PHASE SHIFT ANALYSIS OF pp SCATTERING AT 95 Mev

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Submitted to JETP editor October 6, 1960

J. Exptl. Theoret. Phys. (U.S.S.R.) 40, 1106-1111 (April, 1961)

A five-parameter analysis of the experimental data on pp-scattering at 95 Mev (cross section, polarization, depolarization) is performed by a new numerical method (the "ravine" method). We obtain a broad complex range of solutions, which cannot be described by specifying the local minima and error matrices as in the well known "local" technique. The region obtained can be divided into two comparatively small regions by including some data on rotation of polarization R, obtained by extrapolating from energies of 150, 210, and 310 Mev.

1. PHASE-SHIFT ANALYSIS PROCEDURE

At the present time the universally adopted procedure for processing of experimental data, particularly the phase-shift procedure, is as follows. We minimize the square of the deviation

$$\chi^2(\delta) = \sum \left[\frac{y(\delta) - y_e}{\Delta_e} \right]^2, \tag{1}$$

where $y(\delta)$ are the theoretical data (cross sections, etc.) as functions of certain parameters (phase shifts) $\delta = \delta_1, \delta_2, \ldots, \delta_n$, subject to determination from corresponding experimental data y_e (Δ_e — experimental errors); the sum extends over different points on the specified curves. The problem thus reduces formally to a determination of local minima of the function (1) in many-dimensional space of the parameters δ (the phase space), for which we usually employ the method of gradient descent from a series of randomly drawn points. The local minima obtained are called solutions, and their accuracy is specified in terms of an error matrix, which outlines certain many-dimensional ellipsoids in phase space (see, for example, references 1 — 4).

In the simplest cases the solution obtained in this manner can give a correct idea of the phase-space regions that satisfy the experimental data. However, if the number of dimensions and surfaces (1) is large and the surfaces have various undulations and troughs, the method is laborious and does not provide a sufficient assurance that all the regions with small values of χ^2 has been determined. This is not surprising, for two reasons:

1) It is difficult to trace a function that has essentially different values in a tremendous number of points. Thus, for example, in nine-parameter analysis with a range of variation $-\pi/2 \ll \delta \ll \pi/2$, with a spacing of $0.1 - 0.2$, the number of points to be investigated for each phase is $10^{10} - 10^{14}$.

2) In any case of considerable complexity, the method of gradient descent is unjustifiably cumbersome, since it forces us to trace the most minute details of the relief, details which usually have no physical meaning, and we cannot extricate ourselves from individual 'cavities' in which we "get stuck." In addition, the method is essentially suitable for finding those individual cavities (local minima) which have usually the sense of a sort of 'fine structure' in broader regions containing low values.

Thus, instead of finding local minima it is desirable to carry out a direct "probing" of the phase space in order to find the entire region with low values of χ^2 and to obtain a more complete and accurate idea of the possible values of the phase shift. However, a direct "probing" of an entire phase space with a large number of dimensions is impossible in practice, and we must therefore use a method which works predominantly in regions with low values of χ^2. In this problem we use for this purpose a new numerical method (the method of "ravines") proposed by Gel'fand (more details about this method will be published separately). A characteristic feature of this method are "jumps" of finite length along the "ravines" of low values of χ^2. If the network of "ravines" is not too badly tangled up, this method yields relatively rapidly all regions with low values of χ^2. This situation obtains in "well organized" functions, such as the many-dimensional functions usually encountered in practical problems.

To complete the analysis we must choose some criterion for solving the problem. We use the simplest method of drawing the obtained points, assuming the solution to include all the regions of the phase space with values $\chi^2(\delta) \le \chi^2_{max}$. In this problem we assume $\chi^2_{max} = 2\overline{\chi^2}$ where $\overline{\chi^2}$ is the mean mathematical expectation. By plotting the theoretical curves corresponding to the phase shifts obtained in this manner we can ascertain that they agree sufficiently well with the experimental points.

We do not give a more detailed analysis of the region of solutions for the following reasons. Regions with low values of χ^2 are large and complex, and therefore any attempt to describe them by specifying the local minima and the error matrix, as is customary in the old procedure, may lead to a loss of large regions in which the true solution may be located. Upon sufficient improvement of the experimental data (improvement of the set of data through supplementary polarization experiments or by reducing the errors), the form of the surface (1) apparently becomes simpler and is converted into a certain small number of sufficiently narrow, clearly separated "troughs" of paraboloidal type. In this case it would be easy to investigate in detail the regions for solution and to find reliable limits. Further, the existing experimental data contain many systematical errors. Thus, new data on the cross section for 98 Mev (Harwell) differ greatly in a certain range of angles from those used in this investigation (Harvard). An analogous situation takes place for depolarization at 150 Mev. This leads to a certain distortion and to a shift in the level lines in phase space.

In making up the squared deviation (1), all the phases corresponding to large orbital momenta were considered in the one-meson approximation (with meson-nucleon constant $g^2 = 14.5$), as proposed in reference 5. The choice of values of momenta, starting with which the phase becomes "fixed" in the one-meson approximation, is based on estimates of the two-meson corrections obtained by Galanin et al.[6]

2. ANALYSIS OF DATA FOR 95 — 98 Mev

We processed the data on the cross section $\sigma(\theta)$ and polarization $P(\theta)$ for fourteen scattering angles[7] jointly with data on depolarization $D(\theta)$ for five scattering angles.[8] The independently varied parameters were five proper phase shifts with allowance for the Coulomb interaction (called BB shifts by Stapp et al.[2]), viz: $\delta_0(^1S_0)$, $\delta_2(^1D_2)$, $\delta_1^0(^3P_0)$, $\delta_1^1(^3P_1)$, and $\delta_1^2(^3P_2)$.

In the determination of the phase shifts $\epsilon_2(^3P_2 - {}^3F_2)$ and $\delta_3^2(^3F_2)$, connected with δ_1^2, it is necessary to take into account the fact that the one-meson approximation gives only the real parts of the scattering matrix. For this reason we specified in the one-meson approximation the following matrix elements[5]

$$\xi_2 = \mathrm{Re}\,[S^2_{1,3}/2i], \qquad \eta_3^2 = \mathrm{Re}\,[(S_3^2 - e^{2i\Phi_3})/2i],$$

(Φ_3 — Coulomb phase), which leads to the relations*

$$\mathrm{tg}\,\varepsilon_2 = 2\xi_2/[\sin 2\delta_1^2 - 2\eta_3^2 - \sin 2\Phi_3],$$

$$\sin 2\delta_3^2 = 2\eta_3^2 + \sin 2\Phi_3 - 2\xi_2\,\mathrm{tg}\,\varepsilon_2.$$

Starting with the state $^3F_4$, all the contributions were taken into account in the form of a closed sum (one-meson amplitudes), and the imaginary parts of the scattering matrix were thus neglected.

As a result of the analysis we obtained a large complex region of solutions, which cannot be described by specifying the local minima and error matrices. We obtained several hundreds of points with $\chi^2 \le 2\overline{\chi^2}$ ($\overline{\chi^2} = 28$), which lie within the limits

$$-25° \leqslant \delta_0 \leqslant 25°, \quad -4° \leqslant \delta_2 \leqslant 9°, \quad -20° \leqslant \delta_1^0 \leqslant 35°,$$
$$-13° \leqslant \delta_1^1 \leqslant 9°, \quad 9° \leqslant \delta_1^2 \leqslant 18°.$$

In an examination of the topography of the resultant region, we see that it has a tendency to split, in accordance with the phase shifts $^1S_0$ and $^3P_0$, into two regions (which we designate I and II), each of which is subdivided in turn into two others, corresponding to $^3P_1$ and $^1D_2$. In places where the proposed separations lie, there are low 'mountain ranges' with $\chi^2 \approx 3\overline{\chi^2}$. Figure 1 and the table show some of these points: 1 — 4 from region I and 5 and 6 from region II. These points were chosen from among all those obtained so as to include as fully as possible the entire resultant region of solutions.

Figures 2, 3, and 4 show the curves corresponding to the chosen points for the depolarization

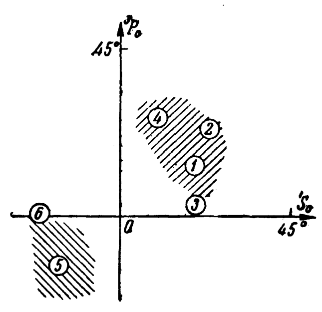

FIG. 1

*tg = tan.

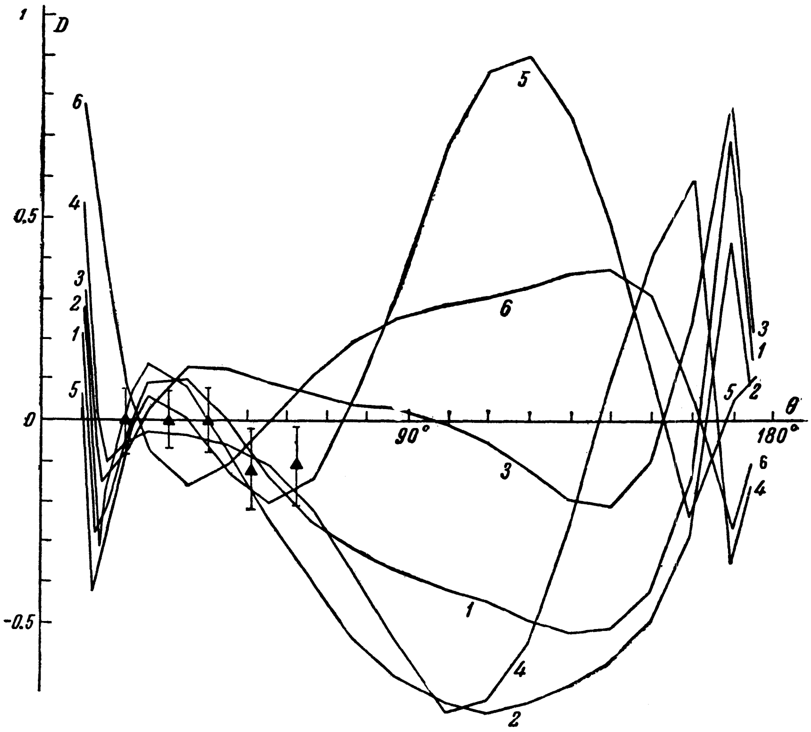

FIG. 2

Certain solutions (points) from the resultant region of phase space (in degrees)

Point	χ^2	$^1S_0$	$^1D_2$	$^3P_0$	$^3P_1$	$^3P_2$
1	25	20.3	6.0	14.4	—11.5	13.2
2	44	23.5	4.0	24.1	—8.7	12.5
3	59	20.2	6.8	3.7	—14.9	11.8
4	24	9.8	—3.5	26.0	—2.6	15.3
5	24	—16.9	7.2	—13.5	7.0	15.8
6	37	—20.5	—1.7	0.5	—10.6	13.8

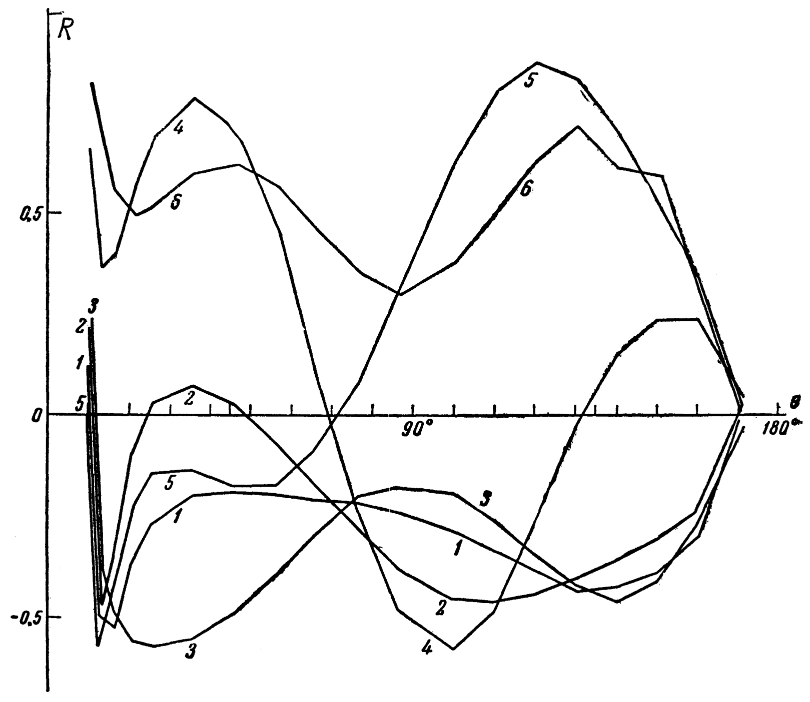

FIG. 3

D (θ) and rotations of polarization R (θ) and A (θ). Figure 2 shows also the experimental curves.[8] We do not give the curves for the cross section and polarization, for in practice the various solutions merge, within the limits of experimental errors, and differ somewhat in the interference region $\theta \approx 10°$.

Figures 2—4 show that a measurement of D (θ) for $\theta \approx 120°$ and R (θ) for $\theta \approx 30$ and 120° would bring us much closer to a single-valued solution with relatively low tolerances in all the phases. Extrapolating the available data for R (θ) at energies 150, 210, and 310 Mev, we can assume that for 95 Mev the rotation of the

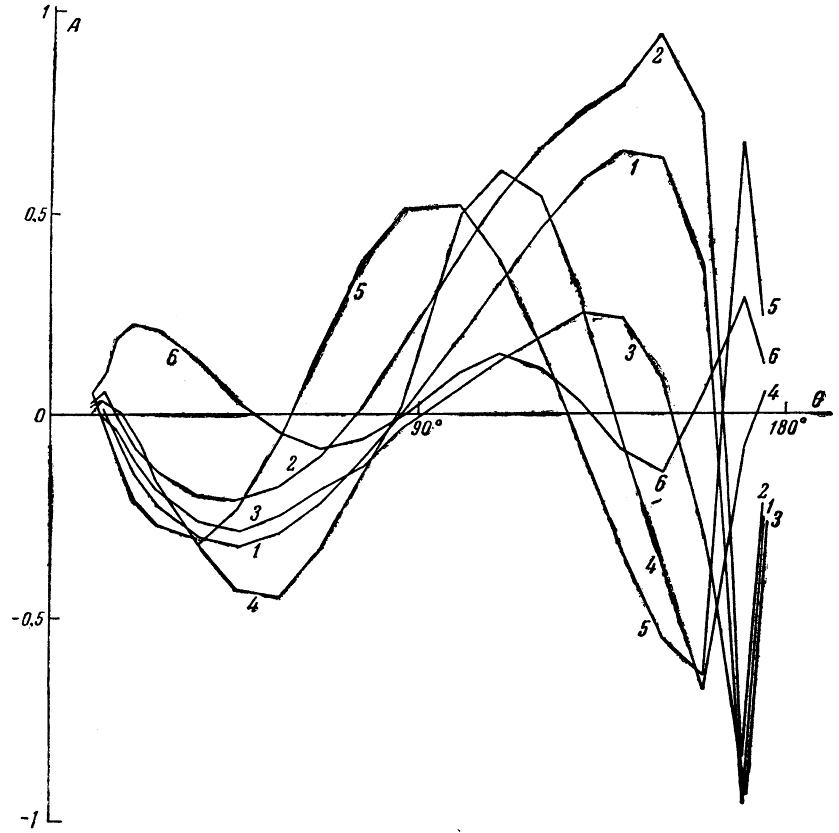

FIG. 4

polarization R ≈ − 0.2 at $\theta \approx 30°$. The addition of such data would shrink and sharply demarcate the foregoing regions and would yield two relatively small regions (two solutions):

I. $\delta_0 \approx 20°$, $\delta_2 \approx 6°$, $\delta_1^0 \approx 15°$, $\delta_1^1 \approx -12°$, $\delta_1^2 \approx 13°$;

II. $\delta_0 \approx -17°$, $\delta_2 \approx 7°$, $\delta_1^0 \approx -13°$, $\delta_1^1 \approx 7°$, $\delta_1^2 \approx 16°$.

In particular, only points with a positive ^1D phase shift, $\delta_2 \approx 4-8°$, would remain. We note that both solutions differ little in cross section and in polarization in the interference region.

For the foregoing solutions we obtained continuations in the region of high energies — solutions I and II for 150 Mev[9] and solutions I and II for 310 Mev,[3] respectively.

In carrying out the phase-shift analysis, we also undertook to ascertain whether it is possible to obtain from the available experimental data the "peripheral" phases, for which the main contribution should be made by the one-meson approximation, and to verify thereby the correctness of the estimate of the accuracy of the one-meson approximation, obtained by calculating the two-meson phases (see reference 6). For 95 Mev we can already regard as peripheral the F phase shifts and the mixing parameter ξ_2, which for this reason were 'fixed' in the one-meson approximation. However, the analysis was repeated with varying these phase shifts (four additional parameters). With this, the dimensions of the solution region increased and very low values $\chi^2 \approx 10$ were obtained at the minima (with $\overline{\chi^2} = 24$). To illustrate the permissable tolerances in the ^3F phase shifts we give one point with $\chi^2 = 26$:

$$\delta_0 = 21.1°, \quad \delta_2 = 6.1°, \quad \delta_1^0 = 3.2°, \quad \delta_1^1 = 10.1°,$$
$$\delta_1^2 = 8.9°, \quad \xi_2 = 4.3°\,(-3.4°), \quad \eta_3^2 = -11.8°\,(0.8°),$$
$$\delta_3^3 = 6.3°\,(0°), \quad \eta_3^4 = 2.3°\,(0.3°).$$

(the parentheses contain also the theoretical values of ξ_2 and ^3F phase shifts assumed in the five-parameter analysis).

The results obtained allow to conclude that it is impossible to obtain with any degree of reliability the values of ξ_2 and ^3F shifts from the available data, and it can only be stated that the one-meson values for these phase shifts do not contradict the experiment.

We note in conclusion that of the two regions of solutions indicated above, region I is preferred since it is characterized by positive values of the ^1S phase shift, and this agrees better with the positive ^1S phase at lower energies, where the effective scattering length approximation can be employed.

The authors are grateful to M. A. Evgrafov and I. I. Piatetskii-Shapiro for help in the development of the procedure, to Ya. A. Smorodinskii for discussions and useful remarks, and to S. L. Ginzburg for help in the calculations.

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Translated by J. G. Adashko
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