$, and to angle points for level lines of the function $f(\mathbf{n})$.

Thus, there are angle points of the level lines for the function $f(\mathbf{n})$. If $f(\mathbf{n})$ does not have these singularities, then $\Delta(\mathbf{n})$ is defined everywhere. Now let the function $f(\mathbf{n})$ be known. We shall take the level line γ_a of the function $f(\mathbf{n})$. For each point of $\mathbf{n}$ of the line γ_a we construct a circle $C(\mathbf{n})$ and find the envelope of the resultant family of circles. According to what has been pointed out above, this is also the level line Γ_a of the function $\Delta(\mathbf{n})$.

In experiment, the level line γ_a will be a certain broken curve consisting of segments of large circles. For each segment of the circle it is necessary to construct its center on the sphere. Joining all the points constructed in this fashion, we obtain an approximation of the level line Γ_a of the function $\Delta(\mathbf{n})$.

It is necessary to give separate consideration to the case in which there is an angle point on γ_a. In this case, for sufficiently detailed measurement, the two neighboring segments of circles on the line approximating γ_a intersect at an angle which is strongly different from π. The two points on the line Γ_a corresponding to them will be sufficiently far removed from one another. According to what has been observed in connection with the examples considered above, it is not necessary to join such points, since they form the boundary of a "white spot."

What has been said above needs to be made more precise. For experiments close to the angle point, the line γ_a will be the characteristic situation schematically drawn in Fig. 3. In this case, it is necessary to draw the line through the experimental points so that only a single angle is obtained that differs strongly from π. Evidently, after the initial measurements, it will be necessary to carry out more detailed measurements close to the points which can be shown to be angle points.

As has been shown,[1] for superconductors with a Fermi surface of the corrugated plane type, Eq. (1) is found to be invalid even at temperatures satisfying condition (2) in that region of directions of sound propagation which are not parallel to any normal to the Fermi surface. In this case, the following formula holds:

$$\ln(\alpha_s/\omega_s) \approx -\Delta_0/T. \qquad (3)$$

This means that $f(\mathbf{n})$ is constant in some region on the sphere. Reconstruction of $\Delta(\mathbf{n})$ in terms of $f(\mathbf{n})$ is carried out in this case also by the method described above.

The situation becomes complicated if the Fermi surface is not singly connected. In this case, to reconstruct $\Delta(\mathbf{n})$ by means of $f(\mathbf{n})$, it is necessary to apply the procedure that has been described, but it is not possible to show to just which of the unconnected parts of the Fermi surface the level lines that have been found belong.

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[1] V. L. Pokrovskii, JETP **40**, 898 (1961), Soviet Phys. JETP **13**, 628 (1961).

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