ON THE THEORY OF FERMION MASSES

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It is shown that the four-fermion interaction to any order will not yield masses for particles that have no bare mass.

SEVERAL works have of late developed the point of view that the masses of particles result from their interactions with (or transformations into) other or similar particles (or fields).[1-3] The only example, however, of a definite prediction of a fermion mass is the two-component neutrino theory with an identically vanishing mass for any interaction.[4-6]

One may use the analogy with the neutrino theory to establish important restrictions on the kind of interaction which may lead to a finite particle mass. It turns out that a finite mass cannot be obtained from a theory with several different types of fermions, all with vanishing bare mass, if one uses the electromagnetic interaction and a four-fermion interaction of any kind (that is, S, V, T, A, or P, and no derivative interaction).

If some of the particles of a several-fermion theory have a finite bare mass (or a mass obtained from some other interaction), then those fermions whose bare mass vanishes can be given a finite mass by a four-fermion interaction involving those with mass. This is possible, however, only by violating the Gell-Mann—Feynman postulate,[7] which will lead to a polarization in decay differing from v/c.

Let us prove this assertion. To do this we first give a somewhat different formulation of the usual Dirac equation for a free particle possessing mass.

As is well known, a four-component wave function can be written in terms of two two-component spinors

$$\psi = \begin{vmatrix} \varphi \\ \dot{\chi} \end{vmatrix},$$

each of which (that is φ and $\dot{\chi}$) separately transforms by an irreducible representation of the proper Lorentz group. For vanishing mass the Dirac equation decomposes into two independent equations for φ and $\dot{\chi}$, or in other words describes separately "right" and "left" particles with mass zero and velocity equal to that of light, each of which has a fixed spin component equal to either $+\frac{1}{2}$ or $-\frac{1}{2}$ along the direction of motion.

In this manner of speaking one may regard the mass as a coupling constant between the "right" and "left" particles, since in the Lagrangian it appears as a multiplicative factor in the expression

$$m(\varphi^{*}\dot{\chi} + \dot{\chi}^{*}\varphi).$$

Similarly, the mass enters on the right side (giving rise to inhomogeneity) of the equations for φ and $\dot{\chi}$, namely

$$L_1\varphi = m\dot{\chi}, \qquad L_2\dot{\chi} = m\varphi,$$

where

$$L_{1,2} = i\partial/\partial t \pm \sum_{k=1,2,3} \sigma_k \partial/\partial x_k.$$

Thus, whereas in the case of vanishing mass we are dealing with independent "right" and "left" particles, a finite mass is a quantity which determines the probability for spontaneous transitions of a "right" into a "left" particle, and vice versa. This situation is reminiscent of the Pais-Piccioni scheme, in which the K^0 and $\overline{K}^0$ can be freely transformed into each other, so that the vacuum contains particles which are the linear combinations (superpositions) K_1^0 and K_2^0. In the same way particles with finite mass may be considered superpositions of right and left particles.*

In order for an interaction to lead to a particle mass, this interaction must give rise to mutual transformations of right and left particles. In other words, it must be possible with this interaction to construct Feynman diagrams with a single incoming line corresponding, for instance, to φ, and a single outgoing line corresponding to $\dot{\chi}$. Let us see whether it is possible to obtain such a result through the electromagnetic or four-fermion interaction.

*It should be emphasized that we are dealing with right and left particles which are not each other's antiparticles. On the contrary, we are assuming that there exist right and left particles all of whose charges (i.e., all superselection-rule quantum numbers) are the same, and such that their mutual transformation is not forbidden.

We shall introduce the electromagnetic interaction by using the principle of minimum interaction, which means that we shall assume that the free Lagrangian contains only terms of the form $\varphi^* \partial\varphi/\partial t$ and $\varphi^*\sigma_k \partial\varphi/\partial t$, and similar bilinear terms involving $\dot{\chi}^*$ and $\dot{\chi}$. Then by performing the replacements

$$\frac{\partial}{\partial t} \to ieA_4 + \frac{\partial}{\partial t}, \qquad \frac{\partial}{\partial x_k} \to ieA_k + \frac{\partial}{\partial x_k},$$

we obtain only vertices in which the incoming and outgoing lines are of the same kind ($\varphi \to \varphi$, $\dot{\chi} \to \dot{\chi}$); thus the electromagnetic interaction gives no contribution to the transformation $\varphi \to \dot{\chi}$.

Turning now to the four-fermion interaction, we remark that from φ^* and φ (as from $\dot{\chi}^*$ and $\dot{\chi}$) we can construct only a four-vector, whereas from the different spinors φ^* and $\dot{\chi}$ (or $\dot{\chi}^*$ and φ) we can construct a scalar and a tensor.* Therefore a scalar can be constructed of four two-component quantities in only five ways: three scalars are the products of pairs of four-vectors formed of four φ or four $\dot{\chi}$ or of two φ and two $\dot{\chi}$; one scalar is the product of two scalars formed of two φ and two $\dot{\chi}$; and finally one scalar is the product of two tensors (formed of two φ and two $\dot{\chi}$).

The four two-component spinors refer to different particles, but we shall not write out in detail the corresponding expressions. What is important is that in any of the versions the number of particles of each kind, that is right (undotted function) or left (dotted function), can change only by an even number, namely 0 or 2. This is obviously true to any order, no matter how many vertices we deal with. It is therefore impossible to connect any number of vertices so as to obtain a diagram in which the number of right particles changes by -1, while the number of left particles changes by $+1$. This proves the assertion. We note that this remains true when one introduces scalar form-factors depending on the (invariant) square of the momentum transfer.

We have dealt here with four-fermion expressions of the form $\psi^*\psi^*\psi\psi$, where ψ^* and ψ may refer to different kinds of right and left particles. This form corresponds to a vertex with two incoming particles and two outgoing ones; we shall call such a vertex a 2 : 2 vertex.

In addition to such vertices we must consider also those corresponding to expressions of the form $\psi^*\psi\psi\psi$ with three incoming and one outgoing particles (3 : 1 vertices), and $\psi^*\psi^*\psi^*\psi$ (1 : 3 vertices).

Since complex conjugation is equivalent to transforming from left to right particles, in 3 : 1 and 1 : 3 vertices the number of left and right particles changes by an odd number, in contradistinction from the rule we obtained for 2 : 2 vertices.

Consider a diagram with one incoming particle on the left and one outgoing one on the right, and any number of interior vertices. Any 1 : 3 vertex increases the total number of particles by two, while a 3 : 1 vertex decreases it by 2, and a 2 : 2 vertex leaves it invariant. Thus a diagram with one incoming and one outgoing line may contain any number of 2 : 2 vertices, but it is necessary that $n_{1:3} = n_{3:1}$. This means that the total number of 1 : 3 and 3 : 1 vertices, namely $n_{1:3} + n_{3:1}$, is even. Although each of these vertices changes the total number of right and left particles by an odd number, the diagram as a whole again changes the number of right and left particles by an even number, and the assertion remains valid.

In distinguishing between 1 : 3, 2 : 2, and 3 : 1 diagrams, we assume that we have decided beforehand as to what constitutes an "antiparticle" as opposed to a particle, and only then is a direction of propagation associated with each line of the diagram.

According to a remark due to B. L. Ioffe, a scalar interaction between fermions and mesons will also give no fermion mass, and this result is independent of the fermion mass. Indeed, vertices corresponding to such an interaction involve the transformation of a right fermion into a left one, or vice versa, with the simultaneous creation or annihilation of the meson. In a diagram containing a single incoming and single outgoing fermion line, the number of mesons created is equal to the number of mesons destroyed; thus the total number of meson vertices is even, and it follows thence that the total change of right or left particles is also even.

This theorem breaks down if a particular kind of meson has two different interactions (scalar and vector) with the fermions. We remark that with nonconservation of parity one need not distinguish between scalar and pseudoscalar, or vector and pseudovector.

The theorem also breaks down for three-meson vertices, in which one of the mesons is transformed into two.

If among our fermions there is at least one which has a bare mass m_0, then there exists a diagram involving this fermion in which there is

*We are considering only the proper Lorentz group, so that we need not differentiate between a scalar and a pseudoscalar or between a vector and a pseudovector, etc.

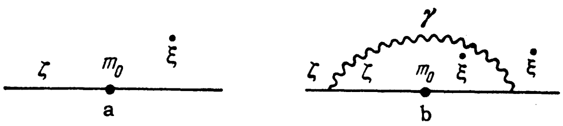

FIG. 1

an incoming right particle (ζ) and an outgoing left one ($\dot{\xi}$), as shown in Fig. 1a. The matrix element corresponding to this diagram is proportional to m_0. By combining this diagram with two electromagnetic diagrams, we obtain a correction to the mass of the particle, and at the same time we see that $\Delta m \sim m_0 e^2$ (see Fig. 1b).

By combining an m_0 diagram for one of the fermions with a four-fermion vertex, one can in principle obtain the transformation of a right particle into a left one for any pair of particles (with zero bare mass), so long as such a transformation is allowed by charge conservation. Then by perturbation theory the particle will end up with a mass $\mu \sim m_0 g^n$, where m_0 is the bare mass of the other particle, g is the four-fermion coupling constant, and n is the number of four-fermion vertices (see Fig. 2). Since g has the dimensions of m^{-2}, the form of this expression predetermines the power to which the cutoff momentum enters into μ.

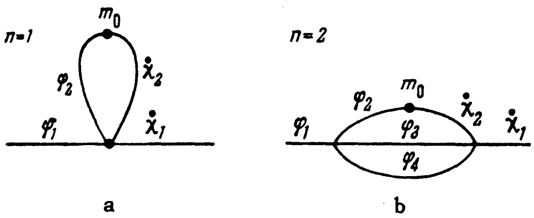

FIG. 2

Consider a given particle with wave function (φ, $\dot{\chi}$). In order that this particle have a finite mass as a result of the four-fermion interaction and the existence of some other particle with bare mass m_0, the original particle must enter into the four-fermion interaction in two ways, namely through φ and through $\dot{\chi}$. This represents a violation of the Gell-Mann—Feynman principle, which states that all particles enter into weak interactions through just a single two-component function. This violation will lead, obviously, to weak-decay production of such particles with longitudinal polarization different from v/c.

It has been remarked by L. B. Okun' that the assertion of the present article can also be proven in the more common language of four-component bispinors if one considers simultaneous γ_5-transformation of the bispinors of all particles in the Lagrangian. Any Lorentz invariant four-fermion interaction is automatically invariant under this transformation, even if any one of the fermions taken separately is not.

I take this opportunity to express my gratitude to L. D. Landau for his repeated insistence that I use two-component quantities, as well as to B. L. Ioffe, L. B. Okun', and I. Ya. Pomeranchuk for discussions.

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97