A RESONANCE MODEL FOR THE REACTIONS $N + \pi \rightarrow N + \pi + \pi$ *AND* $\gamma + N \rightarrow N + \pi + \pi$

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The reaction $N + \pi \rightarrow N + \pi + \pi$ for 300 to 550 Mev incident mesons and the reaction $\gamma + N \rightarrow N + \pi + \pi$ for 450 to 700 Mev γ quanta are studied phenomenologically. It is shown that a simple model, according to which the production of a meson in meson-nucleon collisions proceeds mainly via the transition $D_{3/2} \rightarrow P_{3/2}\, s_{3/2}$, is in agreement with all available experimental data.

THE reaction $N + \pi \rightarrow N + \pi + \pi$ for incident mesons with energies of 300 to 450 Mev has been considered earlier[1] under the assumption that in the final state one of the mesons and the nucleon are created in the resonance state $(^3/_2, {}^3/_2)$ with an energy which is in most cases of the order of the resonance energy. The interaction of the second meson, which requires little energy, can in this case be neglected in comparison with the resonance interaction of the first meson with the nucleon. For incident meson energies above 450 Mev this approach becomes incorrect, since the second meson will require more energy, and the interaction of this meson with the nucleon must also be included. The interaction of the two mesons with the nucleon must be considered also for energies below 450 Mev, if the reaction proceeds in such a way that the first meson and the nucleon do not in most cases interact with an energy of the order of the resonance energy. In this case both mesons have an energy which is somewhat below the resonance energy and interact rather strongly with the nucleon.

In the present paper we consider the reactions $N + \pi \rightarrow N + \pi + \pi$ and $\gamma + N \rightarrow N + \pi + \pi$ for the case when the interaction of both mesons with the nucleon has to be accounted for in the final state. We shall use the approximation of a static nucleon; everywhere in the following we shall consider the mass of the nucleon infinitely large. As in reference 1, we assume that the energy dependence of the matrix element is determined solely by the interaction of the particles in the final state and that the meson-meson interaction is small. We assume further that the additional meson is created in such a way that in the final state one of the mesons and the nucleon are in the resonance state $(^3/_2, {}^3/_2)$, while the other meson has an angular momentum L which is either zero or unity. If the second meson has angular momentum one, it can enter into a resonance interaction with the nucleon. This interaction leads to the appearance of an additional factor in the transition amplitude $q_2^{-2} \sin \delta(q_2)$, where q_2 is the momentum of the second meson, and $\delta(q)$ is the meson-nucleon scattering phase in the state $(^3/_2, {}^3/_2)$.

1. THE REACTION $N + \pi \rightarrow N + \pi + \pi$ FOR MESON ENERGIES OF 300 TO 550 MEV

In this case the following transitions are possible (the notation is the same as in reference 1):

$$D_{3/2} \rightarrow P_{3/2} s_{3/2}, \qquad P_{1/2} \rightarrow P_{3/2} p_{1/2}, \qquad P_{3/2} \rightarrow P_{3/2} p_{3/2}. \tag{1}$$

The matrix element corresponding to these transitions has the form

$$C^{jM}_{\lambda 0 ^1/_2 M} a^t_{jL} q_1^{-2} \sin \delta (q_1)\, q_2^{-2L} \sin^L \delta (q_2) \times \sum_m C^{jM}_{LM-m\, ^3/_2 m} C^{^3/_2 m}_{1 m-\mu\, ^1/_2 \mu} Y_{1 m-\mu} (\mathbf{q}_1) Y_{LM-m} (\mathbf{q}_2); \tag{2}$$

$\mathbf{q}_1$ and $\mathbf{q}_2$ are the momenta of the mesons, t is the isotopic spin of the system, λ is the orbital angular momentum of the incident meson, a^t_{jL} are constants which must be determined by comparison of the calculated expressions for the cross sections with the experimental data. Since L is either 0 or 1, the factor $q_2^{-2} \sin \delta(q_2)$ comes into play only if $L = 1$.

The connection between the imaginary and real parts of a^t_{jL} can be found in the same way as, for example, in reference 2, by using the unitarity of S matrix:

$$\mathrm{Im}\,(a^t_{jL}) = a^t_{jL} \exp(-i\eta^t_{j\lambda}) \sin \eta^t_{j\lambda}, \tag{3a}$$

where $\eta^t_{j\lambda}$ is the meson-nucleon scattering phase. As intermediate states we used here only one-meson states. It follows from (3a) that

$$a^t_{jL} = b^t_{jL} \exp(i\eta^t_{j\lambda}), \tag{3b}$$

where b^t_{jL} is a real quantity.

ε	$E_{\pi\ \mathrm{lab}}$, Mev	$E_{\gamma\ \mathrm{lab}}$, Mev		I_2	I_3	I_4	I_5
2,65	290	450	0.2	0.005	0.03	0.042	0,154
2.9	340	500	0,4	0.018	0.08	0.09	0,30
3.1	380	540	0,9	0.049	0.20	0.24	0,66
3.3	420	575	1.3	0.110	0.35	0,43	0,86
3,5	470	620	1.6	0.190	0.55	0.70	0,90
3.7	510	660	1.7	0.200	0.60	1.00	0,76

With the help of computations analogous to those done by us in reference 1, we obtain from (2) the following expression for the angular distribution of the mesons in the center-of-mass system (c.m.s.)

$$4\pi d\sigma / d\Omega = b_0 + b_1 \cos\theta + b_2 (3\cos^2\theta - 1)/2;$$

$$b_0 = (A_1 + A_2) I_1 + B I_2, \quad b_1 = C I_3, \quad b_2 = D I_1 + E I_2. \qquad (4)$$

The quantities I_1, I_2, and I_3 depend only on the total energy of the system. Their values are given in the table; there we also list the value of the energy of the incident meson or γ quantum in the laboratory system. The constant coefficients A_1, A_2, ..., E depend on the type of reaction and on the meson whose angular distribution we are considering. All these constants for all reactions $N + \pi \rightarrow N + \pi + \pi$ are expressed in terms of six real parameters b^t_{jL}. The explicit form of the coefficients for various reactions is given in the Appendix.

We can also obtain the energy distribution of the mesons,

$$\frac{d\sigma}{d\omega} = \Big(A_1 \frac{\sin^2\delta(\omega)}{(\omega^2-1)^2} + A_2 \frac{\sin^2\delta(\varepsilon-\omega)}{((\varepsilon-\omega)^2-1)^2}$$

$$+ B\frac{\sin^2\delta(\omega)\sin^2\delta(\varepsilon-\omega)}{(\omega^2-1)^2((\varepsilon-\omega)^2-1)^2}\Big) \frac{\sqrt{1+k^2}\sqrt{M^2+k^2}}{k^2(\sqrt{1+k^2}+\sqrt{M^2+k^2})}$$

$$+\omega(\varepsilon-\omega)\sqrt{\omega^2-1}\sqrt{(\varepsilon-\omega)^2-1}, \qquad (5)$$

where $\omega = \sqrt{1+q^2}$ is the energy of the meson under consideration, $\epsilon = \omega_1 + \omega_2$ is the total energy of the two mesons, and k is the momentum of the incident meson in the c.m.s. As in formula (4), the coefficients A_1, A_2, and B depend on the type of reaction and on the meson whose energy distribution we are considering.

The behavior of the cross section (4) as a function of the energy is determined by the coefficients b_0, b_1, and b_2. These coefficients depend on the energy only through the functions I_1, I_2, and I_3. In the region of 300 to 450 Mev the behavior of I_1, I_2, and I_3 is the same as that of the analogous functions introduced in reference 1; therefore, the behavior of the coefficients b_0, b_1, and b_2 in this energy region will also be the same as that of the analogous coefficients in reference 1. Formula (4) will thus also be in good agreement with the available experimental data in the region of 300 to 450 Mev, just as the corresponding formulas of reference 1. For higher energies there are not yet any data available with the help of which the energy dependence of the coefficients b_0, b_1, and b_2 could be tested.

It is seen from the foregoing and from formula (4) that our present formulas for the total cross sections are in complete agreement with the formulas of reference 1, if the main contribution to the meson creation process comes from the transition $D_{3/2} \rightarrow P_{3/2}\, s_{3/2}$. In this case and also for higher energies, the same relations between the total cross sections of the various processes must obtain as in reference 1 [formula (4)]. It follows from these relations that the ratio of the total cross sections of the processes

$$\pi^+ + p \rightarrow p + \pi^0 + \pi^+ \text{ and } \pi^+ + p \rightarrow n + \pi^+ + \pi^+$$

is equal to three. This does not contradict the experimental value $1.5^{+1.5}_{-0.5}$ (reference 3). This can, perhaps be regarded as evidence that the process mainly involves the transition $D_{3/2} \rightarrow P_{3/2}\, s_{3/2}$ also at higher energies.

Below we shall treat in more detail the case when the main contribution to the cross section comes from the transition $D_{3/2} \rightarrow P_{3/2}\, s_{3/2}$. In this case the cross sections for the five reactions $N + \pi \rightarrow N + \pi + \pi$ are expressed in terms of the two unknown real parameters $b^{1/2}_{3/2,0}$ and $b^{3/2}_{3/2,0}$ and the meson-nucleon scattering phases $\eta^{1/2}_{3/2,2}$ and $\eta^{3/2}_{3/2,2}$ in a wide region of energies (300 to 550 Mev). The phases enter in the cross section in the form $\cos(\eta^{1/2}_{3/2,2} - \eta^{3/2}_{3/2,2})$. The phase $\eta^{3/2}_{3/2,2}$ is small; the same cannot be said of the phase $\eta^{1/2}_{3/2,2}$ since there is a resonance in the state $D_{3/2}$ at the energy 650 Mev. However, according to Burrowes et al.,[4] $\cos\eta^{1/2}_{3/2,2}$ is not smaller than 0.85 for the energies under consideration. We shall therefore regard $\cos(\eta^{1/2}_{3/2,2} - \eta^{3/2}_{3/2,2})$ as equal to unity.

The two parameters $b^{1/2}_{3/2,0}$ and $b^{3/2}_{3/2,0}$ can be determined from the data of Willis[3] on the summed cross section of the reactions $\pi^+ + p \rightarrow p + \pi^0 + \pi^+$ and $\pi^+ + p \rightarrow n + \pi^+ + \pi^+$ and from the data of

Perkins et al. on the cross section of the reaction $\pi^- + p \rightarrow n + \pi^- + \pi^+$ (reference 5). Then the cross sections of the possible reactions $N + \pi \rightarrow N + \pi + \pi$ in the energy interval 300 to 550 Mev are given by the following expressions (cross sections in millibarns):

$$\sigma(\pi^+ + p \rightarrow n + \pi^+ + \pi^+) = 0.5 I_1,$$
$$\sigma(p + \pi^- \rightarrow n + \pi^+ + \pi^-) = 2.3 I_1,$$
$$\sigma(\pi^+ + p \rightarrow p + \pi^0 + \pi^+) = 1.6 I_1,$$
$$\sigma(p + \pi^- \rightarrow n + \pi^0 + \pi^0) = 3.2 I_1,$$
$$\sigma(p + \pi^- \rightarrow p + \pi^- + \pi^0) = 2.1 I_1. \quad (6)$$

Of further interest is the ratio of the number of fast mesons over the number of slow mesons for various reactions and various energies. This ratio can be obtained with the help of formula (5). It is equal to unity for the π^+ in the reaction $\pi^+ + p \rightarrow n + \pi^+ + \pi^+$ and for the π^0 in the reaction $\pi^- + p \rightarrow n + \pi^0 + \pi^0$. For the other reactions we have from (5):

$$N(p + \pi^+ \rightarrow p + \pi^0 + \pi^+;\ \pi^0) = (I_4 + 0.35 I_5)/(0.35 I_4 + I_5),$$
$$N(p + \pi^- \rightarrow n + \pi^- + \pi^+;\ \pi^-) = (0.71 I_4 + 1.7 I_5)/(1.7 I_4 + 0.71 I_5),$$
$$N(p + \pi^- \rightarrow p + \pi^- + \pi^0;\ \pi^-) = (1.3 I_4 + 0.8 I_5)/(0.8 I_4 + 1.3 I_5). \quad (7a)$$

If $\epsilon = \omega_1 + \omega_2$, the summed energy of the two mesons in the c.m.s., then $N(\pi^+ + p \rightarrow p + \pi^0 + \pi^+;\ \pi^0)$ is the ratio of the number of neutral mesons with an energy larger than $\epsilon/2$ over the number of neutral mesons with an energy smaller than $\epsilon/2$ in the reaction $\pi^+ + p \rightarrow p + \pi^0 + \pi^+$. For the π^+ mesons this ratio is given by the expression

$$N(\pi^+ + p \rightarrow p + \pi^0 + \pi^+;\ \pi^+) = 1/N(\pi^+ + p \rightarrow p + \pi^0 + \pi^+;\ \pi^0). \quad (7b)$$

I_4 and I_5 are functions of the total energy only. It follows from the data of Batusov, Bogachev, and Sidorov (see reference 5, p. 81) that for π^- and π^+ in the reaction $\pi^- + p \rightarrow n + \pi^+ + \pi^-$ at the energy 290 Mev, these ratios are equal to 1.0 ± 0.5 and 0.45 ± 0.25, respectively. Formulas (7) give the values 1.5 and 0.65 for these ratios, which is in agreement with the experimental data.

2. THE REACTION $\gamma + N \rightarrow N + \pi + \pi$ FOR γ QUANTUM ENERGIES OF 450 TO 700 MEV

Two mesons can be created by photoproduction in the following reactions:

$$\gamma + p \rightarrow p + \pi^+ + \pi^-, \qquad \gamma + n \rightarrow n + \pi^- + \pi^+,$$
$$\gamma + p \rightarrow n + \pi^+ + \pi^0, \qquad \gamma + n \rightarrow p + \pi^- + \pi^0,$$
$$\gamma + p \rightarrow p + \pi^0 + \pi^0, \qquad \gamma + n \rightarrow n + \pi^0 + \pi^0. \quad (8)$$

Since one of the mesons and the nucleon are necessarily in a state with the total isotopic spin $^3/_2$, the transition amplitudes corresponding to these six reactions can be expressed in terms of the three independent amplitudes with total isotopic spin $^1/_2$ and projection $^1/_2$, total isotopic spin $^1/_2$ and projection $-^1/_2$, and total isotopic spin $^3/_2$ (reference 6). With this assumption, each of these reactions can proceed via the following transitions:

$$E1 \rightarrow P_{3/2} s_{3/2}, \quad (9a)$$
$$M1 \rightarrow P_{3/2} p_{1/2}, \qquad M1,\ E2 \rightarrow P_{3/2} p_{3/2}, \qquad E2 \rightarrow P_{3/2} p_{5/2}. \quad (9b)$$

The expression for the differential cross section can be obtained in the usual way; we find

$$4\pi d\sigma / d\Omega = A I_1 + B I_2 + C I_3 \cos\theta + (D I_1 + E I_2)(3\cos^2\theta - 1)/2. \quad (10)$$

According to (8) and (9), the constants A, B, ..., E will in this process be expressed in terms of 12 constants. It is meaningless to express the coefficients A, B, ..., E in terms of such a large number of unknown constants. The constants A, B, ..., E should therefore be considered independent for each of the reactions (8). Formula (10) allows us only to determine the energy dependence of the cross sections.

Sellen, Cocconi, Cocconi, and Hart[7] have measured the cross section of the reaction $\gamma + p \rightarrow p + \pi^+ + \pi^-$ in a wide energy interval. They obtained small coefficients in front of $\cos\theta$ for the angular distributions of the π^- and π^+ mesons in the region of 500 to 700 Mev. This indicates, perhaps, that the process goes mainly via the transition $E1 \rightarrow P_{3/2} s_{3/2}$. Estimates show that the magnitude of the coefficient in front of $\cos\theta$ in the angular distribution of the mesons can be explained by a ratio of 0.2 between the contributions from the transitions (9b) and the total cross section.

If the process goes mainly via the transition $E1 \rightarrow P_{3/2} s_{3/2}$, the total cross section has the form

$$\sigma = A I_1. \quad (11)$$

The coefficients A for the various processes will be expressed in terms of the three constants d_t (t are the isotopic spin variables). The explicit dependence of the coefficients A on d_t is given in the Appendix. The imaginary and real parts of the coefficients d_t are connected by a relation that follows from the unitarity condition.

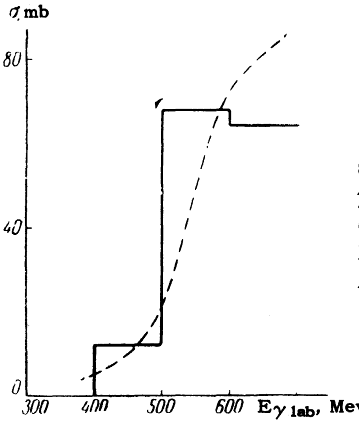
By a method analogous to that leading to (3), we obtain

$$d_t = c_t \exp(i\varphi_t), \qquad \sin\varphi_t = c_t^{-1}\gamma^t_{3/2\,2} b^t_{3/2\,0} \sin\eta^t_{3/2\,2}. \qquad (12)$$

The quantities $b^t_{3/2,0}$ and $\eta^t_{3/2,2}$ were introduced in (3), and c_t is a real constant. We have written the matrix element for the photoproduction of a single meson in the state $D_{3/2}$ in the form

$$\langle \gamma \mid N\pi \rangle = \gamma^t_{3/2\,2} \sin\eta^t_{3/2\,2} \exp(i\eta^t_{3/2\,2}).$$

Thus the six photoproduction cross sections (8) are in this case expressed in terms of three real parameters.



Solid curve: total cross section of the reaction $\gamma + p \rightarrow p + \pi^+ + \pi^-$;[7] dashed curve: cross section calculated by formula (11) with A = 49 mb.

Formula (11) gives satisfactory agreement with the experimental data,[7] as is seen from the figure. The expression for the cross section obtained by formula (11) leads to a value which is smaller than the experimental value by a fourth in the interval 500 to 600 Mev and larger by a fifth in the region 600 to 700 Mev.

In the present paper and in reference 1 we consider a rather simple model according to which the process $N + \pi \rightarrow N + \pi + \pi$ goes mainly through the transition $D_{3/2} \rightarrow P_{3/2}S_{3/2}$. This model predicts correctly the energy dependence of the coefficients in the angular distribution of the created particles.[1] Moreover, in this model all cross sections are expressed in terms of only two unknown parameters $b^{1/2}_{3/2,0}$ and $b^{3/2}_{3/2,0}$. In this model it is therefore possible to express all total cross sections and energy distributions of the created particles solely in terms of known functions of the energy, I_1, I_2, etc., once two arbitrary different experimental quantities are known. The three quantities predicted in this way (the ratio of the cross sections for $p + \pi^+ \rightarrow n + \pi^+ + \pi^+$ and $p + \pi^+ \rightarrow p + \pi^+ + \pi^0$, the ratio of the number of fast π^+ mesons over the number of slow π^+ mesons and the ratio of the number of fast π^- mesons over the number of slow π^- mesons in the reaction $p + \pi^- \rightarrow n + \pi^+ + \pi^-$ for an incident meson energy of 290 Mev) are in agreement with the experimental values.

The energy dependence of the total cross section for the reaction $\gamma + p \rightarrow n + \pi^+ + \pi^-$ is in qualitative agreement with the predictions of the model. However, owing to the paucity of the experimental data on the photoproduction of two π mesons, it is impossible to make a detailed comparison.

APPENDIX

We give the expressions for the coefficients A_1, A_2, ..., E entering in formula (4) for the π mesons denoted in reference 1 by the symbol $\pi(1)$:

$$A_1 = \frac{2}{5}\sum_{tt'} \alpha_t\alpha_{t'} \operatorname{Re}(a^{t*}_{3/2\,0} a^{t'}_{3/2\,0}),$$

$$B = \sum_{tt'}\Big\{\big(\tfrac{1}{3}\alpha_t\alpha_{t'} + \tfrac{2}{9}\alpha_t\beta_{t'} + \tfrac{1}{3}\beta_t\beta_{t'}\big)\operatorname{Re}(a^{t*}_{1/2\,1} a^{t'}_{1/2\,1}) + \big(\tfrac{2}{3}\alpha_t\alpha_{t'} - \tfrac{8}{9}\alpha_t\beta_{t'} + \tfrac{2}{3}\beta_t\beta_{t'}\big)\operatorname{Re}(a^{t*}_{3/2\,1} a^{t'}_{3/2\,1})\Big\},$$

$$C = \sum_{tt'}\Big\{\Big(-\tfrac{2}{3}\sqrt{\tfrac{2}{15}}\alpha_t\beta_{t'} - 2\sqrt{\tfrac{2}{15}}\beta_t\beta_{t'}\Big)\operatorname{Re}(a^{t*}_{1/2\,1} a^{t'}_{3/2\,0}) + \Big(-\frac{8}{15\sqrt{3}}\alpha_t\beta_{t'} + \frac{4}{5\sqrt{3}}\beta_t\beta_{t'}\Big)\operatorname{Re}(a^{t*}_{3/2\,1} a^{t'}_{3/2\,0})\Big\},$$

$$D = \sum_{tt'}\tfrac{2}{5}\alpha_t\alpha_{t'}\operatorname{Re}(a^{t*}_{3/2\,0} a^{t'}_{3/2\,0}),$$

$$E = \sum_{tt'}\Big\{\Big(-\tfrac{4}{3}\sqrt{\tfrac{2}{5}}\alpha_t\alpha_{t'} + \tfrac{4}{9}\sqrt{\tfrac{2}{5}}\alpha_{t'}\beta_t - \tfrac{2}{9}\sqrt{\tfrac{2}{5}}\alpha_t\beta_{t'} - \tfrac{2}{3}\sqrt{\tfrac{2}{5}}\beta_t\beta_{t'}\Big)\operatorname{Re}(a^{t*}_{1/2\,1} a^{t'}_{3/2\,1}) + \Big(-\tfrac{2}{15}\alpha_t\alpha_{t'} + \tfrac{32}{45}\alpha_t\beta_{t'} - \tfrac{8}{15}\beta_t\beta_{t'}\Big)\operatorname{Re}(a^{t*}_{3/2\,1} a^{t'}_{3/2\,1})\Big\}. \qquad (A.1)$$

A_2 is obtained from A_1 by replacing α_t and $\alpha_{t'}$ by β_t and $\beta_{t'}$. The numbers α_t and β_t for various processes are given in reference 1.

We give now the expression for A in formula (11) in terms of d_t:

$$A = \frac{1}{5}\sum_{tt'}(\alpha_t\alpha_{t'} + \beta_t\beta_{t'})\operatorname{Re}(d^*_t d_{t'}). \qquad (A.2)$$

α_t and β_t are numbers which for the various processes are equal to

$$\alpha_t = C^{t\tau}_{1\mu\,3/2\,\lambda+\nu} C^{3/2\,\lambda+\nu}_{1\nu\,1/2\,\lambda}, \qquad \beta_t = C^{t\tau}_{1\nu\,3/2\,\lambda+\mu} C^{3/2\,\lambda+\mu}_{1\mu\,1/2\,\lambda}.$$

t and τ are the total isotopic spin and its projection on the z axis in the final state, μ and ν are the isotopic spins of the mesons, and λ is the isotopic spin of the nucleon in the final state.

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Errata

Volume	No.	Author	page	col.	line	Reads	Should read
10	5	Bogachev et al.	872	1	21	± 0.3 cm	± 0.7 cm
11	6	Gol'danskii et al.	1229	r	Eq. (13)	$\frac{1}{4\pi^2}\ \frac{h}{Mc}$	$\frac{1}{4\pi^2}\ \frac{h}{Mc}$
			1331	r	4	$\ldots + \frac{1}{4} + \frac{\gamma_a}{2}$	$\ldots + \frac{1}{4}\cos\ + \frac{\lambda}{2}$
12	2	Moroz and Fedorov	210	1	Eq. (7)	$\ldots \frac{\sin k_0 x_0}{k_0} e^{i\mathbf{kx}} d^3k,$	$\ldots \frac{ik_0 \delta(k^2)}{\lvert k_0 \rvert} e^{ikx}\, d^4k.$
			212	1	Eq. (39)	$\ldots = 4\pi\hbar c \ldots$	$\ldots = -4\pi\hbar c \ldots$
			212-3	r-1	Eqs. (44) and (39)	$\ldots + \frac{1}{2} iel\, \nabla_k \psi_4(x) \ldots$	$\ldots + \frac{1}{2} iel \nabla_k \psi_4''(x) \ldots$
			213	r	Eq. (51), line 2	$\ldots \frac{iel}{2} \int \nabla_m \psi_4(x) \ldots$	$\ldots \frac{iel}{2} \int \nabla_m \psi_4''(x) \ldots$
			213	r	Eq. (53)	$\ldots e^{-ik_0 \lvert x_0 - x_0' \rvert} e^{i\mathbf{k}(\mathbf{x}-\mathbf{x}')} \frac{d^3k}{k_0}.$	$\ldots e^{ik(x-x')} \frac{d^4k}{2\pi i (k^2 - i\varepsilon)}$
12	3	Nikishov	530	1	Eq. (10)	—	$\mu^{(2)} = \frac{1}{2\beta_2\zeta} \ln\left[\frac{y_1 - 1}{y_1 + 1} \cdot \frac{-y_2 - 1}{-y_2 + 1}\right]$
			533	r	Fig. 4	The dashed curve of Fig. 4 has been incorrectly calculated (corrections to μ^+ scattering on electrons). Its value ranges from −6 to −8.	
12	1	Anisovich	72, 75		Eqs. (4a), (4b), (11)	$\sigma(\pi^+ + p \to n + \pi^+ + \pi^+)$	$2\sigma(\pi^+ + p \to n + \pi^+ + \pi^+)$
	5	,,	948		Eq. (6)	$\sigma(\pi^- + p \to n + \pi^0 + \pi^0)$	$2\sigma(\pi^- + p \to n + \pi^0 + \pi^0).$