SUPERFLUIDITY OF NUCLEAR MATTER

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Using Bogolyubov's method, we treat the possibility of super-fluidity of nuclear matter and of He^3 for the case of realistic interactions between fermions in S states. We show that in the normal state, near the Fermi surface, repulsion rather than attraction predominates for a pair of nucleons with opposite momenta and spins. However, because of renormalization of the repulsive term in the compensation equation, nuclear matter may become superfluid. The reason for the absence of superfluidity for He^3 is given.

1. INTRODUCTION

In recent years there have appeared a whole series of papers in which Bogolyubov's method[1] is applied to the study of the problem of superfluidity of the nucleus and of nuclear matter.[1-3] The essential feature of this method is the use of ordinary perturbation theory. However, as was shown by Brenig,[4] for an arbitrarily small attractive potential the wave function for a pair of nucleons with energy near the Fermi surface and with opposite momenta has a singularity, so that the use of matrix elements of the potentials computed using plane waves is not possible.

On the other hand, Brueckner's theory,[5,6] which takes account of correlations in the many body system, cannot be simply combined with Bogolyubov's method by replacing the potential V by the K matrix, since the latter has a singularity for a pair of nucleons with opposite momenta near the Fermi surface. At the same time, the correctness of the final result of Bogolyubov's theory, which gives such a good description of superconductivity in metals, permits one to hope that with some modernization this method can also be applied to the problem of superfluidity of nuclei.

Actually, both for Brenig and Bogolyubov the Pauli principle plays the main role in the formation of the energy gap. But it is included entirely differently in the two methods. While in Brenig's method it appears via the singularity of the K matrix and the pairing of real nucleons with energy close to E_F, in Bogolyubov's method it comes from the compensation of the dangerous diagrams when we use second quantization with Fermi quasi-amplitudes.

Continuing this comparison, one may hope to use Bogolyubov's theory for studying superfluidity of nuclear matter by replacing the interaction potential by the K matrix and calculating it so that the correlations near E_F are not included twice. In particular, the K matrix near the Fermi energy E_F can be calculated disregarding the Pauli principle. In the theory of superconductivity of metals such a replacement of the potential by the K matrix disregarding the Pauli principle gives nothing new, since for the relatively weak interactions in the metal the matrix element of the potential is not drastically different from the K matrix omitting the Pauli principle. But in the nuclear case this replacement enables us to treat some interactions which cannot be handled in the usual Bogolyubov scheme, for example the case of an infinite repulsion between nucleons at small distances.

According to the results of Bogolyubov and coworkers,[1] the equation of compensation of dangerous diagrams has a nontrivial solution, which corresponds to a superfluid state, if: a) the interaction is attractive and weak; b) the interaction is attractive and localized on the Fermi sphere; c) the interaction between nucleons is repulsive and highly singular, so that the derivative of the matrix element of the interaction, averaged over angles, is sufficiently large.

Interactions between nucleons of type c) are not known at present. As will be shown in Sec. 2, case b) apparently is also not realized in the nucleus.

In the work of Solov'ev[2] and Dotsenko[3] on the superfluidity of nuclear matter, it is postulated that case a) is satisfied, and by means of a formalism analogous to that developed in reference 1 they obtain an energy gap between the superfluid and normal states. From this they conclude that a superfluid state is favored for nuclear matter; however no specific features of the nucleus appear at all in these papers.

As we shall show, in the general case of the "normal" state a short range repulsion and attraction, for the case of a pair of particles with arbitrary relative orbital angular momenta, affect the wave function of the pair completely differently, so that their influence does not reduce to an effective attractive potential.

For the special case of nuclear matter, the criterion of superfluidity was rewritten by Bogolyubov in the form:

$$|E(k) - E_F|\psi(\mathbf{r}) + V(\mathbf{r})\psi(\mathbf{r}) = E\psi(\mathbf{r}). \qquad (1)$$

If by a suitable choice of ψ we can obtain $E < 0$ from (1), the superfluid state is more favorable than the normal state, which is the criterion for superfluidity.

Equation (1) was used by Cooper, Mills, and Sessler[7] for investigating the possible superfluidity of nuclear matter, by choosing trial functions containing variational parameters. The authors concluded that there was no superfluidity for their class of trial functions. Naturally, however, they could not conclude that superfluidity of nuclear matter is impossible. Besides, as we shall see in Sec. 3, the trial functions used in their work are far from the actual solution of Eq. (1).

2. THE POSSIBILITY OF REPLACING NUCLEAR POTENTIALS BY AN EFFECTIVE ATTRACTION

The fundamental difficulty in the analysis of the interaction of two nucleons is the simultaneous inclusion of the strong repulsion at small distances (which we shall call the "hard core") and the short range attraction.

In most present day work on nuclear theory, it is assumed that the forces between nucleons in the nucleus are almost the same as the forces between isolated nucleons, concerning which we can get information from scattering experiments. This question has been discussed repeatedly in the papers of Weisskopf, Bethe, Brueckner et al., who have succeeded, by starting from this assumption, in explaining a whole host of properties of nuclei.[5,6] There is therefore no reason for supposing that the inetraction of nucleons near the Fermi surface is different from the interaction between isolated nucleons and consequently is no different from that of the other nucleons in the nucleus. An argument in favor of this is the agreement between the parameters of the real part of the optical potential for nucleons having an energy greater than E_F with the parameters of the self-consistent potential for nucleons with energy below E_F. We shall therefore use the Gammel-Thaler[8] potentials for the interaction.

In the theory of superfluidity of nuclear matter, n-n and p-p interactions are of particular interest. In fact, because of the large difference in E_F for neutrons and protons, a neutron and proton with equal and opposite spins and momenta cannot simultaneously lie near the same Fermi surface. But in the interaction of a pair of identical particles, only the central singlet forces are important for the production of superfluidity. In fact, a pair of identical particles ($T = 1$) with even l can occur only in the singlet state, where the tensor forces are equal to zero. In triplet states with $l = 1, 3, 5, \ldots$, where tensor forces are important, the superfluid state is not formed because the interaction of the nucleons in this state will be repulsive.[8]

In many papers on nuclear theory and superfluidity of nuclear matter, it is assumed that the interaction of two nucleons can be approximated by an effective attractive potential. We shall show that in the "normal" state of nuclear matter such a replacement is not possible, i.e. case a) of Sec. 1 does not occur.

In the presence of central forces alone, the wave function for a pair of nucleons with opposite momenta and spins, corresponding to a relative orbital angular momentum l, has the form

$$\psi_l(r) = j_l(kr) + 4\pi \int_0^\infty G_l(r, r') V(r') \psi_l(r') r'^2 dr', \qquad (2)$$

where $G_l(r, r')$ is the Green's function. In defining the Green's function, we must consider the following points, which enable us correctly to select the "normal" state. Far from k_F one could without any contradiction combine the Brueckner procedure with that of Bogolyubov and replace the potential V by the K matrix. Near k_F, a precise inclusion of the Pauli principle leads to a singularity in the K matrix. Therefore in order not to take account of the Pauli principle twice, as pointed out in the introduction, we should in this region replace the potential V by the K matrix and disregard the Pauli principle. Thus for the study of the normal state we must construct Green's function which, in the region far from k_F would coincide with the exact Green's function, which was found in the paper of Brueckner and Gammel,[9] and near k_F would not give a singular K matrix and in particular would not take account of the Pauli principle.

Starting from these considerations, we choose $G_l(r, r')$ in the form

$$G_l(r, r') = (\mu^* k / 4\pi\hbar^2)\, n_l(kr_>)\, j_l(kr_<) \exp\{-\alpha(r_> - a)\}, \quad (3)$$

where μ^* is the effective mass, which takes into account the self-consistent field ($\mu^* = 0.6$ m); n_l and j_l are the spherical Neumann and Bessel functions of half-integral order; a is the radius of the core; $r_>(r_<)$ denotes the larger (smaller) of r, r′; α is a variational parameter which takes account of the effect of the Pauli principle, and is determined from the condition that the total energy be a minimum. This parameter is approximately equal to the reciprocal of the "healing distance".[10]

The Green's function (3) satisfies the following physical conditions: 1) it describes correctly the behavior of the wave function far from k_F; 2) it gives the correct binding energy of the whole system; 3) for $\alpha \to 0$, the function $G_l(r, r')$ goes over into the ordinary Green's function for a pair in a medium, in the "effective mass" approximation in which one disregards the Pauli principle. In this paper, we choose the value $\alpha = 0.8\, k_F$ in agreement with the work of Weisskopf and coworkers.[10] All the following calculations are done for the two cases: $\alpha = 0$ and $\alpha = 0.8\, k_F$.

Let us consider a potential of the type

$$U(r) = \begin{cases} \infty, & r \leqslant a \\ V(r), & r > a \end{cases}. \quad (4)$$

In this case we write (2) in the form

$$\psi_l(r) = j_l(kr) + 4\pi \int_0^a G_l(r, r')\, U(r')\, \psi_l(r')\, r'^2\, dr' + 4\pi \int_a^\infty G_l(r, r')\, V(r')\, \psi_l(r')\, r'^2\, dr'. \quad (5)$$

Following Brueckner and Gammel,[9] to include the effect of the repulsive core we make the following replacement in (5):

$$U(r)\, \psi_l(r) = \lambda \delta(r - a) \quad \text{for} \quad r \leqslant a. \quad (6)$$

Substituting (6) in (5) and using the boundary condition $\psi_l(a) = 0$, we find for λ,

$$\lambda = \frac{j_l(ka)}{4\pi a^2 G_l(a, a)} - \frac{1}{a^2 G_l(a, a)} \int_a^\infty G_l(a, r')\, V(r')\, \psi_l(r')\, r'^2\, dr'. \quad (7)$$

From (5), using (6) and (7) we have

$$\psi_l(r) = S_l(r) + 4\pi \int_a^\infty F_l(r, r')\, V(r')\, \psi_l(r')\, r'^2\, dr', \quad (8)$$

where

$$S_l(r) = j_l(kr)\,[1 - e^{-\alpha(r-a)} \tan h_l(a) / \tan h_l(r)], \quad (9)$$

in which

$$\tan h_l(r) = j_l(kr) / n_l(kr), \quad (10)$$

$$F_l(r, r') = G_l(r, r') - G_l(a, r')\, G_l(r', a) / G_l(a, a). \quad (11)$$

Since $k_F a < 1$ (for the actual density, $k_F = 1.48 \times 10^{13}$ cm^{-1}; a = 0.4 f), expanding $\tan h_l(a)$ in series in $k_F a$ and stopping with the first terms, we obtain

$$\tan h_0(a) = -k_F a, \quad \tan h_1(a) = -(k_F a)^3/3,$$
$$\tan h_2(a) = -(k_F a)^5 / 45. \quad (12)$$

Starting from the relations we have found, we shall give the condition under which one can, in principle, replace the nuclear interaction of the nucleons by an effective attractive potential. It is obvious that the repulsive core is completely compensated by the attractive potential if the wave function $\psi_l(r)$ can be replaced by the free function $j_l(kr)$. Actually this condition is a sufficient criterion for superfluidity, since we then realize case a) of Sec. 1. We can express the mathematical condition for the criterion formulated above by replacing the quantity $\psi_l(r)$ in (8) by the free wave and multiplying the attractive potential by an as yet undetermined function $K_l(r)$, which we shall find later from the condition that the criterion is satisfied. Using (9), we have

$$j_l(kr) = j_l(kr) - n_l(kr) \tan h_l(a)\, e^{-\alpha(r-a)} + 4\pi K_l(r) \int_a^\infty F_l(r, r')\, V(r')\, j_l(kr') r'^2\, dr'. \quad (13)$$

Then using the properties of the Green's function we get for $K_l(r)$,

$$K_l(r) = n_l(kr) \tan h_l(a)\, e^{-\alpha(r-a)} \left[4\pi \int_a^\infty \left\{ G_l(r, r') - \frac{\mu^* k}{4\pi\hbar^2} n_l(r)\, n_l(r') \tan h_l(a)\, e^{-\alpha(r-a)} e^{-\alpha(r'-a)} \right\} \times V(r')\, j_l(kr')\, r'^2\, dr' \right]^{-1}. \quad (14)$$

We shall evaluate $K_l(r)$, as given by (14), for two limiting cases: when $a \le r \le r_0$ and when $r \to \infty$. The second case is equivalent to the requirement that the phase of the scattered wave be zero. For the case of $\alpha \neq 0$, the function $K_l(r) \equiv 0$ for $r \to \infty$. When we omit the effect of the Pauli principle ($\alpha = 0$), the value $K_l(r) \to K_l^\infty$, where

$$K_l^\infty = \left[\frac{\mu^* k}{\hbar^2} \int_a^\infty \{ j_l^2(kr') - \tan h_l(a)\, n_l(kr') j_l(kr') \} V(r')\, r'^2\, dr' \right]^{-1} \times \tan h_l(a). \quad (15)$$

It is obvious that in the intermediate range the value of $K_l(r)$ lies between K_l^0 and K_l^∞. Therefore we should take the larger of these values for the condition for replacement of $\psi_l(r)$ by $j_l(kr)$.

Let us consider the case of $l = 0$. As we see from formula (12), the effect of the repulsive core is largest in this state. For comparison with the work of Cooper, Mills and Sessler,[7] we choose the interaction potential for the nucleons in the form

$$U(r) = \begin{cases} \infty, & r \leqslant a \\ -V_0 e^{-\mu(r-a)}, & r > a \end{cases}, \qquad (16)$$

where $a = 0.4$, $V_0 = 26$ Mev, $\mu = 0.544 \times 10^{13}\ \text{cm}^{-1}$. Substituting (16) in (14) and averaging over the region $a \le r \le 1/\mu$ for $k = k_F$ and arbitrary α, we get

$$K_0^0(\alpha) \approx 11.2 \frac{(\mu+\alpha)^2 + 4k_F^2}{\mu^2 + 4k_F^2} \frac{1+\mu a}{1+\mu a + 2\alpha a}. \qquad (17)$$

$K_0^0(\alpha)$ is a monotonically decreasing function, with

$$K_0^0(0) = 11.2, \quad K_0^0(0.8\,k_F) \approx 9.$$

From this it is clear that the operation of the Pauli principle has little effect on the relation between the attractive and repulsive forces near k_F. For $\alpha = 0$, from (15) we have $K_0^\infty = 2.1$.

The estimate of K_0 which we have given for the potential (16) shows that its attractive part is 9 — 11 times too weak to compensate the repulsion. Moreover, as we shall see in Sec. 3, near k_F, for the state of a pair of identical particles with $l = 0$, the repulsive energy is 2.28 times as great as the energy of attraction. Such nucleons can be kept inside the nucleus only through their interaction with other nucleons having $k \neq k_F$ and with nucleons in triplet states. But for pairs with $k \le k_F$ and in arbitrary relative states ($T = 0,1$; $S = 0,1$; $l \ge 0$) the introduction of an effective attraction is possible only when condition (14) is satisfied. In particular it seems that this replacement can be made for states near k_F, with $l = 2$, since $K_2^0 = 0.4$.

An attempt to introduce an effective attractive interaction potential between nucleons in the shell model was made by Bauer and Moshinsky.[11] Replacing the effect of the repulsive core by the pseudopotential

$$4\pi a\hbar^2\,\delta(\mathbf{r})/\mu^*, \qquad (18)$$

and also replacing the attractive potential V by $-g\delta(\mathbf{r})$, where

$$-g\int \delta(\mathbf{r})\,d\mathbf{r} = \int V(\mathbf{r})\,d\mathbf{r},$$

these authors obtained a total attractive potential of the form

$$-4\pi\hbar^2 a^*\,\delta(\mathbf{r})/\mu^*, \qquad (19)$$

where $-a^* = a - g\mu^*/4\pi\hbar^2$. All the estimates in their paper are made with this potential. For the two-particle potentials which are known from scattering experiments, the relation $a^* \approx -a$ holds, so that we can apply (19) for the calculation of the interaction energy of nucleons in nuclear matter, using the formalism of Huang and Yang,[12] who calculated the energy of the repulsive core with the pseudopotential (18). Then the interaction energy has the form

$$U = \frac{\hbar^2 k_F^2}{2\mu^*} A \left\{ a^* k_F \frac{2}{\pi} + \frac{12}{35}\pi^2(11 - 2\ln 2)(a^* k_F)^2 + \ldots \right\}.$$

The total energy of the system is $E = T + U$, where T is the kinetic energy, and has no minimum for any value of k_F, i.e. a system with such an interaction is unstable.

3. SUPERFLUIDITY OF A SYSTEM OF STRONGLY INTERACTING FERMIONS

Let us apply the apparatus of the Bogolyubov theory to derive the criterion for superfluidity of nuclear matter for the case of realistic nucleon-nucleon interactions. We shall consider only n-n and p-p interactions between pairs with opposite spins and momenta, which are the most important for the appearance of superfluidity.[3] We write the compensation equation, which is equivalent to Eq. (1), for the radial non-symmetric case in the linear approximation:

$$C(k, \Omega_k) = -\frac{1}{2}\frac{1}{(2\pi)^3}\int d\Omega_{k'} \int dk'\, k'^2 J(k, k') \times C(k', \Omega_{k'})/\sqrt{\Delta^2 + \xi^2(k')}, \qquad (20)$$

where the matrix element of the interaction, $J(k, k')$ is defined by the formula

$$J(k, k') = (\psi_k(\mathbf{r}),\ V(\mathbf{r})\,\psi_{k'}(\mathbf{r})). \qquad (21)$$

Here $\psi_{\mathbf{k}}(\mathbf{r})$ is the wave function of the relative motion of the pair:

$$\psi_k(\mathbf{r}) = (e^{i\mathbf{kr}} \pm e^{-i\mathbf{kr}})/\sqrt{2}, \qquad (22)$$

and the sign + (−) holds for even (odd) states; Δ is the width of the energy gap;

$$\xi(k) = k^2\hbar^2/2\mu^* - k_F^2\hbar^2/2\mu^*.$$

We know that the superfluid state appears when there is a nontrivial (i.e., one with $C(k, \Omega) \neq 0$) solution of (20). In the general case, the energy of a single-particle excitation in the superfluid state is given by the formula[1]

$$E_e^s(k) = \sqrt{(E_e^n(k))^2 + C^2(k, \Omega)}.$$

The elementary excitations are separated from the ground state by a gap $\Delta = \min C(k_F, \Omega)$. We expand $C(k, \Omega)$ in spherical harmonics:

$$C(k,\Omega_k)=\sum_{l=0}^{\infty}\sum_{m=-l}^{l}C_{lm}(k)Y_{lm}(\theta_k,\varphi_k). \tag{23}$$

From (20) we get

$$C_{lm}(k)Y_{lm}(\theta_k,\varphi_k)=-\frac{1}{2(2\pi)^3}\int d\Omega_{k'}\int\frac{dk'k'^2}{\sqrt{\Delta^2+\xi^2(k')}}\times J(k,k')C_{lm}(k')Y_{lm}(\theta_{k'},\varphi_{k'}). \tag{24}$$

From now on the calculations will be done only for even states (including the odd states does not change the final appearance of the formulas).

We represent $e^{i\mathbf{k}\cdot\mathbf{r}}$ in the form

$$e^{i\mathbf{kr}}=4\pi\sum_l\sum_{m=-l}^{l}i^l j_l(kr)Y^*_{lm}(\theta_k,\varphi_k)Y_{lm}(\theta_r,\varphi_r). \tag{25}$$

For the function of a pair in an even state, we then have

$$\psi_k(\mathbf{r})=\frac{4\pi}{\sqrt{2}}\sum_l\sum_{m=-l}^{l}i^l[1+(-1)^l]j_l(kr)Y^*_{lm}(\theta_k,\varphi_k)Y_{lm}(\theta_r,\varphi_r). \tag{26}$$

In this case the matrix element of the interaction (22) takes the form

$$J(k,k')=\sum_{l,m}\frac{(4\pi)^2}{2}[1+(-1)^l]^2\int_0^\infty j_l(kr)j_l(k'r)V(r)r^2drY^*_{lm}(\theta_{k'},\varphi_{k'})\times Y_{lm}(\theta_k,\varphi_k). \tag{27}$$

Substituting (27) in (24), we get equation (20) in a new form:

$$C_{lm}(k)=-\frac{1}{2}\int\frac{k'^2dk'}{\sqrt{\Delta^2+\xi^2(k')}}K_l(k,k')C_{lm}(k'), \tag{28}$$

where

$$K_l(k,k')=\frac{4}{\pi}\int_0^\infty j_l(kr)j_l(k'r)V(r)r^2dr. \tag{29}$$

Formula (29) enables us to determine $K_l(k, k')$ for any potential, when we replace V by the K matrix calculated disregarding the Pauli principle ($\alpha = 0$). Using the K matrix calculated with the Green's function (3) with $\alpha \neq 0$ leads to no fundamental change in the result.

Since Eq. (28) enables us to find C_{lm} only to within a sign factor, it is also impossible to determine $C(k, \Omega_k)$ in (23) uniquely. We shall therefore choose Δ so that we get Eq. (20) in the radially symmetric case which was treated earlier by Bogolyubov, Tolmachev and Shirkov.[1] To do this we assume that $\Delta = \langle C(k_F, \Omega)\rangle$, where the symbol $\langle\ldots\rangle$ means an average over angles. We then get for the energy gap,

$$\Delta=\frac{1}{4\pi}\int\sum_{l,m}C_{lm}(k_F)Y_{lm}(\theta_k,\varphi_k)d\Omega_k=\frac{1}{\sqrt{4\pi}}C_{00}(k_F). \tag{30}$$

The correction to this value coming from including states with higher l values does not exceed a few percent, because of the smallness of the matrix elements of the interaction in these states.

Let us apply (28) to the solution of the problem of superfluidity of nuclear matter. We consider the case of $l = 0$. First we calculate the matrix element (29) for the potential (16). To get the matrix element for the core potential, we first replace it by a finite barrier of height V_0 and width a, and find the wave functions for a pair, for $r \geq a$ and $r \leq a$:

$$\psi_{\rm I}(r)=Ar^{-1}\sinh\beta r,\qquad r\leqslant a;$$

$$\psi_{\rm II}(r)=B(kr)^{-1}\{\sin kr-D\cos kr\},\qquad r\geqslant a, \tag{31}$$

where $\beta = [\mu^*\hbar^{-2}(V_0 - \hbar^2k_F^2/\mu^*)]^{1/2}$. Matching the solutions at $r = a$ and using the normalization condition, we get for $V_0 \to \infty$:

$$\psi_{\rm I}(r)=(-1)^p\sinh\beta r/\beta\cosh\beta a, \tag{32}$$

$$\psi_{\rm II}(r)=(-1)^p(kr)^{-1}\sin k(r-a), \tag{33}$$

where

$$p=\begin{cases}0, & ka\neq\pi(2n+1)/2\quad ka\neq n'\pi\\ n, & ka=\pi(2n+1)/2.\\ n', & ka=n'\pi\end{cases} \tag{34}$$

The function $\psi_{\rm I}$ has the property (6) which was used by Brueckner and Gammel.[9] In fact,

$$V_0\psi_{\rm I}(r)=\frac{(-1)^p\hbar^2\beta^2}{\mu^*}\frac{\sinh\beta r}{\beta\cosh\beta a}=\begin{cases}0, & r<a\\ (-1)^p\hbar^2\beta/\mu^*, & r=a\end{cases}.$$

Applying (32) to the computation of the matrix element of the core potential gives

$$K_1(k,k')=(-1)^p(4\hbar^2/\mu^*\pi k)\sin ka, \tag{35}$$

where p is determined by the value of k′a. The matrix element (35) for $ka \ll 1$ coincides with the matrix element of the core when calculated with the pseudopotential (18). Since $K_1(k, k')$ from (35) is a discontinuous function having jumps, it cannot be substituted immediately in formula (28). However the analysis of the compensation equation in a form more general than (20) shows that including the points of discontinuity of the matrix element (35) gives a zero contribution to the solution of the equation. We shall therefore in the following consider $K_1(k, k')$ without the factor $(-1)^p$.

The matrix element of the potential (16) under the conditions ka, $k'a \ll \pi/2$ has the form

$$K_2(k,k')=-8V_0\mu/\pi[\mu^2+(k'-k)^2][\mu^2+(k'+k)^2]. \tag{36}$$

From (35) and (36) we see that with increasing k and k′, the matrix element K_2 falls off much faster than K_1. In particular they are equal for

$k = k' = 0.71\, k_F$, while for $k = k' \sim 2k_F$ the matrix element K_1 is ten times as great as K_2. We may therefore assume that the attraction exists only over a certain range $0 < k \leq \omega$, $0 < k' \leq \omega$. An estimate gives $K_1(k_F, k_F) = 34.4 \times 10^{-39}$ Mev cm^3 and $K_2(k_F, k_F) = 15 \times 10^{-39}$ Mev cm^3. Thus near k_F the energy of repulsion of the core in the "normal" state is 2.28 times as great as the energy of attraction.

Let us represent the coefficient $C_{00}(k)$ in Eq. (28) for $l = 0$ in the form

$$C_{00}(k) = \begin{cases} C_1(k), & 0 \leqslant k \leqslant \omega \\ C_2(k), & \omega \leqslant k < \infty \end{cases}. \quad (37)$$

For $C_2(k)$, we find from (28),

$$C_2(k) = -(2\hbar^2 \sin ak/\pi\mu^* k) \int_0^\infty dk' C_{00}(k') k'^2 / \sqrt{\Delta^2 + \xi^2(k')}. \quad (38)$$

Equation (38) has the solution

$$C_2(k) = -(2\hbar^2 \sin ak/\pi\mu^* kN) \int_0^\omega dk' C_1(k') k'^2 / \sqrt{\Delta^2 + \xi^2(k')}, \quad (39)$$

where

$$N = 1 + (2\hbar^2/\pi\mu^*) \int_a^\infty dk' k' \sin ak' / \sqrt{C^2(k_F) + \xi^2(k')}. \quad (40)$$

Substituting (39) in (28), we obtain an equation for $C_1(k)$. Introducing the quantity $C(k) = C_1(k)/4\pi$, we have

$$C(k) = -\frac{1}{2} \int_0^\omega dk' K(k, k')\, C(k') k'^2 / \sqrt{C^2(k_F) + \xi^2(k')}. \quad (41)$$

Thus, by choosing the gap in the form $\Delta = \langle C(k_F, \Omega) \rangle$, we have in fact arrived at a radially symmetric equation in Bogolyubov's form. Here in Eq. (41),

$$K(k, k') = K_2(k_1 k') - (2\hbar^2/N\pi\mu^*)\, k^{-1} \sin ak. \quad (42)$$

From formula (42) we see that the matrix element of the repulsion appears in renormalized form because of the slower decrease in k-space of the matrix element of the core potential as compared to the attractive potential. Since $K(k, k')$ is small near k_F (this can be seen from the value of the parameter ρ) we can obtain an approximate formula by using Bogolyubov's formalism:[1]

$$C(k) = \frac{K(k, k_F)}{K(k_F, k_F)} \tilde{\omega} \mu e^{-1/\rho}, \quad \rho = \rho_2 - \frac{\rho_1}{N}, \quad (43)$$

where

$$\rho_1 = \frac{2\hbar^2 k_F}{\pi\mu^*} \frac{\sin ak_F}{E'(k_F)} = 0.75, \quad \rho_2 = -\frac{K_2(k_F, k_F) k_F^2}{E'(k_F)} = -0.31,$$

$$\mu = 2\sqrt{E_F(\hbar^2\omega^2/2\mu^* - E_F)},$$

$$\ln \tilde{\omega} = -\frac{1}{2} \int_0^\omega \frac{d}{dk'} \left[\frac{K(k_F, k')^2 k'^2}{K(k_F, k_F)^2 k_F^2} \right] \times \ln \left[\frac{\xi(k') + \sqrt{C^2(k_F) + \xi^2(k')}}{C(k_F)} \right] dk'. \quad (44)$$

A nontrivial solution of equation (43) exists for $\rho > 0$. In this case we find for the gap Δ,

$$\Delta = C(k_F) = \mu \tilde{\omega} e^{-1/\rho}. \quad (45)$$

The quantity N is of great importance in the solution of the problem of superfluidity. N depends strongly on ω, and the choice of ω is difficult. However the estimate of the matrix elements given above enables us to assume that ω is near to k_F. Then for $k_F < \omega < 1.5\, k_F$, we get $\rho > 0$, and the superfluid state should occur. In this case, for values of ω sufficiently close to k_F, we can obtain a wide gap, in particular $\Delta \sim 1$ Mev (see reference 13). Here we should remember that matrix elements for the nucleus may differ markedly from the matrix elements for nuclear matter.

Thus we here meet an extremely peculiar situation, where in the "normal" state near the Fermi surface the repulsion predominates, but because of renormalization of the core matrix element in the compensation equation, the criterion for superfluidity is nevertheless satisfied. Consequently nuclear matter is superfluid not because of the anticipated presence of an effective attractive interaction in the "normal" state, but rather because of the specific character of the nuclear interactions; here the expansion parameter has a value $\rho \ll 1$, i.e., the condition for convergence of the perturbation series is satisfied.

Let us now show that superfluidity is impossible for a system of He^3 atoms which interact in S states. For the interaction potential of the atoms we again choose the expression (16), but with different values of the parameters;[7]

$$V_0 = 10°\,\text{K}, \quad \mu = 1.85\, \text{A}^{-1}, \quad a = 2.5\, \text{A}. \quad (46)$$

In this case, because of the very large value of a, the matrix elements of the attraction must be calculated for distorted waves, i.e., by replacing V by the K matrix in which we disregard the Pauli principle. For the distorted wave in $K_2(k, k')$, we use the function (33) with the parameter values in (46). For the parameters $\mu^* = 1.843$ m and $k_F = 0.735\, \text{A}^{-1}$,[14] the matrix element of the attraction for small k and k′ coincides with (36), while it oscillates for large values of k and k′. We may choose π/a as the width of the well. With these values of the parameters we get $\rho_1 = 1.6$, $\rho_2 = 0.7$, and $N < 1$; consequently $\rho < 0$ and the superfluidity criterion is not satisfied. Thus there is no superfluidity of He^3 in this state.

An investigation of the problem of superfluidity of nuclear matter and of He^3 was carried out by Cooper, Mills, and Sessler[7] using the method of trial functions. On the basis of the potential (16), with the parameters (46) which for He^3 give a good approximation to the exact potential of Brueckner and Gammel,[14] they found the required increase in the attractive potential and analyzed the superfluidity criterion in the form (1).

A comparison of the value K = 9 — 11 obtained by us in Sec. 2, with the result of Cooper et al.[7] for nuclear matter (K = 5 — 7), shows that these estimates do not differ markedly from one another. This is a consequence of the fact that the estimates in reference 7 were made using trial functions close to the functions of the normal state. These estimates somewhat overestimate the role of the repulsive core, whose influence in the superfluid state is drastically suppressed as a result of the renormalization. In the case of He^3, the estimate of reference 7 coincides with ours, even though it was actually made with the same omissions as in the case of nuclear matter.*

The similarity of the results obtained here to the results of Brenig[4] confirms the arguments made in the Introduction concerning the applicability of Bogolyubov's method (with the modifications made above) to the investigation of the problem of superfluidity of nuclear matter, and enables us to understand better the mechanism of appearance of the superfluid state.

*Our attention was called recently to a paper in which the same authors, through a choice of modified trial functions, concluded that nuclear matter can be superfluid.[15]

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