SOVIET PHYSICS JETP VOLUME 13, NUMBER 2 AUGUST, 1961

CALCULATION OF SOME EXTENSIVE AIR SHOWER CHARACTERISTICS WITH ALLOWANCE FOR FLUCTUATIONS

L. G. DEDENKO

Nuclear Physics Institute, Moscow State University

Submitted to JETP editor September 3, 1960

J. Exptl. Theoret. Phys. (U.S.S.R.) **40**, 630-636 (February, 1961)

The size spectrum of EAS at 640 g/cm^2 altitude (the Pamirs), produced by 10^{13}, 10^{14}, and 10^{15} ev protons, is calculated with allowance for fluctuations in the number and altitude of the primary-proton collisions. The energy spectrum of protons producing showers of a given size at the observation level is determined. The size spectrum of showers produced by primary protons, α particles, and oxygen nuclei is calculated.

1. INTRODUCTION

THE probability of the production of showers with a given number of particles (shower size) at mountain altitudes (640 g/cm^2) by primary protons of various energies was calculated by the Monte-Carlo method by means of the "Strela" electronic computer. The calculations were carried out under the following assumptions:[1]

1. A proton with an arbitrary energy colliding with an air nucleus conserves a constant fraction α of its energy, and loses an energy fraction $\eta_p = 1 - \alpha$ for the production of π mesons. A proton with an initial energy E_0 will have an energy $E_j = \alpha^j E_0$ after j collisions. If we assume that the interaction mean free path for protons in air $\lambda_0 = 80$ g/cm^2, that the absorption mean free path $\lambda = 120$ g/cm^2, and that the exponent of the primary-proton energy spectrum $\gamma = 1.7$, then, from the relation[2,3] $\lambda_0/\lambda = 1 - \alpha^\gamma$, we find the inelasticity factor to be $\eta_p = 0.47$. For $\pi^\pm$ mesons, $\eta_\pi = 1$ and $\lambda_0 = 80$ g/cm^2.

2. In the collision of a proton or a $\pi^\pm$ meson of energy E with an air nucleus, the effective number of π mesons produced is equal to[4]

$$n_\pi(E) = 1.26\ (E/10^{10})^{0.25} \quad (1)$$

where E is in ev. The π^0 mesons amount to one third of the total number of mesons. The energies of all secondary mesons are assumed to be the same:

$$E_\pi(E) = \eta E/n_\pi(E), \quad (2)$$

where $\eta = 0.47$ for an incident proton and $\eta = 1$ for an incident meson.

3. The number of particles arriving at the observation level at the depth $X_0 = 8$ (nuclear lengths) in a shower produced by the collision of a proton of energy E_j with an air nucleus at a depth X_p in the atmosphere is

$$N_{j+1} = 0.47K\ (E_\pi\ (E_j),\ X_0, X_p)\ E_j E_{ph}^{-1} N\ (E_{ph}, X_0 - X_p) \quad (3)$$

where $K[E_\pi(E_j), X_0, X_p]$ is the coefficient accounting for the energy fraction transferred to π^0 mesons. The equation for calculating the coefficient K is given in reference 1. In deriving this equation, it was assumed that the atmospheric density between X_p and X_0 is equal to the density at X_p. The variation of K with the energy of the mesons produced by the protons is shown in Fig. 1 for different depths X_p of proton interactions with air nuclei in the atmosphere. $N(E_{ph}, X_0 - X_p)$ is the number of charged particles in the pure electron-photon cascade initiated by a photon with energy $E_{ph} = 0.5\ E_\pi(E_j)$. This number was calculated from the approximate equation of Greisen[5]

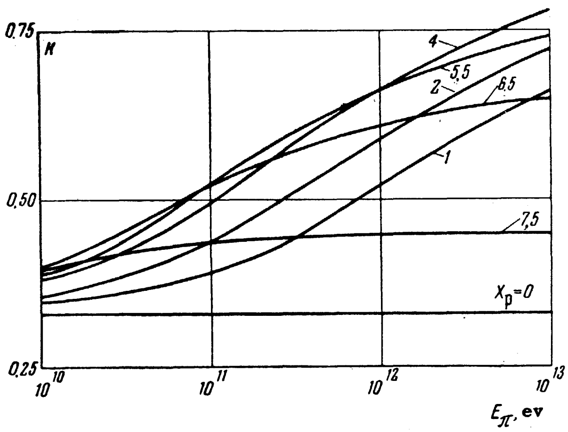

FIG. 1. Energy fraction K transferred to π^0 mesons of various energies. The numbers on the curves denote the depth of interaction X_p of the primary proton in nuclear lengths.

$$N(E_{ph}, X_0 - X_p) \approx 0.31\beta_0^{-0.5}\exp[t(1-1.5\ln s)],$$

where $\beta_0 = \ln(E_{ph}/\epsilon_0)$, $\epsilon_0 = 7.2 \times 10^7$ ev is the critical energy of electrons in air, $t = \rho(X_0 - X_p)$, $\rho = 2.34$ is the ratio of the nuclear unit to the radiation length, and $s = 3t/(t + 2\beta_0)$.

4. In the case where the proton has undergone m collisions before reaching the observation level, the total number of particles in the shower is

$$N = \sum_{j=1}^{m} N_j. \tag{4}$$

2. DISTRIBUTION FUNCTION OF THE PROBABILITY OF A GIVEN SHOWER SIZE AT THE OBSERVATION LEVEL, PRODUCED BY A PRIMARY PROTON OF A GIVEN ENERGY

In order to obtain the probability distribution function of shower sizes, a given number m of collisions of a proton with air nuclei distributed according to the Poisson law with $\overline{m} = 8$ were considered. For each m, the values of the depths of collisions between the protons in the atmosphere were played out using pseudorandom numbers.[6] The probability density of the logarithm of the number of particles $z = \ln N$ at the observation level as a function of the primary-proton energy E_0 is given by

$$\Psi(z \mid y) = \sum_{m=1}^{\infty} P_m \Psi^{(m)}(z \mid y), \tag{5}$$

where P_m is the probability of m collisions, $\Psi^m(z \mid y)$ is the probability density z for m collisions, and $y = \ln(E_0/10^{10})$ (E_0 being in ev).

The probability density $\Psi(z \mid y)$ was found for primary protons with energies 10^{13}, 10^{14}, and 10^{15} ev. The number of showers played for each number m of collisions was 125. Series (5) was terminated at the term for m = 15. The total number of played showers thus amounted to 1875 for each primary-proton energy. The values of the probability density $\Psi(z \mid y)$ obtained with the electronic computer are represented in Fig. 2 by the points. The solid curves represent the approximated functions.

For the approximated functions, we used the fourth-power polynomial

$$\ln \Psi(z \mid y) = \sum_{i=1}^{4} a_i (z - y)^i. \tag{6}$$

The polynomial was determined assuming a quasi-uniform dependence of the probability density $\chi(N \mid E_0)$ on the number of particles in the shower N and on the proton energy E_0. The coefficients a_i are linear functions of y. This made it possible to find the probability density $\Psi(z \mid y)$ for any proton energy in the range 10^{13}—10^{15} ev. The average values of the number of particles in showers produced by primary protons with energies 10^{13}, 10^{14}, and 10^{15} ev are equal to 2.73×10^3, 4.25×10^4, and 5.66×10^5 respectively. The double half-widths of the curves are such that protons with energies 10^{13}, 10^{14}, and 10^{15} ev produce, with a sufficiently high probability, showers whose sizes differ by factors of 5, 3.5, and 2.6 respectively from the above-given values, i.e., approximately 1/5 of the same factors at sea level.[1] With increasing primary-proton energy, the role of the fluctuations decreases. Just as at sea level, this can be explained by the increasing length of the electron-photon showers.

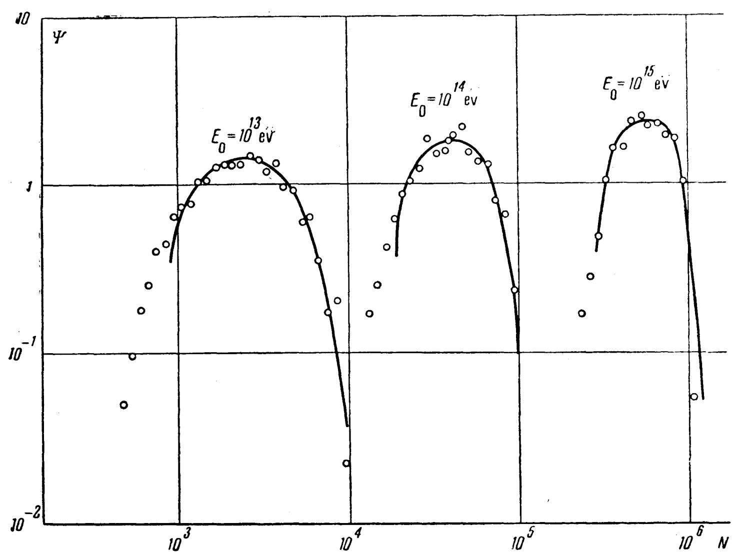

FIG. 2. Distribution function of the probability of a given shower size produced by primary protons of different energies.

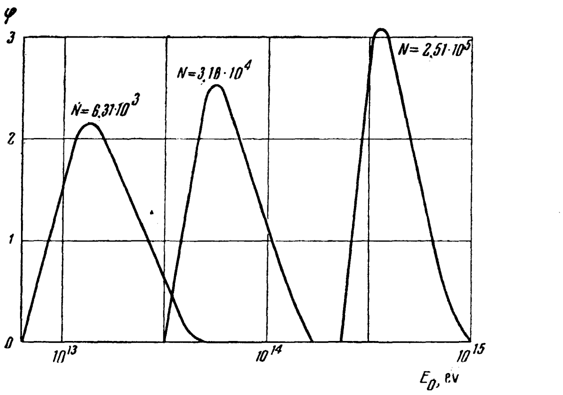

FIG. 3. Energy spectra of protons producing showers of a given size N at the observation level.

3. PROBABILITY DISTRIBUTION FUNCTION OF THE ENERGIES OF PRIMARY PROTONS PRODUCING SHOWERS OF A GIVEN SIZE AT THE OBSERVATION LEVEL

Let $\Psi(z\,|\,y)\,dz$ be the probability of a value z for a given y, and $Be^{-\gamma y}dy$ the energy spectrum of primary protons where B = const and γ = const. Then, from Bayes' theorem,[7] the probability of y for a given z is

$$\varphi(y\,|\,z)\,dy = C^{-1}\Psi(z\,|\,y)\,Be^{-\gamma y}dy, \qquad (7)$$

where the normalized constant C determines the total number of showers with a given value of z

$$C = \int_{y_{\min}}^{\infty} Be^{-\gamma y}\Psi(z\,|\,y)\,dy. \qquad (8)$$

Figure 3 shows the functions $\varphi(y\,|\,z)$, i.e., the energy spectra of protons which, at the observation level, produce showers with a given number of particles N equal to 6.31×10^3, 3.16×10^4, and 2.51×10^5 respectively. The average values of the energy of protons producing showers with such a number of particles are equal to 1.67×10^{13}, 6.69×10^{13}, and 4.21×10^{14} ev respectively, i.e., the energy is less by a factor of 3.2, as compared to sea level.[1] Neglecting the fluctuations, the energies of protons producing showers of a given size at the Pamirs altitude and at sea level differ by a factor of 4.7, i.e., the factor is approximately 1.5 greater than in the former case. The factor $k = \overline{E}_0/N$ relating the number of particles with the average proton energy equals 2.65×10^9, 2.11×10^9, and 1.68×10^9 ev respectively. The double half-widths of the curves are such that showers with 6.31×10^3, 3.16×10^4, and 2.51×10^5 particles are produced with a sufficiently high probability by primary protons with energies differing by factors of 2.95, 2.43, and 2.05 respectively from the above-given values, i.e., approximately ½ to ⅓ of the same factors at sea level.[1]

N	Sea level			Pamirs		
	$6.3 \cdot 10^3$	$3.2 \cdot 10^4$	$2.5 \cdot 10^5$	$6.3 \cdot 10^3$	$3.2 \cdot 10^4$	$2.5 \cdot 10^5$
He^4	2.3	2.2	2	1.5	1.4	1.3
O^{16}	3.2	3	2.6	2	1.8	1.6

Under the assumptions made, it is possible to calculate the average energy of the primary nuclei with atomic weight A producing showers with a given number of particles N. The table shows the ratios of the average energies of He and O nuclei to the energy of protons which produce the same shower size N at sea level and at (640 g/cm^2) altitude.

4. SIZE SPECTRUM AND ALTITUDE DEPENDENCE OF THE SHOWERS

The number of showers with a given number of particles as determined by Eq. (8) can, as is well known, be approximately represented by a power law $C(z)\,dz \sim e^{-\kappa z}dz$, where κ = const. The number of showers with a size greater than a given one is given by

$$C(>z) = C(z)/\kappa. \qquad (9)$$

The value of κ in Eq. (9) is found from

$$\kappa = -d\ln C(z)/dz. \qquad (10)$$

On the other hand, knowing the mean shower size $\overline{N}$ produced by a primary proton of a given energy, we can determine the number of showers of a size that is, on the average, greater than a given one:

$$C(>z) = B\gamma^{-1}e^{-\gamma y}, \qquad (11)$$

where $z = \ln\overline{N} = -1.28 + 1.43y - 0.015\,y^2$.

The size spectrum of the showers can also be calculated[8] for the case where the primary is a nucleus with an atomic weight A, if we accept the hypothesis that the nucleus in the collision with an air nucleus always disintegrates into A independent nucleons with energies E_0/A, where E_0 is the energy of the primary nucleus. Knowing the mean value of the number of particles $\overline{N}$ and the dispersion D of the distribution $\chi(N\,|\,E_0A^{-1})$ for the case where the primary particle is a proton, and using the central-limit theorem of probability theory,[7] we find that the number N_A of particles at the observation level produced by a primary particle with atomic weight A and energy E_0 is distributed according to the normal law with a mean value $\overline{N}_A = A\overline{N}$ and with a dispersion $D_A = AD$. Within the limits of the assumptions made, the size spectrum of showers for a primary nucleus is similarly calculated for the case where

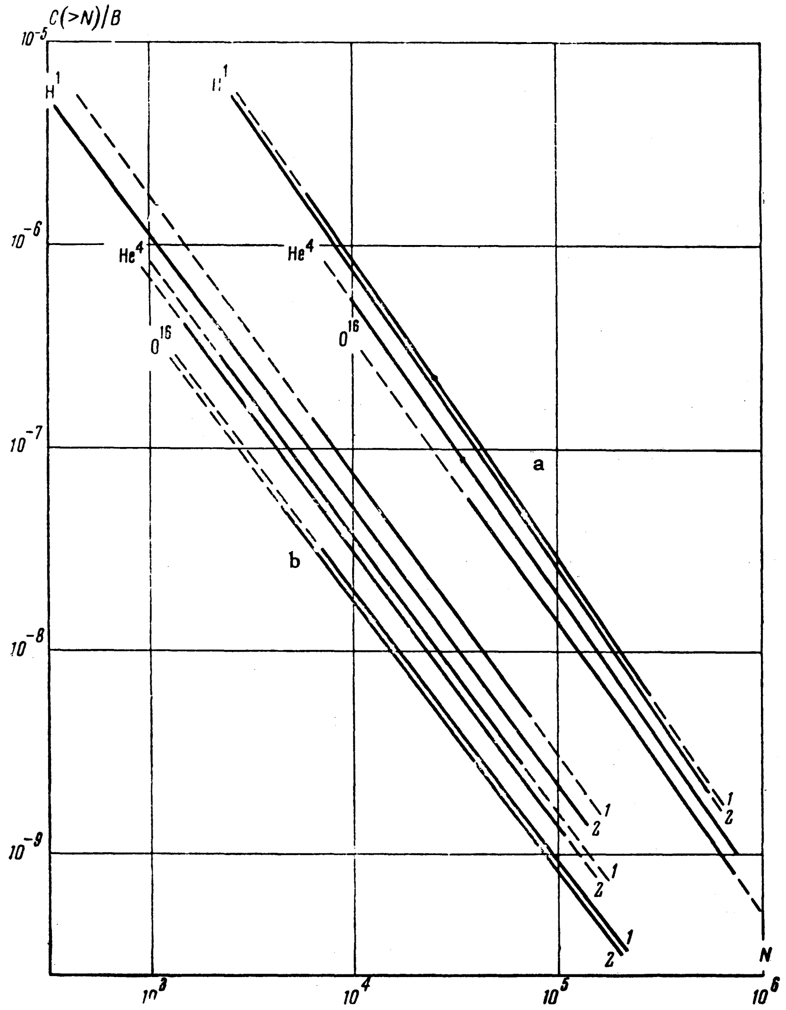

FIG. 4. Size spectra of showers (a – at the Pamirs altitude, b – at sea level) for various primary particles; 1 – taking fluctuations into account, 2 – neglecting fluctuations.

the primary particle is a proton. If the primary particle is a He nucleus, then $A = 4$, and the use of the central-limit theorem is not permissible. However, we are not interested in the exact shape of the function $\chi(N \mid E_0)$ but only in its integral given by Eq. (8). Therefore, small inaccuracies in the shape of the function $\chi(N \mid E_0)$ are irrelevant.

Figure 4 shows, both for sea level[1] and for the Pamirs altitude, the size spectrum of showers taking the fluctuations into account, and the average number of showers produced by primary protons, and He^4 and O^{16} nuclei. For the Pamirs altitude, the curves for primary He and O nuclei are identical whether fluctuations are considered or neglected. For primary protons at the Pamirs altitude, the curves differ by 3—5%. This difference is of the same order of magnitude as errors in the calculations. The value of the exponent κ increases from 1.45 to 1.50 with a change in the number of shower particles from 6.3×10^3 to 2.5×10^5. At sea level, the fluctuations[1] increase the number of showers of a given size by 50% for primary protons, by 16% for He nuclei, and by 5% for O nuclei. The number of showers at sea level[1] with $N > 10^4$ particles differs from that at the Pamirs altitude by factors of 11.5, 13.5, and 17 respectively for primary protons, He nuclei, and O nuclei. The altitude dependence of showers decreases by 10 to 15% with a variation of the number of particles in the showers from 10^4 to 10^5.

5. THE PRIMARY-PROTON SPECTRUM

If we compare the calculated number of showers produced by primary protons at the Pamirs altitude to the number of experimentally-observed showers,[9] we can determine the value of the constant B in Eq. (8). Knowing the value of B and γ, we can calculate the intensity of the primary protons. The primary-proton intensity calculated in such a way differs by only 5% from the proton intensity calculated by the same method from the data for sea level,[1] while it is less by a factor of approximately two than the intensity given by Greisen.[5] If, instead of the data of the Moscow group,[9] we use the experimental data for the size

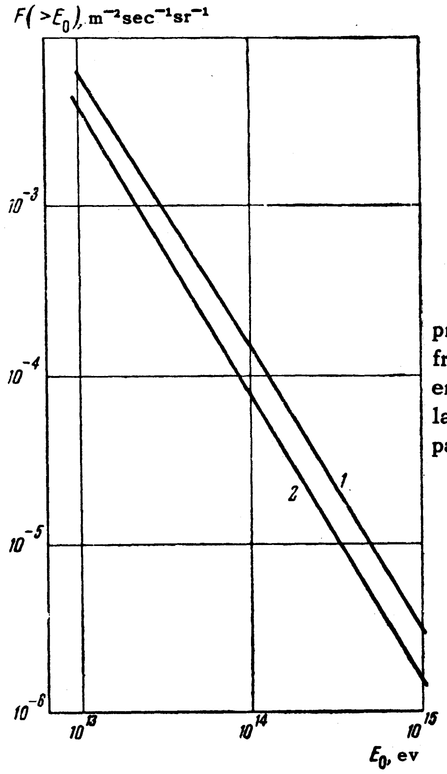

FIG. 5. Spectrum of primary protons; 1 – from the data of reference 5, 2 – as calculated in the present paper.

spectrum given in the review article of Greisen,[5] then the calculated intensity of the primary protons will be smaller yet. Figure 5 shows the intensities of primary protons as reported by Greisen and as calculated in the present paper.

6. DEPENDENCE OF SHOWER CHARACTERISTICS ON THE PROTON-ENERGY SPECTRUM EXPONENT γ

Calculations similar to those described above but for $\gamma = 1.8$ have also been carried out. For this case, in order to conserve the former value of the parameter $\alpha = 0.53$, we obtain a value of $\lambda = 117$ g/cm^2 from the relation $\lambda_0/\lambda = 1 - \alpha^{\gamma}$,[2,3] keeping the former value of $\lambda_0 = 80$ g/cm^2. The increase in γ leads to the increase in κ. The average value of κ increases from 1.47 to 1.55 at the Pamirs altitude and from 1.38 to 1.46 at sea level. With increasing γ, the role of fluctuations in the development of air showers also increases. For $\gamma = 1.8$, the difference between the number of showers calculated taking fluctuations into account and the average number of showers calculated for primary protons increases by 10% at sea level and by 6% at the Pamirs altitude, as compared to the difference for the value $\gamma = 1.7$. The altitude dependence of showers for $\gamma = 1.8$ is greater by approximately 10% than for $\gamma = 1.7$.

In conclusion, the author expresses his thanks to Prof. G. T. Zatsepin for constant help in the work. The author would also like to thank Prof. E. S. Kuznetsov, who kindly made the use of the electronic computer possible, and O. B. Moskalev for his help in programming the computer.

[1] L. G. Dedenko and G. T. Zatsepin, Proceedings of the Moscow International Conference on Cosmic Rays, 2, 222 (1960).

[2] G. T. Zatsepin, JETP **19**, 1104 (1949), Soviet Phys. JETP.

[3] G. T. Zatsepin, Proceedings of the Moscow International Conference on Cosmic Rays, 2, 212 (1960).

[4] G. T. Zatsepin and L. I. Sarycheva, Doklady Akad. Nauk. SSSR **99**, 951 (1954).

[5] K. Greisen, Progress in Cosmic Ray Physics (North-Holland Publishing Co., Amsterdam, 1956) vol. III.

[6] I. M. Sobol', Теория вероятностей и ее применения (Probability Theory and Its Application) 3, 205 (1958).

[7] B. V. Gnedenko, Курс теории вероятностей (A Course in Probability Theory), Gostekhizdat, 1950.

[8] B. Peters, Proceedings of the Moscow International Conference on Cosmic Rays, 3, (1960).

[9] Kulikov, Nesterova, Nikol'skii, Solov'eva, Khristiansen, and Chudakov, Proceedings of the Moscow International Conference on Cosmic Rays, 2, (1960).

Translated by H. Kasha
96