# Letters to the Editor

## *EFFECT OF ANISOTROPY ON THE PROPERTIES OF SEMICONDUCTORS*

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THE theory of superconductivity[1,2] has been developed, as is well known, for an isotropic model of the metal. The present paper is devoted to an investigation of the effect of anisotropy on the properties of superconductors.

The electron-phonon interaction is characterized by the Hamiltonian

$$H' = \sum_{\mathbf{k}'=\mathbf{k}+\mathbf{q}} g(\omega/2V)^{1/2} a^{+}_{\mathbf{k}'s} a_{\mathbf{k}s} b_{\mathbf{q}} + \text{compl. conj.} \qquad (1)$$

where $a_{\mathbf{k}s}$ is the second-quantized amplitude, and g is the coupling constant, the anisotropy of which must be taken into account when solving the problem under consideration.

We start with an expression for the matrix element that characterizes the electron transitions in the metal due to the interaction between the electrons and the lattice (we disregard Umklapp processes)[3]

$$K = -\frac{h^2 i}{m}\left(\mathbf{k}-\mathbf{k}', \int \operatorname{grad} u_{\mathbf{k}} \frac{\partial u^*_{\mathbf{k}'}}{\partial s} d\tau_0\right) - \left[E_{\mathbf{k}} - E_{\mathbf{k}'} - \frac{h^2}{2m}(k^2 - k'^2)\right]\int u_{\mathbf{k}} \frac{\partial u^*_{\mathbf{k}'}}{ds} d\tau_0. \qquad (2)$$

The Coulomb interaction is not considered here. On the basis of symmetry considerations, we can determine the form of the integrals that enter in (2). We then obtain from (2)

$$g_j = g_{0j}\left\{b\frac{\mathbf{k}+\mathbf{k}'}{q}\mathbf{e}_{\mathbf{q}j}\left[E_{\mathbf{k}} - E_{\mathbf{k}'} - \frac{1}{2m}(k^2 - k'^2)\right] + a\frac{\mathbf{q}}{q}\mathbf{e}_{\mathbf{q}j} + c\left(\mathbf{k}+\mathbf{k}', \frac{\mathbf{q}}{q}\right)(\mathbf{k}+\mathbf{k}', \mathbf{e}_{\mathbf{q}j})\right\}, \qquad (3)$$

where $\mathbf{e}_{\mathbf{q}j}$ is the unit vector of phonon polarization ($j = 1, 2, 3$), $q = |\mathbf{k}' - \mathbf{k}|$, and a, b, and c are slowly-varying functions of q, which we assume constant in a layer $h\omega_0$ near the Fermi surface. We neglect terms that contain the unit vectors of the lattice, which in the case of s electrons are smaller than the terms written out here.

Carrying out in the Hamiltonian (1) a canonical transformation after Bogolyubov,[2] we can obtain an equation for the gap in the energy spectrum

$$\Delta(\mathbf{k}) = \frac{1}{2(2\pi)^3} \times \sum_{j=1}^{3}\int g_j^2(\mathbf{k},\mathbf{k}')\frac{\omega_j(q)}{[\omega_j(q) + \tilde{\varepsilon} + \tilde{\varepsilon}']}\frac{\Delta(\mathbf{k}')\,d\mathbf{k}'}{\sqrt{\Delta^2(\mathbf{k}') + \xi^2(\mathbf{k}')}}, \qquad (4)$$

where $g_j$ is determined from (3). The substitution $\omega_j(\omega_j + \tilde{\varepsilon} + \tilde{\varepsilon}')^{-1} \approx 1$ leads to the renormalization of $g_{0j}$.[2]

We first consider the case of a closed Fermi surface with a dispersion law

$$E = k_z^2/2m_1 + (k_x^2 + k_y^2)/2m_2$$

(ellipsoid of revolution). We assume that $m_1 - m_2 = \Delta m \ll m_1$ and $h\omega_0/E_F \ll m_1/\Delta m$, and take account of the fact that $b < a$ and $c < a$. We then solve Eq. (4) and find (for $T = 0$) $\Delta(\vartheta) = \Delta_0 + \Delta_1$ where

$$\Delta_1 = \frac{b+c}{a}\left(\frac{\Delta m}{m_1}\right)^2 \Delta_0\left(\frac{11}{6}\cos^4\vartheta - \cos^2\vartheta + \frac{1}{2}\right), \qquad (5)$$

and $\Delta_0$ is determined by

$$1 = 4\pi g_{01}^2 a^2 \alpha\left[\left(1 - \frac{\Delta m}{m_1}\right)^{-1/2} + \left(\frac{\Delta m}{m_1}\right)^2\frac{b+c}{2a}\right]\ln\frac{2\tilde{\omega}}{\Delta_0},$$

$$\alpha = m_2(2m_2E_F)^{1/2}$$

($\vartheta$ is the angle between the z axis and the vector $\mathbf{k}$).

Calculation for $T \to T_c$ leads to the following result:

$$\Delta(\mathbf{k}, T) = \Delta_1(T)\left[1 + \left(\frac{\Delta m}{m_1}\right)^2\frac{b+c}{a}\left(\frac{11}{6}\cos^4\vartheta - \cos^2\vartheta + \frac{1}{2}\right)\right], \qquad (6)$$

where $\Delta_1(T)$ is given by

$$1 = 4\pi g_{01}^2 a^2 \alpha\left(1 - \frac{\Delta m}{m_1}\right)^{-1/2}\left[\ln\frac{2\omega\gamma}{\pi T}\left(1 + \left(\frac{\Delta m}{m_1}\right)^2\frac{b+c}{2a}\right) - \frac{7}{8\pi^2}\zeta(3)\frac{\Delta_1^2(T)}{T^2}\right], \qquad (6')$$

or

$$\frac{\Delta_1(T)}{T} = 3{,}06\left[\left(1 - \frac{T}{T_\text{к}}\right)\left(1 + \left(\frac{\Delta m}{m_1}\right)^2\frac{b+c}{2a}\right]^{1/2},$$

where $T_c$ is determined from (6') [$\Delta_1(T_c) = 0$].

We consider next the case of the simplest open Fermi surface — cylindrical surface with dispersion law $E = k_\perp^2/2m$. The calculation of $\Delta(\mathbf{k})$ leads to the following result:

$$\Delta = \Delta_0 + \Delta_1(k_z) + \Delta_1(-k_z),$$

$$\Delta_1(k_z) = \frac{b+c}{a} g_{01}^2\Delta_0\left[\frac{1}{3}s^3 + (2k_z^2 - 2k_zk_{z\,\max} - 4k_{\perp F}^2)s + 8k_zk_{\perp F}^2\ln\frac{k_{z\,max} + k_z + s}{k_{\perp F}} + 5k_{\perp F}^3 - 8k_z^2k_{\perp F}\right],$$

$$s(k_z) = [4k_{\perp F}^3 + (k_{z\,\max} + k_z)^2]^{1/2}. \qquad (7)$$

We now calculate the expression for the heat capacity of the superconductors in the cases under consideration. As a result we obtain the following expression for the ellipsoid:

When $T = T_c$

$$\frac{C_s(T_c)}{C_n(T_c)} = 2{,}4 + 1{,}4\frac{b+c}{a}\left(\frac{\Delta m}{m_1}\right)^2. \quad (8)$$

In the low-temperature region we obtain

$$\frac{C_s(T)}{C_n(T_c)} = \frac{1}{\pi^2 T_c}\left(\frac{\pi}{2}\right)^{1/2} T^{-3/2}\Delta^{5/2}(0)\exp\left(\frac{-\Delta(0)+\beta}{T}\right)\sqrt{\frac{\pi T}{2\beta}},$$

$$\Delta(0) = \overline{\Delta(\vartheta)}, \qquad \beta = \frac{b+c}{a}\left(\frac{\Delta m}{m_1}\right)^2\Delta(0). \quad (9)$$

The formula is applicable at temperatures that satisfy the condition

$$T/\Delta(0) \ll (\Delta m/m_1)^2(b+c)/a.$$

Analogous results are obtained also for a cylinder. However, in the absence of a small parameter similar to $\Delta m/m$ (both for an ellipsoid and for a cylinder), the variation of the gap and of the heat capacity near $T_c$ may be more complicated.

It is easy to see that as $T \to 0$ the heat capacity $C_s$ is determined by the smallest gap $\Delta_{min}$ (see also reference 4); on the other hand, near $T_c$ it is determined by the average value of the gap $\Delta(0)$. Therefore $C_s$ diminishes in the anisotropic model more slowly with decreasing temperature than in the isotropic model. This agrees with the experimental data (see references 4 and 5). Since the expression for $C_s$ (and obviously also for the critical magnetic field $H_c$) contains the parameters of the Fermi surface ($m_1$ and $m_2$ for an ellipsoid, $k_{\perp F}$ and $k_{z\,max}$ for a cylinder), $C_s$ and $H_c$ are no longer universal functions of the temperature, as in the isotropic model, and this explains the difference between the experimental curves of $C_s$ and $H_c$ for different superconductors.

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[3] H. Bethe and A. Sommerfeld, Electron Theory of Metals (Russ. Transl.) ONTI, 1938, p. 187. A. H. Wilson, Quantum Theory of Metals, 1st ed., Cambridge, 1936.

[4] A. A. Abrikosov and I. M. Khalatnikov, Usp. Fiz. Nauk **65**, 551 (1958).

[5] H. A. Boorse, Am. J. Phys. **27**, 47 (1959).

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## *MEASUREMENT OF ANGULAR DISTRIBUTIONS IN THE REACTION* $Al^{27}(p, p')Al^{27}$ *AT 616 Mev WITH A MAGNETIC ANALYZER*

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It is well known that the study of angular distributions in inelastic proton scattering is an important source of information about the properties of nuclear states and the mechanism of inelastic scattering. In such investigations, it is necessary to measure angular distributions of groups of particles with nearly the same energy; this can be done quite reliably by magnetic analysis. On the other hand, the use of a magnetic analyzer leads to a significant increase in the difficulty of the experiment. Therefore, in the overwhelming majority of experiments, the angular distributions are measured with scintillation spectrometers, nuclear emulsions, etc.[1] Because of their low resolution, and because of overloading due to elastic protons, these techniques can measure angular distributions only for one or two excited states of the nucleus.

The ease of field measurement[3] and regulation and the relatively high intensity of the magnetic analyzer described previously[2] enabled us to measure angular distributions for the six groups of elastic and inelastic protons from $Al^{27}$ leading to the excited states at 0.840, 1.014, 2.216, 2.743, and 3.000 Mev (the last level is, according to reference 4, a doublet: 2.976 and 2.999 Mev) without excessive increase in the difficulty of the experiment.

The protons were accelerated in the 120-cm cyclotron of the Moscow State University. A monochromatic beam was obtained by placing collimating slits after the focusing magnet. The full energy spread of the beam was about 45 kev.

The spectrometer for analyzing the secondary particles was set on a rotating support. The particle detector was a ZnS scintillation counter with an FÉU-29 photomultiplier.

The measured differential cross sections (in mb/sr) for elastic and inelastic proton scattering on $Al^{27}$ at 6.6 Mev are shown in the table. The accuracy of the cross sections is about 10%.