ATTENUATION OF MAGNETOHYDRODYNAMIC AND MAGNETOACOUSTIC WAVES IN A PLASMA WITH ANISOTROPIC CONDUCTIVITY AND VISCOSITY

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Attenuation of weak magnetohydrodynamic and magnetoacoustic waves in an anisotropic plasma located in an external magnetic field is considered for the case where the wave frequency is smaller than the particle collision frequency and the wave length is larger than the mean free path.

A plasma in a magnetic field has anisotropic properties. The electric conductivity, the dielectric constant, the viscosity, etc., of the plasma are represented as tensors of the second rank. We shall consider the attenuation of magnetohydrodynamic and magnetoacoustic waves in an anisotropic plasma. We assume that the wave frequency is smaller than the collision frequency of the particles ($\omega \ll \nu_{\text{eff}}$) and that the mean free path is smaller than the wave length ($l \ll \lambda$).

The conductivity tensor is hermitian. It can be written in the matrix form (the index 3 corresponds to the direction of the vector **H**)[1]

$$(\sigma_{ik}) = \begin{pmatrix} \sigma_1 & i\sigma_2 & 0 \\ -i\sigma_2 & \sigma_1 & 0 \\ 0 & 0 & \sigma_3 \end{pmatrix}. \tag{1}$$

According to the elementary theory*

$$\sigma_1 = \sum_i \frac{\omega_0^2 \nu_{i\text{eff}}}{4\pi} \frac{1}{\omega_{Hi}^2 + \nu_{i\text{eff}}^2},$$

$$\sigma_2 = \sum_i \frac{\omega_{0i}^2 \nu_{i\text{eff}}}{2\pi} \frac{\omega\omega_{Hi}}{(\omega_{Hi}^2 + \nu_{i\text{eff}}^2)^2}, \qquad \sigma_3 = \sum_i \frac{\omega_{0i}^2}{4\pi\nu_{i\text{eff}}}, \tag{2}$$

where

$$\omega_{0i}^2 = \frac{4\pi e_i^2 N_i}{m_i}, \qquad \omega_{Hi} = \frac{|e_i| H}{m_i c},$$

e_i is the charge, N_i is the concentration, m_i is the mass, and $\nu_{i\text{ eff}}$ is the effective number of collisions of the particles. The index i = 1 refers to the electrons and i = 2, 3, . . . , to the various ions. We assume, for simplicity, that the plasma consists of ions of a single type and electrons.

The sums in (2) depend on the density and temperature of the plasma and on the strength of the magnetic field. For high temperatures, low density, and strong magnetic fields, the electric conduction by the ions is more important than the electric conduction by the electrons, so that the latter can be neglected. In the opposite case, the electric conduction by the ions can be neglected in comparison with the electric conduction by the electrons. Comparing the various terms in (2), we see that the electric conduction by the ions must be included if $\omega_{H_1} \gtrsim \nu_{2\text{ eff}}$. It follows that the electric conduction by the ions in a two-component plasma becomes important when*

$$H \gtrsim 10^{-7}\, nT^{-3/2}, \tag{3}$$

where $n = N_2$ is the number of ions in 1 cm^3, T is the temperature in °K, and H is measured in oersteds. If $\omega_{H_1} \gg \nu_{2\text{ eff}}$, we can neglect the electric conduction by the electrons.

The equations of motion of the plasma also contain the viscosity tensor **Π**. This tensor is symmetric. Its components are given in the monograph of Chapman and Cowling[3] and also (with more exact values of the coefficients) in the paper of Braginskii.[4]

We must start with the system of equations of motion of the plasma in the electromagnetic field containing both of the aforementioned tensors. According to the preceding discussion, the basic system of equations is

*Gurevich[2] has given expressions for the tensor σ_{ik} (with only the electrons taken into account) derived from kinetic theory. He showed that, for $\omega \ll \nu_{\text{eff}}$, the expressions obtained from the kinetic theory differ relatively little from the corresponding expressions derived with the help of the elementary theory. Owing to the symmetry of the equations (2) with respect to the characteristics of the ions and electrons, one can assume that this remains true also in the case when only the electric conduction of the ions (with $\omega \ll \nu_{\text{eff}}$) is taken into account. We are therefore justified in using formula (2).

*In deriving this relation we have assumed that ν_{eff} is determined by the collisions of the electrons with the ions.

$$\frac{\partial \mathbf{H}}{\partial t} = \mathrm{rot}\,[\mathbf{vH}] - \frac{c^2}{4\pi}\mathrm{rot}\,\mathbf{G},$$

$$\rho\left[\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\nabla)\,\mathbf{v}\right] = \frac{-1}{4\pi}[\mathbf{H}\,\mathrm{rot}\,\mathbf{H}] - \mathrm{div}\,\mathbf{\Pi},$$

$$\mathrm{div}\,\mathbf{H} = 0, \quad \frac{\partial\rho}{\partial t} + \mathrm{div}\,(\rho\mathbf{v}) = 0, \quad \delta p = u_0^2\delta\rho, \quad (4)^*$$

where

$$G_i = \alpha_{ik}\,[\mathrm{rot}\,\mathbf{H}]_k, \quad \alpha_{11} = \alpha_{22} = \sigma_1/(\sigma_1^2 - \sigma_2^2) = \alpha_1,$$

$$\alpha_{33} = 1/\sigma_3 = \alpha_3, \quad \alpha_{12} = -\alpha_{21} = i\sigma_2/(\sigma_1^2 - \sigma_2^2) = -\alpha_2,$$

$\alpha_{13} = \alpha_{31} \sim \alpha_{23} \sim \alpha_{32} = 0$, and $u_0^2 = (\partial p/\partial\rho)_S$ is the square of the ordinary velocity of sound.

In order to determine the attenuation of waves, we consider the wave solutions of the linearized system (4), as is usually done in isotropic magnetohydrodynamics.[5] The dissipative terms are retained only up to first order. This corresponds to comparatively weak attenuation ($\gamma \ll \omega$).

As a result we find that the anisotropy has no effect on the velocity of propagation of the waves, i.e., we find the same expressions for the velocities of the Alfvén and magnetoacoustic waves as in the isotropic case.

The attenuation coefficient for Alfvén waves is in first approximation†

$$\gamma = \left\{\frac{c^2}{8\pi}(\alpha_1\cos^2\beta + \alpha_3\sin^2\beta) + \frac{\mu}{1{,}92\,\rho}[b'(2\omega_{H_2})\sin^2\beta + b'(\omega_{H_2})\cos^2\beta]\right\}\frac{\omega^2}{u^2}, \quad (5)$$

where μ is the viscosity coefficient in the absence of a magnetic field; β is the angle between **H** and the wave vector **k**, and u is the phase velocity of the wave. The expressions for the coefficients $b'(\omega_{H_2})$ and $b'(2\omega_{H_2})$ are given by Braginskii.[4]

The first term in (5) is related to the electric conductivity and the second term to the viscosity. Comparing these two terms, we find that the second term can be neglected for $T^4n^{-1} < 10^4$. In the opposite case, the second term can have a larger value only for comparatively weak fields. In the limit of very strong fields (considering the fact that the electric conduction by the ions plays the essential role) we can therefore write for γ

$$\lim_{H\to\infty}\gamma = \frac{\omega^2}{2\nu_{2\,\mathrm{eff}}} = C. \quad (6)$$

It is seen from (5) and (6) that the attenuation of the Alfvén waves becomes weaker as the strength of the magnetic field increases, and approaches a limiting value which depends only on the wave frequency and on the effective collision frequency of the particles, but not on H. In this limiting case, the attentuation is independent of β; for finite H the dependence of γ on β can be easily obtained from (5).

For the magnetoacoustic waves we find from (4)

$$\gamma = \Big(\frac{c^2}{8\pi}(u^2 - u_0^2) + \frac{\mu}{1{,}92\rho}\Big\{[b'(\omega_{H2})\sin^2\beta + 1.28\cos^2\beta] \times [u^2 - u_0^2\sin^2\beta - u_H^2] + [0.32\sin^2\beta + b'(2\omega_{H2})\sin^2\beta + b'(\omega_{H2})\cos^2\beta][u^2 - u_0^2\cos^2\beta] + 2[b'(\omega_{H_2}) - 0.64]\,u_0^2\sin^2\beta\cos^2\beta\Big\}\Big)\frac{\omega^2}{u^2}(2u^2 - u_0^2 - u_H^2)^{-1}, \quad (7)$$

where

$$u_H = H/\sqrt{4\pi\rho}.$$

Here we must include the effect of the viscosity both for $T^4n^{-1} < 10^4$ for accelerated waves in weak fields ($H < \sqrt{4\pi\rho}\,u_0$) and for decelerated waves in strong fields ($H > \sqrt{4\pi\rho}\,u_0$). This is due to the fact that near $\beta = 0$, $u \sim u_0$ in both these cases, so that the first term in (7) reduces to zero. This interval near $\beta = 0$ becomes wider for decelerated waves as the strength of the magnetic field increases (in the limit $H \to \infty$ it approaches the interval $-45° < \beta < 45°$). Thus the viscosity is an important effect for decelerated waves, at least for $H > \sqrt{4\pi\rho}\,u_0$. In the limit of strong fields we have

$$\lim_{H\to\infty}\gamma = C\,\mathrm{tg}^2\beta + C_1, \quad C_1 = \frac{2}{3}\frac{\mu}{\rho}\frac{\omega^2}{u_0^2}. \quad (8)^*$$

An accelerated magnetoacoustic wave becomes similar to an Alfvén wave as the strength of the magnetic field increases, and γ approaches the same limit (6). If $T^4n^{-1} \gtrsim 10^4$, the second term of formula (7) here becomes important even for $H \gtrsim \sqrt{4\pi\rho}\,u_0$ (notwithstanding the fact that the second term decreases as the magnetic field is increased).

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[1] Al'pert, Ginzburg, and Feĭnberg, Распространение радиоволн (Propagation of Radio Waves), Gostekhizdat, p. 326 (1953).

[2] A. V. Gurevich, JETP **30**, 1112 (1956), Soviet Phys. JETP **3**, 895 (1956).

[3] S. Chapman and T. G. Cowling, The Mathematical Theory of Non-Uniform Gases, Cambridge (1939).

[4] S. I. Braginskii, JETP **33**, 459 (1957), Soviet Phys. JETP **6**, 358 (1958).

[5] L. D. Landau and E. M. Lifshitz, Электродинамика сплошных сред (Electrodynamics of Continuous Media), Fizmatgiz (1959).

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*rot = curl, $[\mathbf{vH}] = \mathbf{v} \times \mathbf{H}$, tg = tan.

†In deriving expressions (5) and (7), we have assumed that the ions, and not the electrons, play the essential role in the viscosity tensor.